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Relation between a Function of the Right Ascension and the Angular Distance to the Vertex for Hyades Stars

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Abstract. In this paper, relation was developed for Hyades stars between a function of the right ascensions and the angular distances from the vertex. The precision criteria of this relation are very satisfactory and a correlation coefficient value of $\simeq 1$ was found which proves that the attributes are completely related linearly. The importance of this relation was illustrated through its usages as:

- a criterion for membership of the cluster,
- a generating function for evaluating some parameters of the cluster,
- a generating function for the initial values of the vertex equatorial coordinates which could then be improved iteratively using the procedure of differential corrections.

Key words. Moving clusters—Hyades cluster—open clusters—Hyades astrometric and kinematic.

1. Introduction

The open star cluster known as the Hyades occupies a unique place in the history and literature of astronomy. It is one of the few star clusters to have been recognized by the ancients, and shares with the Pleiades and the Coma clusters the distinction of being sufficiently close to us for the brighter members to be individually visible to the naked eye. Hyades cluster, with some 300 possible members, a total mass of some 300–400$M_\odot$, and an age of around 600–800Myr, has an extension in the sky of about 20°. Hyades cluster provides a well known example of a moving cluster, that is, a group of stars whose parallel motions in space yield, on the celestial sphere, directions of proper motion that appear to converge to a point called the vertex of the motion (or of the cluster). The determination of the equatorial coordinates of the vertex, that is, its right ascension and declination (A, D) is one of the most important problems in the kinematical and physical studies of moving clusters (Wayman 1965; Hanson 1975; Eggen 1984; Gunn *et al.* 1988; Sharaf *el al.* 2000). In particular, the color-magnitude diagram of the Hyades cluster has been of prime importance in establishing the Zero Age Main Sequence (ZAMS) as well as in calibrating luminosity criteria which permit the determination of the absolute magnitudes of the stars from observable features in their spectra (Perryman *et al.* 1998; de Bruije *et al.* 2001). The kinematic distance of the Hyades derived from a combination of proper motions and spectroscopic radial velocities, has been one of the fundamental starting points for the calibration of the photometric distance scale (Hanson 1975; Gunn *et al.* 1988; Schwan 1991). In fact, the availability of the final results of the Hipparcos astrometry mission, provide a radical improvement in astrometric data on all stars in the Hipparcos observing programme, including approximately 240 candidate Hyades members. The first detailed study of the distance, structure, membership, dynamics and age of the Hyades cluster, using Hipparcos data was by Perrymant *et al.* (1998). Recently (Sharaf 2003) a relation was established between the apparent magnitude and the parallax for Hyades stars using the best rational approximation technique. The precision criteria of such a relation were very satisfactory and some utilizations of the relation were also given.

In the present paper, a relation was developed between a function of the right ascensions and the angular distances from the vertex. As a test for the existence of such a relation for Hyades cluster, we used 133 stars of Schwan's table (Schwan 1991). Although these data are not the most accurate as compared with those of Hipparcos, the precision criteria of the relation are very satisfactory and a correlation coefficient value of $\simeq 1$ was found which proves that the attributes are completely related linearly. The importance of this relation was illustrated through its usages as: a criterion for membership of the cluster, generating function for evaluating some parameters of the cluster, and the initial values of the vertex equatorial coordinates which could then be improved iteratively using the procedure of differential corrections (Sharaf *et al.* 2000).

2. Basic formulations

The material of this section is a summary of the corresponding section of Sharaf *et al.* (2000), and is given through the following subsections:

2.1 *Determination of the vertex of a moving cluster*

If we know the right ascensions α_i, declinations δ_i and the components $(\mu_\alpha^{(i)}, \mu_\delta^{(i)})$ of the total proper motion $\mu^{(i)}; i = 1, \ldots, N$, where N is the number of the cluster members, then the equatorial coordinates (A, D) of a moving cluster could be determined from

$$A = \tan^{-1}(\eta/\xi), \tag{1}$$

$$D = \tan^{-1}[(\eta^2 + \xi^2)^{-1/2}], \tag{2}$$

where

$$\xi = (T_5 T_2 - T_3 T_4)/\Delta, \tag{3}$$

$$\eta = (T_3 T_2 - T_5 T_1)/\Delta, \tag{4}$$

$$T_1 = \sum_{i=1}^{N} a_i^2, \qquad T_2 = \sum_{i=1}^{N} a_i b_i, \qquad T_3 = \sum_{i=1}^{N} a_i c_i,$$

$$T_4 = \sum_{i=1}^{N} b_i^2, \qquad T_5 = \sum_{i=1}^{N} c_i b_i, \tag{5}$$

$$a_i = \mu_\alpha^{(i)} \sin\delta_i \cos\alpha_i \cos\delta_i - \mu_\delta^{(i)} \sin\alpha_i,$$

$$b_i = \mu_\alpha^{(i)} \sin\delta_i \sin\alpha_i \cos\delta_i + \mu_\delta^{(i)} \cos\alpha_i,$$

$$c_i = \mu_\alpha^{(i)} \cos^2\delta_i. \tag{6}$$

2.2 *Differential corrections to the vertex coordinates*

The differential corrections ΔA and ΔD to the vertex coordinates A and D are given as:

$$\Delta A = (G_5 G_2 - G_3 G_4)/E, \tag{7}$$

$$\Delta D = (G_3 G_2 - G_5 G_1)/E, \tag{8}$$

where

$$E = G_2^2 - G_4 G_1, \tag{9}$$

$$G_1 = \sum_{i=1}^{N} \Psi_i^2, \qquad G_2 = \sum_{i=1}^{N} \Psi_i \Phi_i, \qquad G_3 = \sum_{i=1}^{N} \Psi_i \Delta\theta_i,$$

$$G_4 = \sum_{i=1}^{N} \Phi_i^2, \qquad G_5 = \sum_{i=1}^{N} \Phi_i \Delta\theta_i, \tag{10}$$

$$\Psi_i = \sin^2\theta_{\text{cal}}^{(i)}[\cos\delta_i \tan D \cos(A - \alpha_i) - \sin\delta_i]/\sin^2(A - \alpha_i), \tag{11}$$

$$\Phi_i = -\sin^2\theta_{\text{cal}}^{(i)}[\cos\delta_i \sec^2 D]/\sin(A - \alpha_i), \tag{12}$$

$$\Delta\theta_i = \theta_{\text{obs}}^{(i)} - \theta_{\text{cal}}^{(i)}, \tag{13}$$

$$\theta_{\text{obs}}^{(i)} = \tan^{-1}(\mu_\alpha^{(i)} \cos\delta_i/\mu_\delta^{(i)}), \tag{14}$$

$$\theta_{\text{cal}}^{(i)} = \cos^{-1}[(\sin D - \sin\delta_i \cos\lambda_i)/\cos\delta_i \sin\lambda_i], \tag{15}$$

θ is the position angle of the total proper motion and λ_j is the angular distance of the jth star to the vertex and is given by

$$\lambda_j = \cos^{-1}(\sin\delta_j \sin D + \cos\delta_j \cos D \cos(A - \alpha_j)). \tag{16}$$

Now having obtained the corrections ΔA and ΔD, one can determine the corrected values of the coordinates of the vertex, A^* and D^* from

$$A^* = A + \Delta A, \tag{17}$$

$$D^* = D + \Delta D. \tag{18}$$

This process of corrections could be repeated in an iterative manner until the desired accuracy is thus achieved, for instance, $|\Delta A| \leq \in_1$ and $|\Delta D| \leq \in_2$, where $\in_1$ and $\in_2$ are two given tolerances.

2.3 *Velocity components V_α, V_δ and ρ*

Let V_α, V_δ and ρ be the components of the velocity $\mathbf{V}$ (as a basic assumption, all members of a moving cluster have the same $|\mathbf{V}|$) along coordinate system whose center is a star and consisting of three mutually perpendicular unit vectors $(\hat{\alpha}, \hat{\delta}, \hat{\mathbf{r}})$ defined as follows:

- The unit vector $\hat{\alpha}$ tangent to the circle of constant declination and pointing in the direction of increasing right ascension.
- The unit vector $\hat{\delta}$ tangent to the circle of constant right ascension and pointing towards the north celestial pole.
- The unit vector $\hat{\mathbf{r}}$ lying on the radius vector, which joins the sun to the star, so ρ is the radial velocity of the star. These components are given as,

$$V_\alpha = 4.738\mu_\alpha \cos\delta/p, \tag{19}$$

$$V_\delta = 4.738\mu_\delta/p, \tag{20}$$

$$V_t^2 = V_\alpha^2 + V_\delta^2, \tag{21}$$

where p is the parallax. Also we have

$$V_t = V \sin\lambda, \tag{22}$$

$$\rho = V \cos\lambda, \tag{23}$$

V is the velocity of the cluster.

3. Numerical applications

3.1 *Data*

For the present applications we used $133 (= N)$ stars of Schwan's table (Schwan 1991). Using these data and the algorithm of section 2.1 we get for A and D (both in degrees) together with their probable errors the values

$$A = 96.533 \pm 0.563, \tag{24}$$

$$D = 6.5735 \pm 0.226. \tag{25}$$

3.2 *Relation between H and λ*

Using the data of Schwan's table and the formulations of section 2 together with the numerical values of equations (24) and (25) we get very simple and a very significant

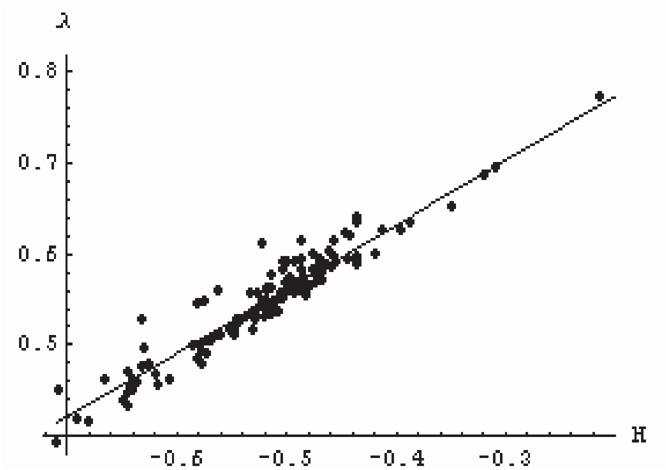

Figure 1. Graphical representation between the raw and the fitted data.

relation between the right ascension α and the angular distance to the vertex λ, this relation is given as;

$$\lambda = c_1 + c_2 H, \tag{26}$$

$$H = \cos\alpha - \sin\alpha, \tag{27}$$

where the coefficients and their probable errors are

- $c_1 = 0.912854 \pm 0.00522729$.
- $c_2 = 0.703744 \pm 0.010081$.
- The probable error of the fit is $e = 0.0117353$.
- The average squared distance between the exact solution and the least squares solution (Kopal and Sharaf 1980) is $Q = 0.00028344$.
- The linear correlation coefficient between (H, λ) is $r = 0.971705$.
- The graphical representation between the raw and the fitted data is given in Fig. 1:

4. Utilizations of the relation

Assuming equation (26) as a given relation for Hyades cluster, it could be utilized in generating some important knowledge about the cluster. They are the following:

4.1 *Right ascension criterion*

Equation (26) may provide an additional membership criterion of Hyades cluster as follows. Assume that the right ascension α_0 of a star, suspected of being a member of the cluster was known. Now from this value of α_0, the spherical distance λ_0 of the star from the vertex could be obtained from equation (26). With the spherical distance λ_c calculated by means of equation (16) and, if $\mid \lambda_0 - \lambda_c \mid$ is reasonably small, we can conclude that possibly the star is a member of the cluster. This is what we may call *right ascension criterion.*

4.2 *Generating function*

Equation (26) may also be used as a generating function for evaluating some important parameters of the cluster. Assuming that the tangential velocities $V_t^{(i)}$ are known together with $\alpha_i, \delta_i, \mu_\alpha^{(i)}$ and $\mu_\delta^{(i)}$ for L (say) stars; $i = 1, 2, \ldots, L$, then from equation (26) we get the corresponding spherical distances λ_i. Having obtained λ's, some parameters could then be evaluated, as in the following.

- *The velocity of the cluster*

  Equation (22) can be considered as an equation of condition for determining the velocity V of the cluster and we derive

$$V = \sum_{i=1}^{L} V_t^{(i)} \sin \lambda_i \Big/ \sum_{i=1}^{L} \sin^2 \lambda_i . \tag{28}$$

- *The radial velocities of the cluster stars*

  From equations (22) and (23) the radial velocities $\rho_i, i = 1, 2, \ldots, L$ follow from

$$\rho_i = V_t^{(i)} \cot \lambda_i . \tag{29}$$

- *The parallaxes of the cluster stars*

  The total proper motions $\mu_i; i = 1, 2, \ldots, L$ are computed from

$$\mu_i = \sqrt{(\mu_\alpha^{(i)} \cos \delta_i)^2 + (\mu_\delta^{(i)})^2},$$

  then the parallaxes $p_i; i = 1, 2, \ldots, L$ are computed from

$$p_i = 4.738 \mu_i / V_t^{(i)} . \tag{30}$$

- *The absolute magnitudes of the cluster stars*

  If the apparent magnitudes $m_i, i = 1, 2, \ldots, L$ of the cluster stars are also known, then their absolute magnitudes can be found from

$$M_i = m_i + 5 + 5 \log p_i , \tag{31}$$

  where p_i are given from equation (30).

- *The center of the cluster*

  The center of the cluster (x_c, y_c, z_c) can be derived by the simple method of finding the equatorial coordinates of the center of mass for a number of discrete objects, so

$$x_c = \left[\sum_{i=1}^{L} \cos \delta_i \cos \alpha_i \Big/ p_i \right] \Big/ L, \tag{32}$$

$$y_i = \left[\sum_{i=1}^{L} \cos \delta_i \sin \alpha_i \Big/ p_i \right] \Big/ L, \tag{33}$$

$$z_i = \left[\sum_{i=1}^{L} \sin \delta_i \Big/ p_i \right] \Big/ L, \tag{34}$$

where p_i are given from equation (30).

- *The distance of the cluster*

  The distance of the cluster is given by

$$d = L \Big/ \left[\sum_{i=1}^{L} p_i \right], \tag{35}$$

where p_i are given from equation (30).

4.3 *Initial values of the vertex coordinates*

Select a few N_0 (say) stars which are adopted as Hyades members. In what follows we shall illustrate the usage of equation (26) in generating initial values of the vertex equatorial coordinates A_0 and D_0. These values could then be improved iteratively using the procedure of differential corrections as mentioned in section 2.

1. Compute $\theta^{(i)}_{\text{obs}}$; $i = 1, 2, \ldots, N_0$ from equation (14).
2. Compute λ_i; $i = 1, 2, \ldots, N_0$ from equation (26).
3. Compute $D^{(i)}_0$ and $A^{(i)}_0$; $i = 1, 2, \ldots, N_0$ from

$$D^{(i)}_0 = \sin^{-1}(\cos \theta^{(i)}_{\text{obs}} \cos \delta_i \sin \lambda_i + \sin \delta_i \cos \lambda_i),$$

$$A^{(i)}_0 = \alpha_i + \cos^{-1}[\{\cos \lambda_i - \sin \delta_i \sin D^{(i)}_0\}/ \cos \delta_i \cos D^{(i)}_0].$$

4. Compute A_0 and D_0 from

$$A_0 = \sum_{i=1}^{N_0} A^{(i)}_0 \Big/ N_0,$$

$$D_0 = \sum_{i=1}^{N_0} D^{(i)}_0 \Big/ N_0.$$

As a simple example let us consider the first five stars of Schwan's table, then from the above computational sequence we get

$$A_0 = 96^\circ.3741,$$

$$D_0 = 6^\circ.75983,$$

which are very reasonable initial values for the differential corrections procedure.

In concluding the present paper, a relation was developed for Hyades stars between a function of the right ascensions and the angular distances from the vertex. Precision criteria of this relation which are the uncertainties of its coefficients, probable error of the fit, and the Q value are all very satisfactory, also a correlation coefficient of value $\simeq 1$ was found, which proves that the attributes are completely related linearly. The importance of this relation was illustrated through its usages as:

- A criterion for membership of the cluster which we may call *right ascension criterion*.
- A generating function for evaluating some parameters of the cluster for example, the velocity and the center of the cluster, also the parallaxes, radial velocities and the absolute magnitudes of the cluster stars.
- Finally, the relation could be utilized to generate initial values of the vertex equatorial coordinates which could then be improved iteratively using the procedure of differential correction.

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