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Spectra of Quasiperiodic Oscillations of Galactic X-ray Sources – Dynamical Regimes from a Simple Model

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Abstract. We suggest that the dynamical regime(s) underlying quasi-periodic oscillations observed in the spectra of bright galactic-bulge X-ray sources are nonlinear with a mixed phase space. The important feature of such regimes is that they are generic among nonlinear Hamiltonian and nearly Hamiltonian systems of more than two degrees of freedom. We give a simple example of such chaotic (deterministic) systems whose spectra share a number of features with those observed for quasiperiodic oscillations of such sources.

Key words. Binaries: close—stars: oscillations—galaxy: centre—X-rays: stars.

1. Introduction

Recent observations of bright galactic-bulge X-ray sources have stimulated a great deal of research. The interest centres mainly on a number of novel features exhibited by the observed power spectra of these sources. Among these are the localized power in the shape of a broad peak (hence the name quasiperiodic oscillations) and various modes of correlation between the peak frequency and the source intensity.

Various attempts have been made to construct a variety of physical models to understand the properties of quasiperiodic oscillations in these sources. However, there is no comprehensive model to account for various patterns of correlated temporal and spectral behaviour of these sources (see review by van der Klis 1989).

Our aim here is not to put forward a new specific model, but rather to suggest that the underlying regime(s) operative in these sources may possess divided phase spaces, comprising of regions of stochasticity and islands of periodicity. Such regimes are appealing for a number of reasons. Generally they have the important property of generecity, in the sense that Hamiltonian systems with more than two degrees of freedom are in general neither purely integrable nor stochastic. More specifically, such regimes share some of the spectral features observed in QPOs such as the existence of broad peaks and the presence of low frequency noise. In addition because they are commonly fragile (Tavakol & Ellis 1988; Coley & Tavakol 1992) in the sense of having different qualitative types of behaviour under small perturbations, both in the system itself or its initial conditions, they can potentially account for the observed diversity in the spectral behaviour of such sources within one theoretical framework.

In the following we give a simple example of a system with a divided phase space which shares some of the observed features of these sources.

2. Stochastic oscillations

As an example of such a system we consider one of the simplest systems which arises in the study of nonlinear oscillations. This is a two-dimensional discrete system (referred to as Chirkov-Taylor or standard map) of the form:

$$\begin{aligned} X_{n+1} &= X_n + k \quad \sin(Y_n) \\ Y_{n+1} &= Y_n + X_{n+1} \quad \mod 2\pi, \end{aligned} \tag{1}$$

where (X, Y) are action-angle variables, $n = 0, 1,...$ denote the discrete times or the number of iterations of the map and k is a real control parameter. Despite its simplicity the above system arises in a number of important physical settings. For example system (1) describes the motion of a charged particle in a uniform magnetic field subject to a periodic potential applied within a small region of the particle's trajectory (Zaslavskii & Chirkov 1972). More generally the standard mapping (1) can be visualized as a kicked rotor that describes the motion near the seperatrix of a fairly general nonlinear resonance (Chirkov 1979). This setting is relevant for the modelling of QPOs, keeping in mind the beat frequency scenario that has been successful in accounting for QPOs in many of the galactic X-ray sources

We shall not delve deeply into the detailed properties of the system (1) but mention briefly that depending on the value of the parameter k it can have extremely varied and complicated types of behaviour ranging from purely regular at $k = 0$ to extremely chaotic at large k. For intermediate values of k (≈ 1), (system (1) has a complicated divided phase space containing both periodic components (in the shape of islands m a hierarchy of scales on which the motion is regular) and chaotic regions. This type of chaotic motion can be viewed as a state of transition from regular to truly chaotic regimes.

A great deal is known about the behaviour of the system (1) in these intermediate k-regimes. In particular, studies have been made of the spectral properties of the above system in its state of transition. The importance of this type of divided phase space, as far as we are concerned here, is that even the trajectories in the chaotic regions are affected by the presence of the islands. It has been argued (Beloshapkin & Zaslavskii 1983) that the presence of islands gives rise to a number of effects: (**i**) the occurrence of localized spectra with an effective frequency band Ω which increases with increasing k according to a relation of the form

$$\text{In } \Omega = a + bk, \tag{2}$$

where a and b are constants. This amounts to a correlation between the central frequency of the band and the parameter k; (**ii**) An anomalously large amount of power in the low-frequency region of the spectra.

As an example, we have calculated the power spectra for the system (1) at a number of values of the parameter k for a set of initial conditions. Figures 1(a–c)

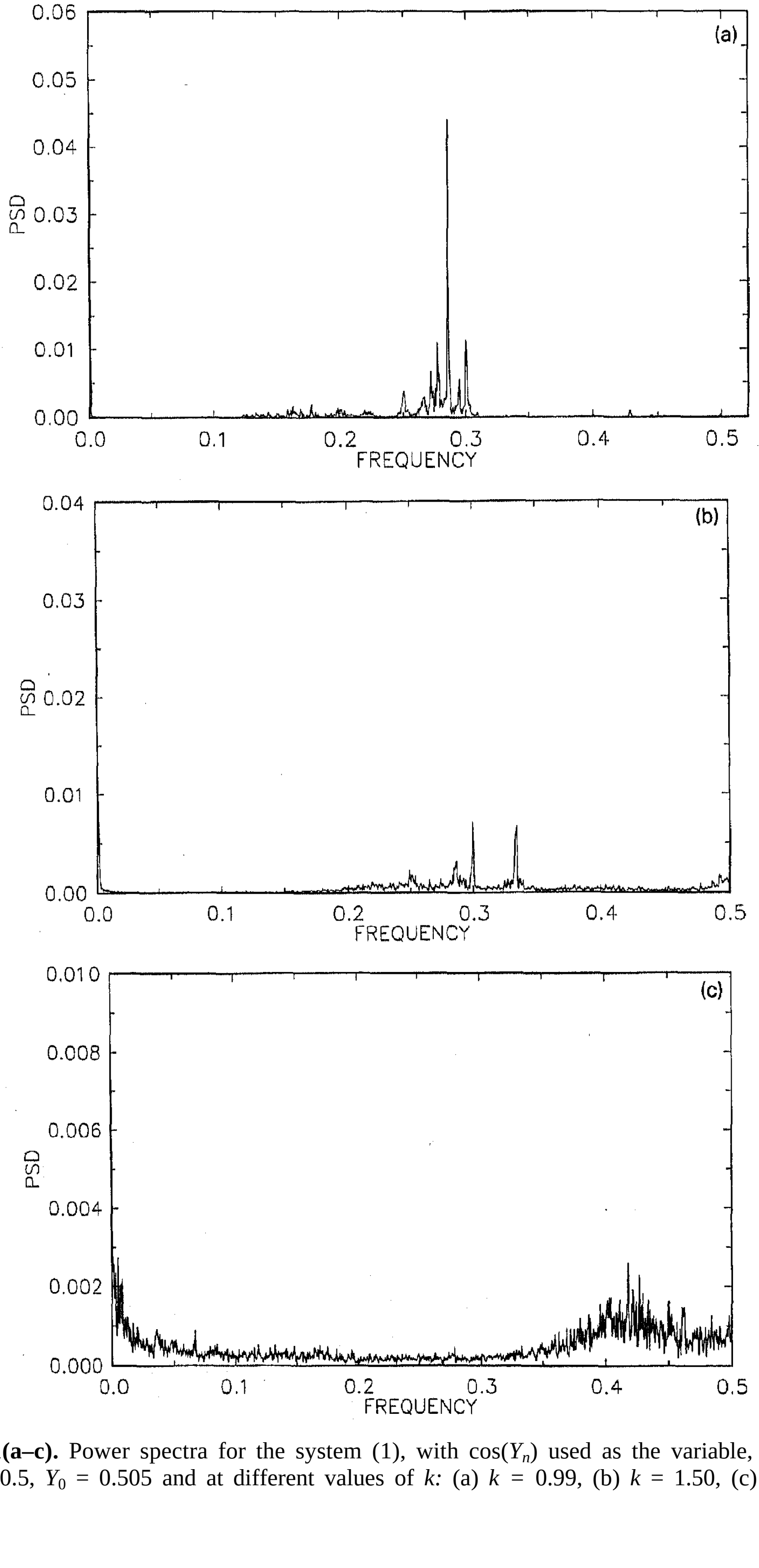

Figure 1(a–c). Power spectra for the system (1), with $\cos(Y_n)$ used as the variable, calculated with X_0=0.5, Y_0 = 0.505 and at different values of *k:* (a) k = 0.99, (b) k = 1.50, (c) k = 2.99.

show the spectra corresponding to the parameter values k = 0.99, k = 1.50, and k = 2.99 respectively. All spectra were calculated using 16384 data points with cos (Y_n) taken as the variable. This choice of variable is made because it represents the potential and hence may be more meaningful physically, otherwise it does not alter the basic results. As can be seen, the three spectra show localized power. In addition the central frequency increases and the frequency band widens as k increases. We should also mention that as k is further increased, the system becomes more chaotic, the spectrum tends towards uniformity and the localization of power disappears, another feature observed in the behaviour of QPOs.

3. Results and discussion

We have seen that despite its simplicity, system (1) can give rise to spectra which share some of the features observed in the spectra of QPOs of the galactic-bulge X-ray sources, namely (**i**) power is effectively localized in a frequency band, (**ii**) both the position of the central frequency and the width of the frequency band increase with increasing k, (**iii**) presence of substantial amount of low frequency noise, and (**iv**) flattening of the spectrum (disappearance of the band structure) at high values of k

Of course there are other features of the QPOs that the above simple model does not share, such as the correlation between the frequency and the source intensity observed for some sources. One could attempt to complicate system (1) in an attempt to account for such features. As an example note that system (1) is conservative. The inclusion of a small amount of noise or dissipation might be considered as a physically meaningful addition. To allow for the latter, we modified system (1) thus:

$$\begin{aligned} X_{n+1} &= X_n + k \quad \sin(Y_n) \\ Y_{n+1} &= (1-\epsilon)\, Y_n + X_{n+1} \quad \mathrm{mod}\, 2\pi, \end{aligned} \tag{3}$$

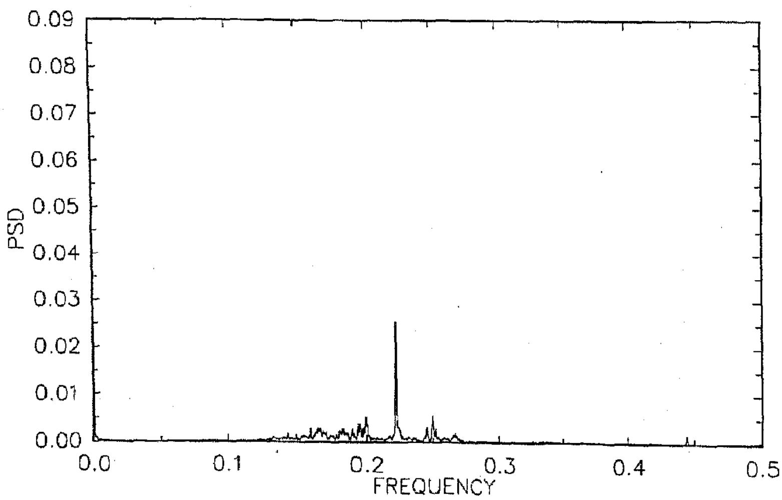

Figure 2. Power spectra for the system(3), with cos(Y_n) used as the variable, calculated at parameter values X_0 = 0.5, Y_0 = 0.505, k = 0.99 and ϵ = 0.000005.

where ϵ determines the amount of dissipation present and system (3) reduces to system (1) for $\epsilon = 0$. Figure 2 shows the effect of a small nonzero ϵ on the spectrum of Fig. 1(a). Clearly the spectra can vary drastically by such small modifications, a question we hope to return to in future.

Whatever the merits or limitations of the simple system we have considered here, we wish to emphasize the potential importance of such generic settings within which a simple and unified qualitative understanding can be gained regarding the underlying phenomena operative in QPOs. It is also worthwhile to point out that the type of behaviour observed for the system (1) is by no means unique but a generic property of simple systems with divided phase spaces. In this respect it would be of value to study the width of the frequency band as a function of the intensity of the QPO sources by employing the observational data and comparing these with relation (2) given above. Perhaps the most important feature of the systems of the type considered here is their capacity to produce extremely varied and complicated behaviour as a function both of their control parameters and their initial conditions. This is particularly of value considering the ever-increasing complexity in the detailed picture of QPOs which seems to be brought about by new observations. Finally we should add that our considerations here might also be relevant to the study of other astrophysical objects where nonlinearities are present.

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