J. Astrophys. Astr. (2004) **25,** 203–211

An Apparent Descriptive Method for Judging the Synchronization of Rotation of Binary Stars

Li Lin-sen
Department of Physics, Northeast Normal University, Changchun 130 024, China.

Received 2003 August 22; accepted 2004 November 6

Abstract. The problem of the synchronous rotation of binary stars is judged by using a synchronous parameter Q introduced in an apparent descriptive method. The synchronous parameter Q is defined as the ratio of the rotational period to the orbital period. The author suggests several apparent phenomenal descriptive methods for judging the synchronization of rotation of binary stars. The first method is applicable when the orbital inclination is well-known. The synchronous parameter is defined by using the orbital inclination i and the observable rotational velocity $(V_{1,2} \sin i)_M$. The method is mainly suitable for eclipsing binary stars. Several others are suggested for the cases when the orbital inclination i is unknown. The synchronous parameters are defined by using $a_{1,2} \sin i$, $m_{1,2} \sin^3 i$, the mass function $f(m)$ and semi-amplitudes of the velocity curve, $K_{1,2}$ given in catalogue of parameters of spectroscopic binary systems and $(V_{1,2} \sin i)_M$. These methods are suitable for spectroscopic binary stars including those that show eclipses and visual binary stars concurrently. The synchronous parameters for fifty-five components in thirty binary systems are calculated by using several methods. The numerical results are listed in Tables 1 and 2. The statistical results are listed in Table 3. In addition, several apparent descriptive methods are discussed.

Key words. Binary stars—synchronization of rotation—judgement of apparent phenomenal descriptive methods.

1. Introduction

In binary system the rotational period of components and its orbital period usually show a synchronous phenomenon. This is the secular evolutional result arising from tidal friction in binary systems. Investigation of this synchronous phenomenon is very meaningful for exploring the evolutional process of binary systems. Therefore, recently some authors have studied this subject by using various methods. Some authors explore the mechanism of synchronous rotational phenomenon of binary stars from theory, such as Zahnn (1966, 1975, 1977), Tassoul (1987, 1988). Some authors explore this subject from observation, such as Tan Hui-song *et al.* (1985, 1989, 1995) and Pan Kai-ke (1996, 1997). Some authors explore this subject from an apparent phenomenal descriptive method, such as this author's work (Li 1997, 1998). The author had earlier defined the ratio of rotational velocity of primary stars with orbital velocity of components as the synchronous parameter (Li 1997). He also defined the ratio of theoretical calculated value of the period of advance of apsidal line to its observational value as the synchronous parameter (Li 1998). But the first method studies the synchronous phenomenon in the evolutional process. The later method needs a great number of the observational data of apsidal line motion of binary stars. So far, however, we have a few observational data of the velocity or period of advance of apsidal line of binary stars. So this method brings certain difficulty for judgement. Hence the author further explores how one can use a great deal of the observational data such as $a_{1,2} \sin i$, $m_{1,2} \sin^3 i$, $K_{1,2}$ and $f(m)$ in tables of binary stars to judge synchronization of rotation of binary stars by using apparent phenomenal descriptive methods. These methods are not only suitable to spectroscopic binary stars, but also eclipsing and visual binary stars. This paper will divide the case of inclination i of binary star into well known and unknown and suggest five apparent phenomenal descriptive methods to study this problem.

2. Definition of the synchronous parameters

The character of apparent phenomenal descriptive methods needs the definition of synchronous parameter for judging the synchronization of binary stars to start with. This paper defines a synchronous parameter Q as the ratio of the rotational period P_{rot}, of binaries (primary or secondary) to the orbital period of component

$$Q = P_{\mathrm{rot}}/P. \tag{1}$$

It reflects the multiple relation between the rotational period of primary or secondary stars with the orbital period of component apparently.

While $Q = 1$, $P_{\mathrm{rot}} = P$, is called as complete synchronism, $Q \to 1$, $P_{\mathrm{rot}} \to P$, is called as approachable synchronism, and when Q differs from 1, it is called as non-synchronism.

In general, the orbital period $P >$ the rotational period, P_{rot}, so, $Q < 1$ usually. But as the binary system evolves the orbital period shortens and the rotational period lengthens, so, $P_{\mathrm{rot}} \to P$ or $Q \to 1$. From the evolutional point of view, P_{rot} cannot exceed P, so we cannot have Q greater than 1. However the observational data may give $Q > 1$, *i.e.*, $P_{\mathrm{rot}} > P$, due to errors in the estimated parameters of the binary system.

In discussing the problem of synchronization of binary stars, one author defined $\omega \leq 1.5\Omega$ (ω: primary rotational velocity, Ω: orbital velocity of component) as the range of the synchronous judgement (Plavec 1970; Levato 1976; Giuricin *et al.* 1984, Li 1998), but another author defined $\omega \leq 1.3\Omega$ as the synchronous judgement according to the estimation of the range of error (Pan Kai-ke *et al.* 1996, 1997). Therefore this paper adopted this judgement. However, this is the case when $P > P_{\mathrm{rot}}$ or $Q < 1$. For the case: $P < P_{\mathrm{rot}}$ or $Q > 1$, we take $\Omega \leq 1.3\omega$. Further $Q = 1$ represents complete synchronization. Then we have the following five cases:

Case A:	$0.95 \leq Q \leq 1.05$	– Almost complete synchronization.
Case B:	$0.80 \leq Q < 0.95$	– Approaching synchronization.
	$1.05 < Q \leq 1.20$	– Approaching synchronization.
Case C:	$0.70 \leq Q \leq 0.80$	– Critical synchronization.
	$1.20 < Q \leq 1.30$	– Critical synchronization.

Case D: $Q < 0.70$ – Non-synchronization.

Case E: $Q > 1.3$ – Slow rotators (slow rotation).

Case E is not a product of evolution.

3. Apparent phenomenal descriptive methods for judging the synchronization of rotation of binary stars

(1) When inclination is well-known, we adopt the following method, if binary star is an eclipsing binary.

Let $(V_{1,2} \sin i)_M$ be the observed rotational velocities of two components observed from spectral lines. i is the inclination. P_{rot} is the rotational period, $R_{1,2}$ are the radii of two components, then apparent rotational velocities of two components are written as

$$(V_{1,2} \sin i)_M = \left(\frac{2\pi R_{1,2}}{P_{\text{rot}}} \right) \sin i. \tag{2}$$

Substituting P_{rot} into expression (1), we obtain

$$Q = \frac{2\pi R_{1,2} \sin i}{P \left[V_{1,2} \sin i \right]_M}. \tag{3}$$

We denote $(V_{1,2} \sin i)_M$ in the units of km/s, radii $R_{1,2}$ by R_θ (solar radius), the orbital period by d (day), the expression (3) becomes

$$Q_{e,e'} = 50.6139 \frac{R_{1,2} \sin i}{P \left[V_{1,2} \sin i \right]_M} \tag{4}$$

$Q_{e,e'}$ denotes Q_e and Q'_e, *i.e.*, synchronous parameters of primary and secondary stars.

The formula (4) is suitable to the visual or eclipsing binary stars.

(2) When the inclination is unknown, we adopt the following three methods, if the table of spectroscopic binary gives $a_{1,2} \sin i$, $m_{1,2} \sin^3 i$ or K_1 and K_2 for double-lined binaries.

The first method: The method uses $a_{1,2} \sin i$.

The expression (3) or (4) is written as

$$Q_{1,1'} = \frac{2\pi R_{1,2} (a_{1,2} \sin i)}{P \left[V_{1,2} \sin i \right]_M a_{1,2}}, \tag{5}$$

$Q_{1,1'}$ denotes Q_1 or Q'_1, *i.e.*, synchronous parameters of two components.

According to the motion of center of mass and Kepler's third law, we have,

$$a_{1,2} = \left(\frac{G}{4\pi^2} \right)^{1/3} \frac{M_{2,1}}{(M_1 + M_2)^{2/3}} P^{2/3}. \tag{6}$$

Substituting (6) into (5), let $q_1 = M_2/M_1 = m_2/m_1, q_2 = M_1/M_2 = m_1/m_2$, $q_{1,2}$ denotes q_1 or q_2 and $m_{1,2}$ denotes m_1 or m_2, $M_1 = m_1 M_\odot$, $M_2 = m_2 M_\odot$, we obtain,

$$Q_{1,1'} = 1.7290 \times 10^{-5} \frac{R_{1,2}(1+q_{1,2})^{2/3}(a_{1,2}\sin i)}{P^{5/3} q_{1,2} m_{1,2}^{1/3} (V_{1,2}\sin i)_M}. \quad (7)$$

The second method: The method uses $m_{1,2}\sin^3 i$.

Formula (3) or (4) is written as

$$Q = \frac{2\pi R_{1,2}(M\sin^3 i)^{1/3}}{P(V_{1,2}\sin i)_M M^{1/3}}.$$

The units are the same as in the previous method, then the above expression can be written as:

$$Q_{2,2'} = 50.6139 \frac{R_{1,2}(m_{1,2}\sin^3 i)^{1/3}}{P m_{1,2}^{1/3}(V_{1,2}\sin i)_M}. \quad (8)$$

The third method: The method uses K_1 and K_2.

Some catalogues of the orbital elements of spectroscopic binary systems gives the following formula (Batten *et al.* 1989, Kopal 1959)

$$m_{1,2}\sin^3 i = 1.0385 \times 10^{-7}(1-e^2)^{3/2}(K_1+K_2)^2 K_{1,2} P.$$

Substituting the formula for $m_{1,2}\sin^3 i$ into (8) and putting $K = K_1 + K_2$, we obtain

$$Q_{3,3'} = 0.2377 \frac{R_{1,2}(1+q_{1,2})^{2/3}(1-e^2)^{1/2} K_{1,2}}{P^{2/3} q_{1,2} m_{1,2}^{1/3}(V_{1,2}\sin i)_M}, \quad (9)$$

where $K_{1,2}$ denotes the semi-amplitudes of the velocity curves of primary and secondary components, e is the orbital eccentricity.

The above three methods are mainly suitable for the double-lined spectroscopic binary stars.

The fourth method: The method uses mass function $f(m)$ for single-lined spectroscopic binaries. We use the following method, only if the spectrum of primary component appears, the mass function $f(m)$ is written as

$$f(m) = \frac{m_2^3 \sin^3 i}{(m_1+m_2)^2} = 1.0385 \times 10^{-7}(1-e^2)^{3/2} K_1^3 P.$$

The above formula can be written as

$$m_2 \sin^3 i = f(m)(m_1+m_2)^2/m_2^2.$$

Substituting it into (8), and putting $q_1 = m_2/m_1$, we get

$$Q_4 = 50.6139 \frac{R_1 (1+q_1)^{2/3} [f(m)]^{1/3}}{P q_1 m_1^{1/3} [(V_1 \sin i)]_M}, \tag{10}$$

where Q_4 is the synchronous parameter for single-lined spectroscopic primary stars. Therefore this method is suitable for single-lined spectroscopic binary stars or eclipsing binary stars concurrently.

$a_{1,2} \sin i$, $m_{1,2} \sin^3 i$, $K_{1,2}$ and $f(m)$ in formulas (7)–(10) are taken from the catalogue of spectroscopic binary systems (Batten *et al.* 1989).

4. Calculation for the synchronous parameters of fifty-five components in thirty binary systems

We calculate the synchronous parameters for eclipsing binary stars and double- and single-lined spectroscopic binary stars by using previous methods. In computation, $(V_{1,2} \sin i)_M$ are quoted from the date given by (Tan 1985, 1989, 1995). The synchronous parameters of eclipsing binary stars are calculated by using the formula (4) (the inclination is well-known). The periods, stellar radius and mass are quoted from the catalogue of parameters of eclipsing binary stars (Brancewicz *et al.* 1980). The calculated results are listed in Table 1. The synchronous parameters of double-lined and single-lined spectroscopic binary stars are calculated by using formulas (7)–(10) (the inclination is unknown). The orbital periods P, $a_{1,2} \sin i$, $m_{1,2} \sin^3 i$, $K_{1,2}$ and $f(m)$ are quoted from the catalogue of the orbital elements of spectroscopic binary systems (Batten *et al.* 1989). Because the stellar mass and radius are not given in the catalogue, the calculated binary systems are all spectroscopic binary stars which are also eclipsing binary stars concurrently, so the stellar mass and radius are quoted from the catalogue of parameters of eclipsing binary stars. The calculated results are listed in Table 2, which also indicates the categories A, B, C, D, E, as defined earlier.

5. Statistical results for synchronization of rotation of fifty-five components

Table 3 which shows the statistical results for synchronization of rotation of fifty-five components can be obtained from Tables 1 and 2.

It can be seen from the statistical table 3 that 14% components have achieved almost complete synchronism (A), 33% are approaching synchronism (B), 18% are at critical synchronism (C) and 31% are non-synchronous (D). The remaining two components are slow rotators (E) which need theoretical explanation. Further among the binaries containing category A stars GT Cep and AH Cep can be considered to have complete synchronism for both components.

6. Discussions and summary

- The characteristic of this paper is that it introduces a synchronous parameter Q. It not only judges directly synchronization of two components, but also analyses

Table 1. The calculated results for synchronous parameters of twenty components in ten eclipsing binary systems by using the apparent descriptive method of formula (4).

Name		Sp type	P(d)	i(deg)	$R_{1,2}(R_\odot)$	$(V_{1,2}\sin i)_M$ (km/s)	$Q_{e,e'}$	$P_R \gtrless P$	Synch
GT Cep	1	B2V	4.9087	78	4.64	42	1.11	$P_R > P$	B
	2	A0			6.04	58	1.05	$P_R > P$	A
My Cyg	1	Am	4.0051	89	2.32	26	1.13	$P_R > P$	B
	2	Am			2.06	47	0.55	$P_R < P$	D
V451 Oph	1	B9V	2.1966	87	2.48	48	1.19	$P_R > P$	B
	2	A0			1.98	45	1.01	$P_R > P$	A
U Oph	1	B4.5	1.6773	86.6	3.27	110	0.89	$P_R < P$	B
	2	B5.5			3.02	100	0.91	$P_R < P$	B
V505 Sgr	1	A2v	1.8287	75	2.30	100	0.95	$P_R < P$	A
	2	F81V			2.35	50	1.94	$P_R > P$	E
CV Vel	1	B2V	6.8925	88.2	4.27	50	0.63	$P_R < P$	D
	2	B2V			4.16	85	0.36	$P_R > P$	D
CD Tau	1	F7V	3.4351	88	1.49	34	0.64	$P_R < P$	D
	2	F7V			1.50	34	0.65	$P_R > P$	D
V1143 Cyg	1	F5V	7.6407	87	1.34	9	0.98	$P_R < P$	A
	2	F5V			1.04	20	0.34	$P_R < P$	D
ZZ Boo	1	F2V	4.9917	88	1.76	12	1.48	$P_R > P$	E
	2	F2V			1.70	25	0.69	$P_R < P$	D
AS Cam	1	B8V	3.4309	89	2.06	40	0.7	$P_R < P$	C
	2	A0V			2.47	30	1.21	$P_R > P$	C

the evolutionary progress of binary systems. Because the synchronous parameter Q indicates the evolutionary status of the binary system in that the variation of the value Q tends to unity due to tidal friction. In the end the binary system arrives at the synchronous phenomenon of the orbital and rotational periods.

- There are different suitable methods for several apparent descriptive methods suggested in this paper. The method of formula (4) suits only eclipsing binary stars including visual binary stars for which inclination i is well-known. It is not suitable for spectroscopic binary stars as their inclinations are unknown. Although some spectroscopic binary stars are eclipsing binary stars concurrently also, this method can be used, but some spectroscopic binary stars are not eclipsing binary stars. Therefore, this method cannot be used. However, for the spectroscopic binary star for which inclination i is unknown, we can make use of $a_{1,2}\sin i$, $m_{1,2}\sin i^3 i$, $k_{1,2}$ and $f(m)$ in the table of spectroscopic binary stars by using the formulas (7)–(10). But the mass is unknown for spectroscopic binary stars. However the mass may be well-known, if the spectroscopic binary stars are eclipsing binary stars concurrently or visual binary stars. Because this paper selects two types of spectroscopic binary stars as eclipsing binary stars concurrently, so, its masses are quoted from the data in the table of eclipsing binary stars.

Table 2. The calculated results for synchronous parameters of thirty-five components in twenty single-lined and double-lined spectroscopic binary systems by using four apparent descriptive methods.

Name		Sp type	P(d)	$R_{1,2}$ ($R_\odot$)	$m_{1,2}$ ($m_\odot$)	$q_{1,2}$	$(V_{1,2}\sin i)_M$ (km/s)	$a_{1,2}\ \sin^3 i$ (km)	$Q_{1,1}$	$P_R \lesseqgtr P$	Synch
The first method calculated by using $a_{1,2}\sin i$											
AG Per	1	B5pV	2.0287	3.02	5.08	0.89	105	4.53	0.69	$P_R < P$	D
	2	F6		2.78	4.52	1.1	120	4.97	0.55	$P_R < P$	D
V448 Cyg	1	O9.5	6.5179	10.00	22.25	0.73	95	$1.92\times10^{+1}$	1.08	$P_R > P$	B
	2	BII		17.34	16.24	1.37	92	$1.50\times10^{+1}$	1.10	$P_R > P$	B
GK Cep	1	A2V	0.9362	2.67	2.67	0.93	106	2.21	1.29	$P_R > P$	C
	2	A4		2.45	2.48	1.08	100	2.41	1.27	$P_R > P$	C
EI Cep	1	FI	8.4397	2.99	1.80	0.95	15	9.42	1.25	$P_R > P$	C
	2	FI		2.31	1.71	1.05	10	8.92	1.31	$P_R > P$	D
AH Cep	1	O8	1.7748	6.44	16.52	0.87	175	6.08	1.08	$P_R > P$	B
	2	O9		5.98	14.37	1.15	160	6.91	1.02	$P_R > P$	A
The second method calculated by using $m_{1,2}\sin^3 i$											
Name		Sp type	P(d)	$R_{1,2}$ ($R_\odot$)	$m_{1,2}$ ($m_\odot$)	$q_{1,2}$	$(V_{1,2}\sin i)_M$ (km/s)	$m_{1,2}\ \sin^3 i$ ($m_\odot$)	$Q_{2,2}$	$P_R \lesseqgtr P$	Synch
EK Cep	1	A1V	4.4278	1.59	2.11	0.55	45	2.0	0.63	$P_R < P$	D
	2	G5		1.19	1.16	1.82	14	1.1	1.02	$P_R > P$	B
NY CeP	1	B01V	15.2767	10.56	16.51	0.67	80	$2.7\times10^{+1}$	0.52	$P_R < P$	D
	2	B01 V		8.66	11.06	1.49	71	$1.3\times10^{+1}$	0.43	$P_R < P$	D
RX Her	1	B9.5 V	1.7786	2.37	2.86	0.86	73	2.7	0.93	$P_R < P$	B
	2	A0.5 V		2.05	2.50	1.07	60	2.3	0.94	$P_R < P$	B
Z Vul	1	B4 V	2.4454	4.54	5.42	0.43	98	5.4	0.95	$P_R < P$	A
	2	A3III		4.57	2.33	2.32	115	2.2	0.79	$P_R < P$	C
λ Tau	1	B3V	3.9529	5.64	6.04	0.24	85	6.4	0.87	$P_R < P$	B
	2	A4IV		4.33	1.45	4.16	60	1.7	0.97	$P_R < P$	A

Table 2. (*Continued*)

Name		Sp type	P(d)	$R_{1,2}$ ($R_\odot$)	$m_{1,2}$ ($m_\odot$)	$q_{1,2}$	e	$(V_{1,2}\sin i)_M$ (km/s)	$m_{1,2}\sin^3 i$ ($m_\odot$)	$Q_{3,3}$	$P_R \lesseqgtr P$	Synch
The third method calculated by using $k_{1,2}$												
EG Ser	1	A0	9.9473	2.17	3.05	0.78	0	48	75.8	0.22	$P_R < P$	D
	2	A2		1.93	2.37	1.29	0	68	83.8	0.12	$P_R < P$	D
V624 Her	1	A_{3m}	3.8950	2.93	2.23	0.86	0	36	96.6	1.02	$P_R > P$	A
	2	A_{4m}		2.32	1.90	1.17	0	36	117.2	0.84	$P_R < P$	B
Sig Aq	1	B_3V	1.9503	4.12	6.48	0.86	0	108	16.42	0.90	$P_R < P$	B
	2	B_3V		3.44	5.58	1.16	0	125	208	0.71	$P_R > P$	C
WW Air	1	A7V	2.5250	1.93	1.81	0.96	0	35	115.6	1.09	$P_R > P$	B
	2	A7V		1.92	1.74	1.04	0	35	127.7	1.15	$P_R > P$	B
RZ Cha	1	F5IV	2.8321	2.94	1.90	0.84	0	39	108.2	1.34	$P_R > P$	D
	2	F5IV		3.19	1.59	1.19	0	39	107.6	1.27	$P_R > P$	C
The fourth method calculated by using $f(m)$												
Name		Sp type	P(d)	R_1 ($R_\odot$)	m_1 ($m_\odot$)	q_1		$(V_1\sin i)_M$ (km/s)	$m_1\sin^3 i$ ($m_\odot$)	Q_4	$P_R \lesseqgtr P$	Synch
UW Vir_1		A_4	1.8107	1.52	1.77	0.24		76	1.9×10^{-2}	0.59	$P_R < P$	D
TW Dra_1		A_5V	2.8067	2.06	2.24	0.43		123	5.9×10^{-2}	0.59	$P_R < P$	D
QS Aql_1		B5V	2.4968	2.65	6.82	0.17		75	2.7×10^{-2}	0.74	$P_R < P$	C
DV Aql_1		LateA	1.5755	2.16	2.20	0.59		92	1.4×10^{-1}	0.70	$P_R < P$	C
UX Her_1		A3	1.5489	1.80	1.92	0.30		61	2.1×10^{-2}	0.85	$P_R < P$	B

Table 3. The statistical results for synchronization of rotation of fifty-five components in binary systems.

Type	No. of systems	No. of components	$Q < 1$ $P_R < P$	$Q > 1$ $P_R > P$	A	B	C	D	E
EB	10	20	11	9	4	6	2	6	2
DSB	15	30	16	14	4	11	6	9	0
SSB	5	5	5	0	0	1	2	2	0
Sum (total)	30	55	32	23	8	18	10	17	2

EB: Eclipsing binary stars.
DSB: Double-line spectroscopic binary stars.
SSB: Single-line spectroscopic binary stars.

References

Batten, A. H. *et al.* 1989, *Publ. Dom. Astrophys,* **17**, 1–127.
Brancewicz, H. K., Dworak, T. Z. 1980, *Acta Astronomica*, **30**(4), 501.
Giuricin, G. *et al.* 1984, *A & A*, **131**, 152; **135**, 393.
Kopal, Z. 1959, *Close binary system* (London: Chapman and Hall) p. 470.
Levato, H. 1976, *Ap J*, **203**, 680.
Li, Lin-sen, *Acta Astrophys. Sinica*, 1998, **18**, 77; 1977, **17**, 407.
Pan Kai-ke 1996, *Acta Astrophys. Sinica*, **16**, 291; 370.
Pan Kai–ke 1997, *A & A*, **321**, 202.
Pan Kai–ke *et al.* 1997, *Acta Astrophys. Sinica*, **17**, 386.
Pan Kai-ke *et al.* 1997, Publ Yunnan Obs, No. 2, 87.
Plavec, M. 1970, in *Slettebok Rotation*, (ed.) A. Slettebok (Dordrecht: Reidel) p. 133.
Tan Hui-song *Acta Astron. Sinica*, 1989, **30**, 135; 1985, **26**, 226.
Tan Hui-song *et al.* 1995, *Acta Astrophys. Sinica*, **15**, 57.
Tassoul, J. L. 1988, *Ap J*, **324**, L71; 1987, **322**, 856.
Zahn, J. P. 1966 *Ann. Ap.*, **29**, 489.
Zahn, J. P. 1978 *A & A*, **67**, 162; 1977, **57**, 383; 1975, **41**, 329.