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Massive Compact Dwarf Stars and C-field

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Abstract. The effect of C-field in high density matter has been studied. We find that the negative energy and negative pressure of the C-field helps in formation of massive compact stable neutron stars of mass ~ 0.5 solar mass which is in the range of 0.01 to 1.0 solar mass of recently observed dwarf stars.

Key words: Dwarf stars—neutron stars—C-field.

1. Introduction

There has recently been reported observations by gravitational microlensing of dark dwarf stars in the mass range of 0.01–1.0 $M_\odot$ (Alcock *et al.* 1993; Aubourg *et al.* 1993). Simple extrapolations of these observations have led people to speculate (Boughn & Uson 1995) that such bodies could be numerous and make up the 'dark matter' inferred from the rates of rotation of galaxies and superclusters of galaxies. In any case, this observation has opened up the question of what these massive compact objects themselves are. Cottingham, Kalafatis & Vinh Mau (1994), have proposed a model where these objects are identified as quark stars formed after quark-hadron transition.

We, in this work, suggest a different understanding of these objects. The stability of these objects at unusual mass values is indicative of a simultaneous increase in binding and reduction in the internal pressure compared to the normal stars leading to the balance between the two at lower mass values. Driven by such an argument and since the creation field (C-field) of Pryce; Hoyle & Narlikar (1962); and Narlikar (1973) used in the context of steady-state theory has exactly these characteristics of negative pressure and negative energy, we have tried to include its contribution into the description of high density matter in terms of SU (2) chiral sigma model (Sahu, Basu & Datta 993).

Even though, the hot big-bang model of the creation and evolution of the Universe has gained acceptance over the steady-state cosmology, it does have problems associated with linearity of Hubble flow and determination of the age of the Universe from the Hubble Space Telescope data (Narlikar 1993). It is, therefore, in the fitness of things to explore the possibility of an explanation for the problem at hand within the context of an alternative model as has been attempted in other contexts (Weinberg 1972; Arp *et al.* 1990).

The massless C-field was originally introduced by Pryce and later extensively used by Hoyle & Narlikar (1962) and Narlikar (1973) to provide a field-theoretic

2. The model

As stated before, our model is an extension of the chiral sigma model approach (Sahu, Basu & Datta 1993) for the study of high density matter with the inclusion of C-field effect. We also take the approach that the isoscalar vector field necessary to ensure the saturation property of nuclear matter is generated dynamically. The effective nucleon mass then acquires a density dependence on both the scalar and the vector fields, and must be obtained self-consistently. We do this using the mean-field theory wherein all the meson fields are replaced by their uniform expectation values.

The Lagrangian for an SU(2) chiral sigma model that includes an isoscalar vector field (ω_μ), an isotriplet (transforming as vector Iso-spin space) vector field ($\vec{\rho}_\mu$) of mass m_ρ ($\hbar = 1 = c$) and a non-interacting C-field is

$$\begin{aligned} L = {} & \tfrac{1}{2}(\partial_\mu \vec{\pi}\,.\,\partial^\mu \vec{\pi} + \partial_\mu \sigma \partial^\mu \sigma) - \frac{\lambda}{4}(\vec{\pi}\,.\,\vec{\pi} + \sigma^2 - x_0^2)^2 \\ & - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}g_\omega^2(\sigma^2 + \vec{\pi}^2)\omega_\mu\omega^\mu \\ & + g_\sigma \bar{\psi}(\sigma + i\gamma_5 \vec{\tau}\,.\,\vec{\pi})\psi + \bar{\psi}(i\gamma_\mu \partial^\mu - g_\omega \gamma_\mu \omega^\mu)\psi \\ & - \tfrac{1}{4}G_{\mu\nu}G^{\mu\nu} + \tfrac{1}{2}m_\rho^2 \vec{\rho}_\mu.\vec{\rho}^\mu - \tfrac{1}{2}g_\rho \bar{\psi}(\vec{\rho}_\mu.\vec{\tau}\gamma^\mu)\psi - \frac{f}{2}\partial_\mu C \partial^\mu C, \end{aligned} \tag{3}$$

where $F_{\mu\nu} \equiv \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ $G_{\mu\nu} \equiv \partial_\mu\rho_\nu - \partial\nu\rho_\mu$ ψ is the nucleon isospin doublet, $\vec{\pi}$ is the pseudoscalar pion field and σ is the scalar field. The expectation value $\langle \bar{\psi}\gamma_0\psi \rangle$ is identifiable as the nucleon number density, which we denote by n_B.

The interactions of the scalar and the pseudoscalar mesons with the vector boson generates a mass for the latter spontaneously by the Higgs mechanism. The masses for the nucleon, the scalar meson and the vector meson are respectively given by $m = g_\sigma x_0$; $m_\sigma = \sqrt{2\lambda}x_0$; $m_\omega = g_\omega x_0$, where x_0 is the vacuum expectation value of the sigma field. C-field being non-interacting remains massless.

Taking the mean-field approximation $\omega_\mu = \omega_0 \delta_{0\mu}$, the equation of motion for the mean vector field specifies ω_0

$$\omega_0 = \frac{n_B}{g_\omega x^2}, \quad x = (\langle \sigma^2 + \vec{\pi}^2 \rangle)^{1/2}. \tag{4}$$

Note that ω_0 depends on n_B but not on space-time coordinates. The equation of motion for σ written for convenience in terms of $y \equiv x/x_0$ is of the form

$$y(1 - y^2) + \frac{c_\sigma c_\omega \gamma^2 k_F^6}{18\pi^4 M^2 y^3} - \frac{c_\sigma y \gamma}{\pi^2}\int_0^{k_F} \frac{\mathrm{d}k k^2}{(\vec{k}^2 + M^{*2})^{1/2}} = 0, \tag{5}$$

where m^* ym is the effective mass of the nucleon and $c_\sigma \equiv g_\sigma^2/m_\sigma^2$; $c_\omega \equiv g_\omega^2/m_\omega^2$.

At high densities typical of the interior of neutron stars, the composition of matter is asymmetric nuclear matter with an admixture of electrons. The concentrations of protons and electrons can be determined using conditions of beta equilibrium and electrical charge neutrality. We include the interaction due to isospin triplet ρ-meson in Lagrangian for the purpose of describing neutron-rich matter. The equation of motion for $\vec{\rho}_\mu$, in the mean field approximation, where $\vec{\rho}_\mu$ is replaced by its uniform value ρ_0^3 (here superscript 3 stands for the third component in isopin space), gives $\rho_0^3 = (g_\rho/2m_\rho^2)(n_p - n_n)$. The symmetric energy coefficient that follows from the semi-empirical nuclear mass formula is

$a_{sym} = (c_\rho k_F^3/12\pi^2) + (k_F^2/6(k_F^2 + m^{*2})^{1/2})$, where $c_\rho \equiv g_\rho^2/m_\rho^2$ and $k_F = (6\pi^2 n_B/\gamma)^{1/3}$. Here $n_B = n_p + n_n$ and γ is the nucleon spin degeneracy factor. We fix the values of c_σ, c_ω and c_ρ by fits of saturation density (0.153 f m^{-3}), the binding energy (–16.3 MeV) and symmetric energy (32 MeV) (Moller *et al.* 1988) in the absence of C-field as a first approximation. These give $c_\sigma = 6.20\, f\, m^2$; $c_\omega = 2.94\, f\, m^2$; $c_\rho = 4.6617\, f\, m^2$.

The diagonal components of the conserved total energy-momentum stress tensor $T_{\mu\nu}$ ($T_{00} = \varepsilon$ and $T_{ii} = -3P$ by definition) corresponding to the Lagrangian given by equation (3) together with the equation of motion for the fermion field (and a mean field approximation for the meson fields) provide the following identification for the total energy density (ε) and pressure (P) for neutron star system:

$$\varepsilon = \frac{m^2(1-y^2)^2}{8c_\sigma} + \frac{\gamma^2 c_\omega (k_p^3 + k_n^3)^2}{72\pi^4 y^2} + \frac{\gamma^2 c_\omega (k_p^3 - k_n^3)^2}{72\pi^4 y^2} + \frac{\gamma}{2\pi^2}\sum_{n,p,e}\int_0^{k_F} \mathrm{d}k\, k^2 (\vec{k}^2 + m^{*2})^{1/2} - \frac{f}{2}\dot{C}^2. \tag{6}$$

$$P = -\frac{m^2(1-y^2)^2}{8c_\sigma} + \frac{\gamma^2 c_\omega (k_p^3 + k_n^3)^2}{72\pi^4 y^2} + \frac{\gamma^2 c_\omega (k_p^3 - k_n^3)^2}{72\pi^4 y^2} + \frac{\gamma}{6\pi^2}\sum_{n,p,e}\int_0^{k_F} \frac{\mathrm{d}k\, k^4}{(\vec{k}^2 + m^{*2})^{1/2}} - \frac{f}{2}\dot{C}^2. \tag{7}$$

A specification of the coupling constants c_σ, c_ω, c_ρ and f now specifies the EOS.

As of the coupling constant f, since it has the dimension of (mass)^2, we have parameterized it in two ways like, $f = \hat{f}\, m^2$ and $\hat{f}\,(n_B/m)$, where m is typically the nucleon mass, n_B is the baryon density and $\hat{f}$ is dimensionless. The second parameterize-ation employing a linear proportionality between f and n_B, we believe, is more reasonable and physical as the coupling of C-field is likely to grow in strength with increase in n_B. This is because the creation of baryons occurs in association with creation of C-field in the steady-state picture and thus greater baryon density implies stronger interaction strength (f) for the C-field.

The structure of a neutron star is characterized by its gravitational mass (M) and radius (R). These gravitational mass and radius for non-rotating neutron star are obtained by integrating the structure equations, which describe the hydrostatic equilibrium of degenerate stars: (Misner, Thorne & Wheeler 1970)

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{G(\rho + p/c^2)(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)}; \quad \frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho, \tag{8}$$

where p and $\rho(\varepsilon/c^2)$ are the pressure and total mass energy density. G is Newton's gravitational constant and m is mass enclosed in a spherical star of radius r.

To integrate equation (8), one needs to know the equation of state for the entire expected density range of neutron star, starting from high density at the center to the surface densities. The composite equations of state for the entire neutron star density span was constructed by joining the equations of state of high density neutron rich matter (curves(a)–(e) in Fig. 1)to that given by (i) Negele &Vautherin (1973) for density region ($10^{14} - 5 \times 10^{10}$) g cm^{-3}, (ii) Baym, Pethick & Sutherland (1971) for the region ($5 \times 10^{10} - 10^3$)g cm^3 and (iii) Feynman, Metropolis & Teller (1949) for the densities less than 10^3 g cm^{-3}.

For a given EOS, $p(\rho)$ and a given central density $\rho\,(r = 0) = \rho_c$, equation (8) are integrated numerically with the boundary condition $m(r = 0) = 0$ to give R and M. The radius R is defined by the point where $P \simeq 0$, or, equivalently, $\rho = \rho_s$, where ρ_s is the density expected at the neutron star surface. The maximum total gravitational mass for stable configuration is then given by: $M = m(R)$.

3. Result and discussion

The results obtained after the inclusion of the contribution of the C-field are given in table 1 and Fig. 1. In table 1, we have presented the maximum mass (M) and the corresponding radius (R) for a more typical stable star as a function of central density (ρ_c). The equations of state (pressure vs. energy density) for neutron star matter at the above nuclear matter density (2.8×10^{14} g cm $^{-3}$) are given in Fig. 1. It is seen from this figure that the equations of state in case I (curves (b) and (c)) is different from case II (curves (d) and (e)). From equations (6) and (7), one sees that the pressure and energy density depend on the baryon density n_B. With variation of n_B, the equation of state behaves like case III (curve (a)) without inclusion of the C-field. In case I, the $\hat{f}$ is proportional to n_B and hence the negative pressure and the negative energy density due to C-field are added to the equations of state. However, in case II the $\hat{f}$ is a constant, therefore, there is constant subtraction in pressure and energy density due to the C-field and hence a constant shift in equation of state from equation of state case III. At high density in all cases, the system approached the causal limit $P = \varepsilon$, representing the 'stiffest' possible equation of state. It is observed from the figure for both cases I and II that with increase of $\hat{f}$, the EOS becomes softer leading to reduction of stable neutron star gravitational mass.

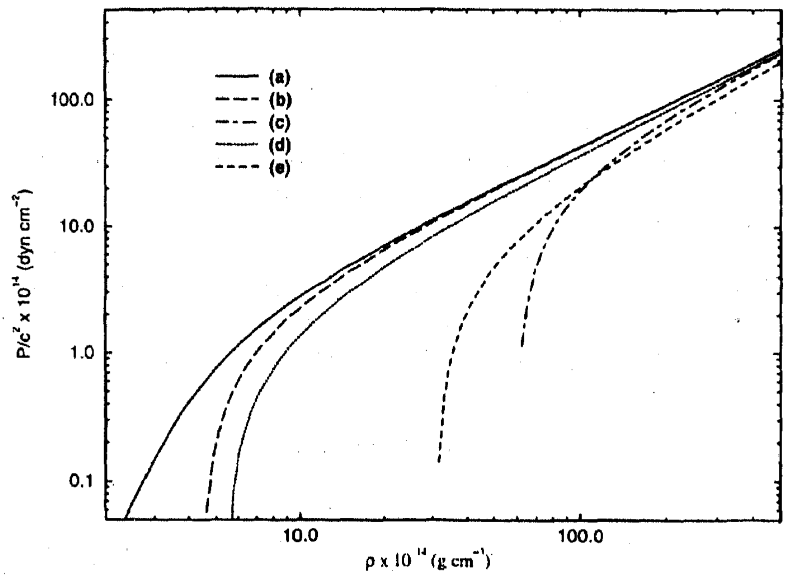

Figure 1. Pressure and energy density curves for three cases: Case I: $\hat{f} = f\,m^2$ with $\hat{f} = 0.001$ (curve *b*) and $\hat{f} = 0.05$ (curve *c*); Case II: $f = \hat{f}\,n_B/m$ with $\hat{f} = 0.5$ (curve *d)* and $\hat{f} = 1.5$ (curve *e*); Case III: $f = 0$ (curve *a*).

Table 1. The maximum mass (M), the corresponding radius (R) and central density (ρ_c) of more typical compact dwarf stars for various values of dimensionless coupling parameter $\hat{f}$ for three cases: Case I: $f=\hat{f}m^2$, Case II: $f = \hat{f}\, n_B/m$ and Case III: $f = 0$.

ρ_c ($\mathrm{g\,cm^{-3}}$)	R (km)	$M/M_\odot$	$\hat{f}$	Cases
2.0×10^{15}	10.96	2.16	0.001	I
4.0×10^{15}	7.61	1.51	0.005	I
7.0×10^{15}	5.94	1.19	0.01	I
2.5×10^{16}	3.12	0.64	0.05	I
3.0×10^{15}	9.10	1.678	0.5	II
7.5×10^{15}	5.66	1.01	1.0	II
2.0×10^{16}	3.61	0.67	1.5	II
1.5×10^{15}	13.65	2.59	0.00	III

A comparison of the values of the radii (R) and gravitational mass (M) for various values of dimensionless coupling parameter $\hat{f}$ with those in the absence of C-field as obtained in an earlier work (Sahu, Basu & Datta 1993) (case III of table 1) reveals that the size and mass of stable neutron star structures have been significantly reduced. This can be understood on the ground that the negative energy provided by C-fields diminishes the effective mass of the star and the negative pressure reduces opposition to gravitational aggregation of matter. Thus, C-field doubly facilitates formation of compact stable neutron star structure of both smaller mass and dimension. In fact, for $\hat{f}$ values near 0.05 and 1.5 in the two schemes of parameterization of dimensional coupling constant f, the stable neutron star mass is about 0.5 $M_\odot$ which is well inside the mass range of dwarf stars recently observed between 0.01 and $1.0M_\odot$ considered to be candidates for the dark matter (Boughn & Uson 1995; Cottingham, Kalafatis & Vinh Mau 1994). Further, the value of $\hat{f}$ being of O (1) for the mass of compact object to lie in the right range implies that the interaction involved is strong in character as is to be expected between the nucleon and scalar field.

Apart from the above agreement, we believe that the study of the effect and role of the C-field in various astrophysical problems need to be taken up in its own merit. This provides scope to investigate the existence of explanations alternative to those found within the ambit of the generally accepted big bang model of the Universe. The present work completes a small programme in that direction.

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