The Mass/Eccentricity Limit in Double Star Astronomy

J. Dommanget Observatoire Royal de Belgique, 3, avenue Circulaire, B-1180 Bruxelles, Belgique.

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Abstract. A research that we conducted in 1963 on the evolution of the binaries based on the available orbital data to obtain a philosophical degree, led to the establishment of an interesting and new diagram between the logarithm of the total mass and a particular parameter $X$, bound to the areal constant. This appeared to have a real physical significance but the basic observational material was insufficiently extended to assure its undeniable existence. In 1981, a new research based on a more extended orbital material, has confirmed this diagram. Presently, another important increase in the orbital material and the availability of highly accurate trigonometric parallaxes produced by the Hipparcos satellite, gave us the opportunity to confirm once more the stability of this diagram. This last research is here described.

Key words. Stellar evolution—statistics—binaries.

1. First research

The discovery by W. Doberck in 1878, of a period/eccentricity correlation amongst the visual double star orbits caught the attention of many astronomers. In 1910, a similar correlation was shown amongst the spectroscopic orbits by W. W. Campbell (1910), H. Ludendorff (1910), Fr. Schlesinger and R. H. Baker (1910), extending the known visual correlation towards the small periods. But the global correlation covering altogether the visual and spectroscopic orbits has been later particularly debated, since the various selection effects existing in the statistical material that appeared were able to explain the observed phenomenon.

Nevertheless, despite the substantial and regular increase of the observational material with time (Fig. 1), the stability of the “plotting” of the correlation appeared evident (J. Dommanget 1963, 1964). It incited us to believe in its reality and to try to explain it from a new and original point of view: the evolution of the double systems under the effect of a secular mass-loss.

We started from the principle that in the two bodies problem, if there exists a secular and isotropic mass-loss, the gravitational force between the two bodies even when decreasing, remains central and the areal constant is conservative. We thus considered this areal constant as the invariable parameter allowing to characterize each binary during its evolution.

To realize this research, we have established a file of the orbits of 212 visual pairs and 98 spectroscopic ones selected for their quality.
Figure 1. Successive period-eccentricity correlations proposed by various authors from 8898 to 1958.
The ignorance of trigonometric or spectroscopic parallaxes for many of these systems – essential for the computation of the semi-axis major and thus of the areal constant in astronomical units – has been the most important stumbling-block.

In order to evade this difficulty, we systematically used the dynamical parallaxes because they are available for each orbit and because they have a uniform accuracy independent of their distance, being uniquely based on the period, the semi-axis major and, through the mass-luminosity relation, on the apparent magnitude and the spectral classification.

We then discovered a particular distribution (Fig. 2) seeming to exist between the logarithm of the total mass: \( \log M_{AB} \) and the expression \( X = \log P - 3 \log C \) which may be written

\[
X = -2 \log M_{AB} - 1.5 \log (1 - e^2) - 3 \log 2\pi, \tag{1}
\]

the areal constant being

\[
C = 2\pi A^2 (1 - e^2)^{1/2}/P. \tag{2}
\]

This result allowed us to conceive an orbital evolution due to mass-loss, from the systems having short period and little eccentricity towards those having large period and showing larger eccentricity. The period-eccentricity correlation should thus have been explained.

2. Second research

Twenty years later, the availability of a more extended visual material (551 visual orbits) confirmed this result (Dommanget 1981) and has permitted definition of the boundary limiting the region of the stellar systems in the diagram (\( \log M_{AB}/X \)) by the equation

\[
e^{2.8} M_{AB} = 3.60. \tag{3}
\]

Its tracing is represented in Fig. 3 and goes along the points given in Table 1.

<table>
<thead>
<tr>
<th>( e )</th>
<th>( \log M_{AB} )</th>
<th>( X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>1.67</td>
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<tr>
<td>0.5</td>
<td>1.40</td>
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<td>-3.93</td>
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<td>0.8</td>
<td>0.83</td>
<td>-3.39</td>
</tr>
<tr>
<td>0.9</td>
<td>0.68</td>
<td>-2.67</td>
</tr>
<tr>
<td>0.95</td>
<td>0.62</td>
<td>-1.04</td>
</tr>
<tr>
<td>0.96</td>
<td>0.606</td>
<td>+0.48</td>
</tr>
</tbody>
</table>

In this second research, the parallaxes once more have been the dynamical parallaxes for the same reasons as explained before.
Figure 2. Diagrams representing on the basis of the orbital material available in 1963: (a) the eccentricity \( e \) as a function of the \( X \) parameter and (b) the log \( M_1M_2 \) values as a function of this same parameter (Dommanget 1963).
Figure 3. Diagrams representing on the basis of the orbital material available in 1981: (a) the eccentricity \( e \) as a function of the \( X \) parameter and (b) the \( \log M_{AB} \) values as a function of this same parameter (Dommanget 1981).
Now, another two decades later, it appeared essential to verify whether the new available orbital material would again confirm this result. This material now contains 1,471 visual binaries (5th catalogue of the USNO, 2001) of which 1,341 have parallaxes (not negative!) observed by Hipparcos (ESA, 1997). When eliminating the astrometric pairs (code 9 in the catalogue) because they do not permit a correct computation of the areal constant as well as of their total mass (photocentric semi axis-major), there remains a file of 1,120 orbits leading to the diagram of Fig. 4.

A quick look at this diagram shows that this file should be considered with great caution. It contains for instance systems having unacceptable masses reaching some 100 to more than 1000 solar units ($\log M_{AB} > 2.0$) probably because too important relative errors on often too small parallaxes or in some cases because unsatisfactory orbital elements. These cases had of course to be rejected.

To eliminate the systems of which parallaxes are too inaccurate, one could either retain only those at less than 100 parsecs, the errors on $\pi$ remaining then in general less than 20%, or suppress those individually showing errors superior than 20%. One should remark that an error such as +50% – classical for many cases of our file – leads to a displacement of the position of the representing point in the diagram ($\log M_{AB}$, $X$) simultaneously of $-0.65$ in $\log M_{AB}$ and of $+1.30$ in $X$. Such a displacement is illustrated by the arrow seen in the upper part of Fig. 4.

As a matter of fact a variation $d\pi$ in the parallax, gives by differentiating the expression of the logarithm of the total mass

$$d \log M_{AB} = -3 \times 0.4343 \ d\pi/\pi = -1.30 \times d\pi/\pi. \quad (4)$$

To eliminate the prohibitive systems, we finally adopted the limit of 20% because it allows to eliminate the most flagrant cases when simultaneously it tolerates keeping an extended file of some 897 cases (when keeping the systems at less than 100 parsecs, the file contains only 708 cases). The resulting diagram is given in Fig. 5.
Now, in order to permit a valuable comparison with our preceding results only based on orbits of good quality and satisfying the mass-luminosity relation (dynamical parallax) we only retained those satisfying this propriety at best. Therefore, we thought we would characterize this quality by comparing the absolute magnitudes of the components computed by the direct way (parallaxes and apparent magnitudes) to those obtained through the use of the orbital elements \( P \) and \( a \) and the concerned relation. The more these values are near each other the more coherent are the observational data and this relation.

The considered equations are the following

1. by the “direct” way (parallax and apparent magnitudes)

\[
M_d = m + 5 + 5 \log \pi, \quad (5)
\]

2. by the “orbital” way (orbit and mass-luminosity relation)

- computation of \( M_{AB} \) by the third law of Kepler,
- computation of the mass ratio by the relation

\[
\log \frac{M_B}{M_A} = -a \Delta M_{bol}, \quad (6)
\]

(with \( a = 0.1117^* \))

- computation of the individual mass of component \( A \) by

\[
M_A = M_{AB}/(1 + \frac{M_B}{M_A}), \quad (7)
\]

and computation of its absolute magnitude by

\[
M_o = M_{bol,A} - C_A = 4.77 - 8.9525 \log M_A - C_A. \quad (8)
\]

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*Coefficient of the Baize-Romani mass-luminosity relation (1943): \( \log M = -0.1117(M_{bol} - 4.77) \). (Any other recent relation does not carry along any noticeable difference in the computations).
$C_A$ being the bolometric correction following Baize (1943). In view of a rough computation and because of the always too many unknown spectral classes for the individual components, we neglected the bolometric correction which in any case generally does not exceed 2 magnitudes. Moreover, we have only taken into account the results found for the principal component.

The difference between these two values of the absolute magnitude may be considered as a parameter of the coherence between the orbital and spectrophotometric data for each case. The histogram of Fig. 6 shows the distribution of the values of this difference for the 897 orbits of our file.

One may see that these are practically all – as absolute values – less than 10 magnitudes. But it appears evident that values of this order are prohibitive and that the smaller they are, the more confident we may be in their corresponding cases. The problem thus consists in choosing the most judicious limiting value for $\Delta M$.

Having tried various values in decreasing the order such as: $\Delta M < 6$ magnitudes (Fig. 7(a) – 858 cases); $\Delta M < 4$ magnitudes (Fig. 7(b) – 784 cases) and $\Delta M < 2$ magnitudes (Fig. 7(c) – 613 cases), we concluded to an acceptable agreement for this last value between the distribution of the representative points and the curve of equation (3) previously found, if we eliminate some non double systems in view to be in the same conditions we imposed in our two preceding researches.

We recall that the selection of the orbits has thus very similarly been made in 1963, 1981 and 2003:

- choice of the best available parallaxes (dynamical in 1963 and 1981; absolute and trigonometric from Hipparcos in 2003);
- orbital elements in agreement with the mass-luminosity relation (dynamical parallaxes in 1963 and 1981; coherence between direct and orbital absolute magnitude in 2003);
- elimination of the known multiple systems.
Figure 7. Diagrams respectively obtained with the conditions $\Delta M < 6$, $\Delta M < 4$, $\Delta M < 2$. 
Doing so, we found the diagram of Fig. 8 where the “boundary” could be expressed by equation

$$e^{3.40} M_{AB} = 3.60,$$

(9)

to be compared with equation (3) which is practically identical in drawings. Actually, its plotting goes then along the points given in table 2.

<table>
<thead>
<tr>
<th>$e$</th>
<th>$\log M_{AB}$</th>
<th>$X$</th>
</tr>
</thead>
<tbody>
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<td>0.470</td>
<td>1.67</td>
<td>-5.57</td>
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<tr>
<td>0.565</td>
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<td>-4.94</td>
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<td>0.920</td>
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<td>-2.01</td>
</tr>
<tr>
<td>0.967</td>
<td>0.606</td>
<td>-1.83</td>
</tr>
</tbody>
</table>

4. Conclusion

Neither the increase of the number of the computed orbits, nor the “switch” from the dynamical parallaxes to the trigonometric ones produced by Hipparcos seems to have perceptively modified their distribution in the diagram ($\log M_{AB}, X$). The boundary expressed by equation (9) appears more and more to have a real physical significance which should thus be considered in any research about the evolution of the binaries even if it is not based on a secular and isotropic mass-loss.
5. Acknowledgement

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References

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