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Power Spectrum Analysis of the Timing Noise in 18 Southern Pulsars

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Abstract. Power spectra of the timing noise observed in 18 southern pulsars have been derived using a novel technique, based on the CLEAN algorithm. Most of the spectra are well described by a single- or double-component power-law model. Some of these spectra can be interpreted in the context of one or more of the current timing noise models. The results combined with those obtained from the time-domain analyses of the timing activity in these pulsars are used to assess the viability of the various theoretical models of pulsar timing noise.

Key words. Pulsars: timing noise—power spectra—neutron star.

1. Introduction

Power spectra of pulsar rotation fluctuations can provide valuable information about the mechanisms responsible for timing noise. In fact, most of the theoretical models of timing noise (e.g., Alpar *et al* 1986; Cheng 1987a,b; Cheng 1989; and Jones 1990) make predictions in terms of a power spectrum of the fluctuations. Furthermore, these models are based on a statistical description of fluctuations in one or more of the three observables – the pulse phase ϕ (PN), frequency ν (FN) or frequency derivative $\dot{\nu}$ (SN), each resulting in a simple power-law spectrum. On the other hand, quasi-periodic oscillations, such as those that may result from free precession or oscillations of the vortex lattice, will produce a narrowband signature in the power spectrum.

The nomenclature that has been used for the three simple "random walk processes", namely PN, FN and SN, is unambiguous in the context of pulsar work, but can be confusing when applied more generally. The general nomenclature emphasises the variable in which the process is stationary, i.e., the one which exhibits white noise properties (Lamb 1981). Hence, PN, FN and SN correspond to processes that produce white noise in ν, $\dot{\nu}$ and $\ddot{\nu}$ respectively. These random walk processes have a "red" power spectrum (i.e., excess power at low frequencies) in the variable ϕ, and can be considered as a repeated integral of white noise in ϕ. Since ϕ, ν and $\dot{\nu}$ are simply related by differentiation, the power spectra are related by factors of $f^{\,2}$, so that $P_\nu(f) \sim f^{\,-2}P_\phi(f)$ and $P_{\dot{\nu}}(f) \sim f^{\,-2}P_\nu(f)$. Deeter & Boynton (1982) use the terminology "*r*-th order red noise", denoting a variable $x(t)$ which is the *r*-fold

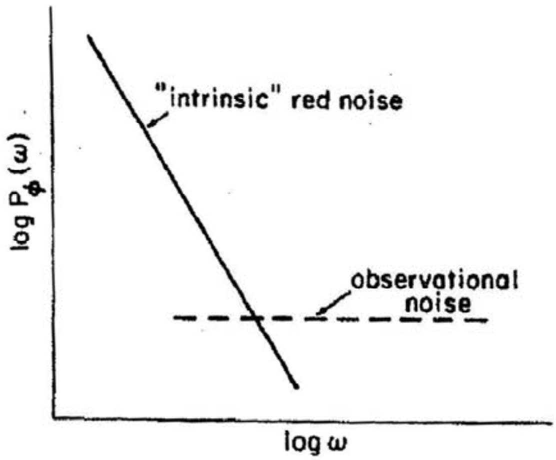

Figure 1. Limitation on the observability of intrinsic red phase noise posed by the measurement process (white noise in the pulse phase); (after Boynton 1981).

integral of white noise. That is, the rth time derivative, x^r (t), reduces to white noise. Hence, the power spectrum of $x(t)$ obeys the law P_x (f) $\sim f^{\sim 2r}$. The orders r = 1, 2 and 3 correspond to phase, frequency and slowing-down noise respectively.

Two features will be evident in a simple power-law spectrum of the phase fluctuations – a steep red noise component at lower frequencies, and a white noise component which may begin dominating at higher frequencies. In practice, the observability of the intrinsic noise process is restricted at high frequencies by this "measurement noise" rather than the Nyquist sampling limit, as shown in Fig. 1. The lower the "signal-to-noise" of the timing activity, the further toward low frequencies one must look in order to detect the red noise in the spectrum.

To investigate successfully all of the proposed noise processes over a frequency range of a decade, a dynamic range of at least six orders of magnitude must be attainable. Conventional Fourier transform (FT) techniques fail when they are used to estimate the spectral power density that is characteristic of red noise processes, particularly from an unevenly sampled time sequence. A basic reason is that there is substantial power "leakage" through the sidelobes of the equivalent power density estimators that can very easily mask any steep variations in the spectrum. While dealing with steep red spectra, simple FT techniques produce meaningless power spectra with a steepest power-law slope of ~ –2. The situation is further complicated by the unevenly sampled time series that inevitably arise from practical astronomical observations. Interpolation of the data does not help much as the spectral contamination resulting from the interpolation seriously affects the spectral power estimation at high frequencies. Hence, the above issues must be considered if one is to correctly recover red noise spectra from the timing data.

We recently developed and tested a new technique of spectral analysis that addresses the problems described above. The technique is based on the CLEAN algorithm and is described in detail elsewhere (Deshpande, D'Alessandro & McCulloch 1996). The main motivation for this work stemmed from the fact that none of the other spectral methods are specifically tailored to producing reliable spectral estimates with a *high dynamic range.* This is an extremely important requirement when obtaining power spectra of pulsar phase residuals.

After the development of our technique based on CLEAN, it was discovered that a similar idea had already been implemented by Roberts *et al.* (1987) for the time-series spectral analysis of unequally spaced data. A variant of their technique has been used by Green *et al.* (1993) to study rapid X-ray variability in active galactic nuclei. It should be stressed, however, that like other methods, the technique developed by Roberts *et al.* does not address the “high dynamic range” requirement that is important in the analysis of pulsar timing noise.

The data analysed and presented in this paper were collected as part of a monthly timing survey of 45 southern pulsars at the Mt Pleasant Observatory, operated by the Physics Department of the University of Tasmania. Details of the observations, data acquisition and reduction have been described elsewhere (D’Alessandro *et al.* 1993). The data were collected at two observing frequencies, 670 and 800 MHz, over a period of up to 7 years from 1987 to 1994. Basic timing and astrometric parameters and the results of a detailed study of the timing noise in all of these pulsars have also been presented elsewhere (D’Alessandro *et al.* 1993, 1995). In this paper, we aim to assess the viability of the various theoretical models of pulsar timing noise by comparing the observed power spectra with those predicted by the theories.

2. Power spectral analyses and results

Although the power density spectrum of the $\dot{\nu}$ fluctuations, $P_{\dot{\nu}}(f)$, is more closely related to the response of the neutron star to torque fluctuations, the derivation of such a spectrum has a number of disadvantages compared to the estimation of the spectrum of the phase fluctuations, $P_{\phi}(f)$ This is because the pulse phase is the quantity directly measured in timing observations of pulsars. For example, the $\dot{\nu}(t)$ estimates invariably result from some sort of “smoothing” process, e.g., from short polynomial fits. This restricts the useful span of the spectrum to low frequencies because fewer independent data points over the time span are available for deriving the spectrum. Also, the contribution of white noise in the pulse phase estimation translates into ”blue” noise in the spectrum of the $\dot{\nu}(t)$ fluctuations. However, in principle, the technique to be described can be applied to any type of pulsar timing residual data set, e.g., phase, frequency or frequency derivative residuals.

Before any analyses were performed, we combined the dual-frequency phase residual data for each pulsar into a single data set. This procedure improves the sensitivity of the data to be analysed. A total of 18 pulsars from the Mt Pleasant sample were found to be suitable for power spectral analysis. These pulsars were selected on the basis that the signal-to-noise of the timing activity was sufficient (typically $\geq$ 10) to obtain a meaningful spectrum.

The CLEAN technique was then applied to the zero-mean, combined phase residual data of the selected 18 pulsars. The power density spectra in ϕ for these pulsars are shown in Figs. 2, 3, and 4. The power spectra are displayed in the conventional manner, i.e., as a log-log plot of power spectral density against frequency. The reasons for such a representation have been discussed by Deeter (1984) and Boynton & Deeter (1986). In the context of the pulsar timing noise investigations being made in this paper, the most important reason is the fact that the log-log representation reveals very readily any power-law behaviour in the power spectrum.

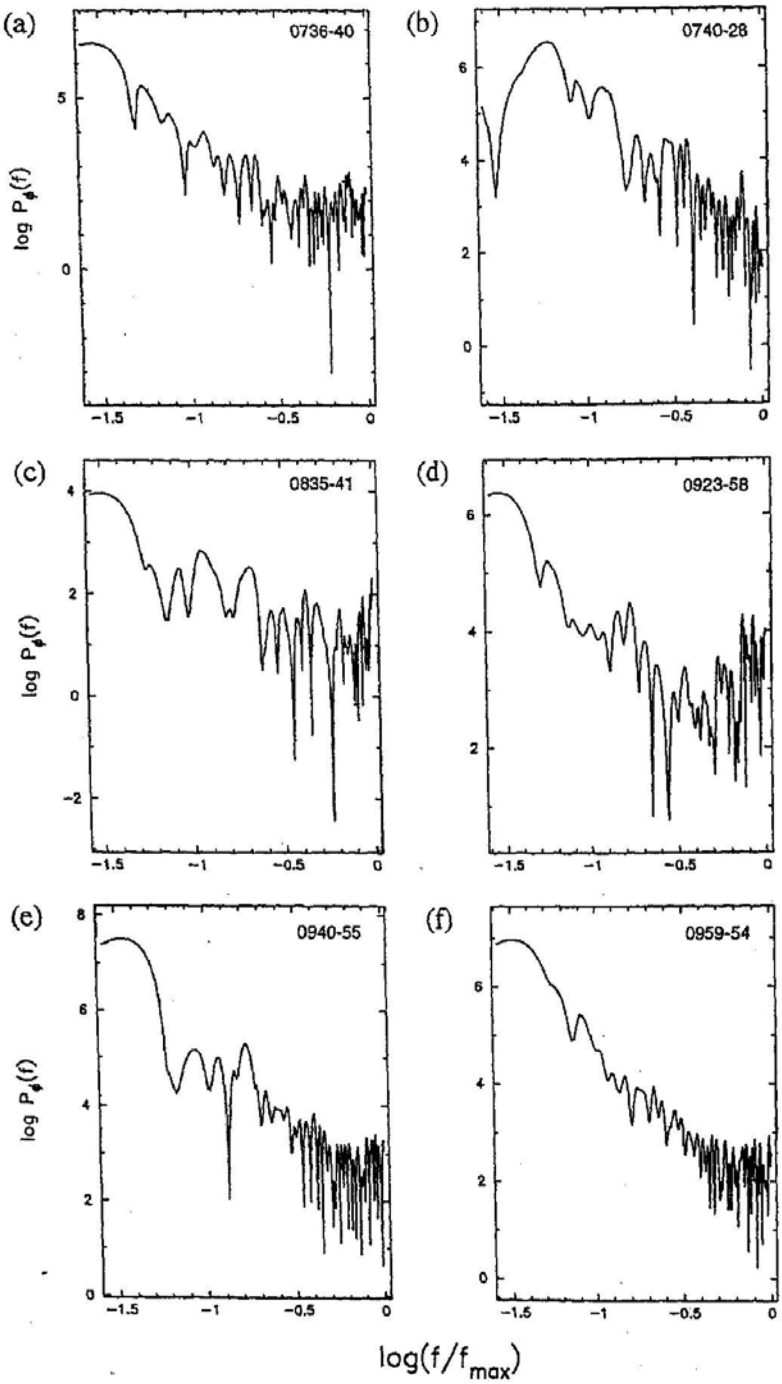

Figure 2. Power density spectra of the phase residuals for six southern pulsars. The units of power density are arbitrary. The frequency scale has been normalized using f_{max}, as defined by the Nyquist limit.

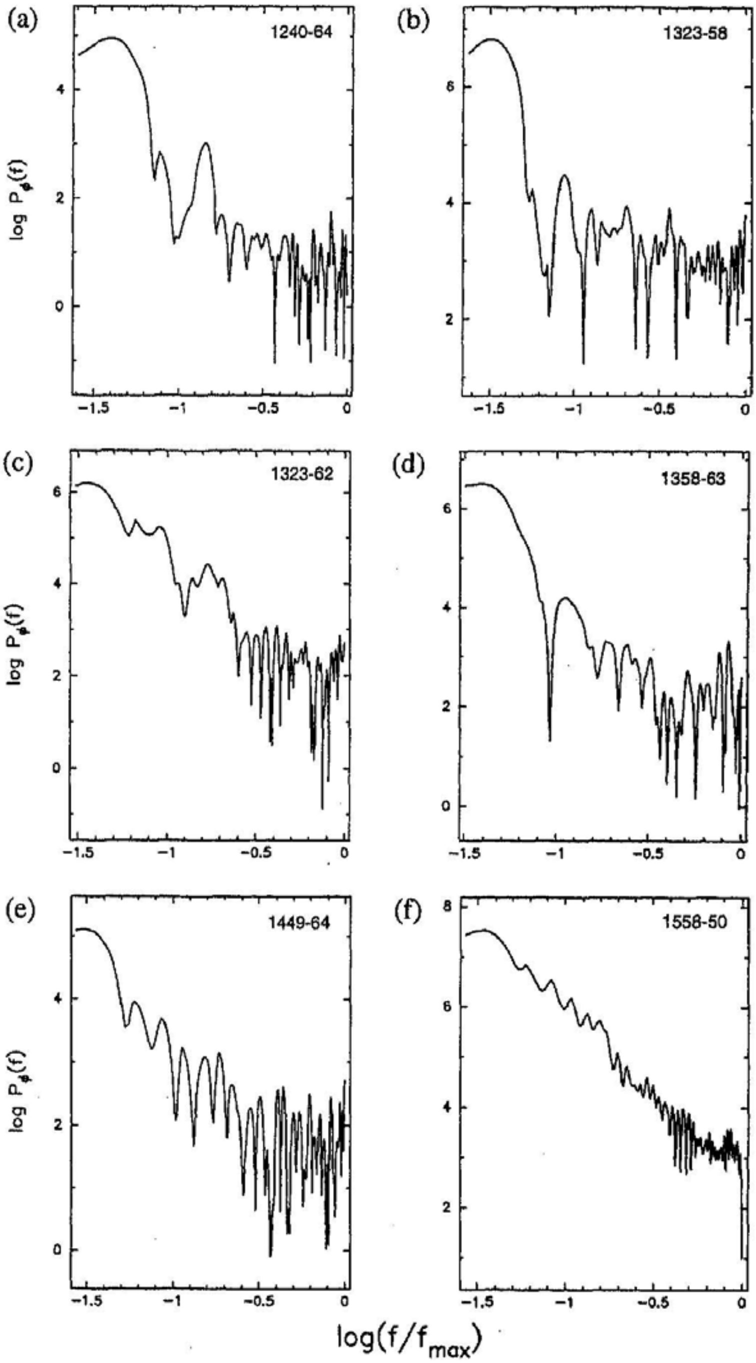

Figure 3. Power density spectra of the phase residuals for six southern pulsars. The units of power density are arbitrary. The frequency scale has been normalized using $f_{\max}$, as defined by the Nyquist limit.

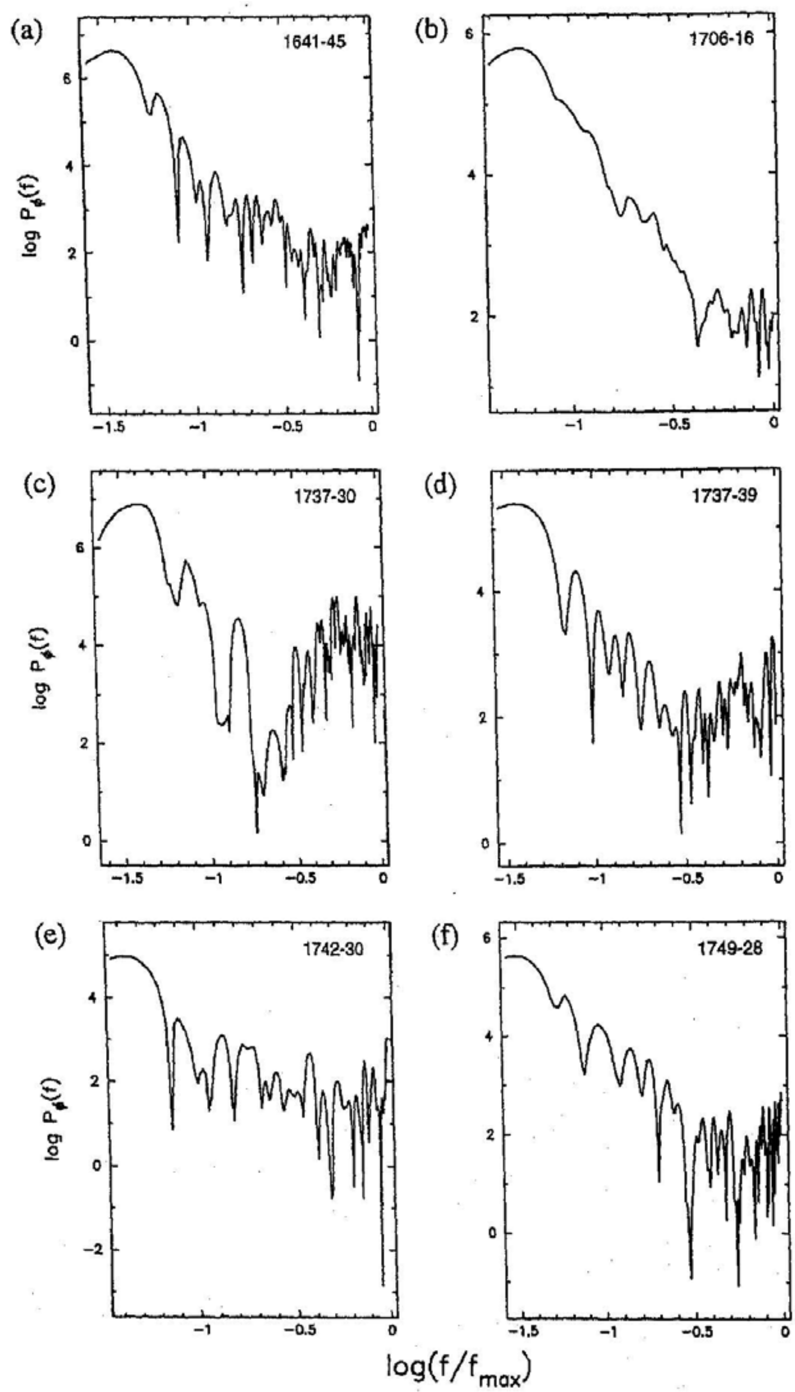

Figure 4. Power density spectra of the phase residuals for six southern pulsars. The units of power density are arbitrary. The frequency scale has been normalized using $f_{\max}$, as defined by the Nyquist limit.

Table 1. Logarithmic slopes obtained from linear least squares fits to the CLEANed power spectra of the timing noise in 18 pulsars.

PSR B	log τ	S/N	Freq. range	Slope	Err	NPPSS	NPTDA
0736-40	6.57	64	−1.6, −0.5	−5.2	0.4	?	FN
0740-28	5.20	42	−1.3, −0.3	−3.4	0.2	?	?
0835-41	6.53	8	−1.6, −0.5	−2.1	0.2	PN	PN
0923-58	6.38	10	−1.6, −1.2	−4.6	0.6	FN	?
			−1.3, −0.6	−2.3	0.3	PN	
0940-55	5.67	80	−1.6, −1.2	−5.0	0.9	SN	FN
			−1.3, −0.5	−2.4	0.2	PN	
0959-54	5.65	151	−1.6, −0.9	−4.8	0.2	?	?
			−0.9, −0.4	−2.7	0.2		
1240-64	6.14	23	−1.6, −0.7	−4.3	0.4	FN	FN
1323-58	6.37	21	−1.6, −0.8	−4.7	0.5	FN	FN
1323-62	5.65	25	−1.6, −0.5	−3.9	0.2	FN	FN
1358-63	5.90	47	−1.6, −0.9	−6.1	0.7	?	FN
			−1.0, −0.5	−3.1	0.3		
1449-64	6.02	12	−1.6, −1.0	−4.0	0.3	?	?
			−1.2, −0.5	−2.6	0.3		
1558-50	5.29	189	−1.6, −0.4	−3.6	0.1	?	?
1641-45	5.55	69	−1.6, −1.0	−5.9	0.5	SN	?
			−1.0, −0.4	−2.3	0.3	PN	
1706-16	6.22	47	−1.3, −0.4	−4.1	0.6	FN	FN
1737-30	4.31	16	−1.6, −0.8	−5.4	0.5	?	?
1737-39	6.62	11	−1.6, −0.5	−3.9	0.2	FN	?
1742-30	5.74	13	−1.6, −0.5	−3.3	0.3	?	?
1749-28	6.04	34	−1.6, −0.4	−4.0	0.2	FN	?

Logarithmic slopes were obtained from the CLEANed and restored spectra by performing linear least squares fits to the spectral estimates over the available frequency range, i.e., where the power exceeded the white noise. In most of the spectra, the crossover point (i.e., the point where the white noise begins to swamp the red noise) occurs at $\log(f/f_{max}) \leq -0.5$. This implies the shortest autocorrelation time that can be probed in the Mt Pleasant pulsar observations is ~200 days. In some cases, a single linear fit did not adequately model the spectrum. A "two-component" linear model was used for these spectra. The results of the linear fits to the spectra are presented in Table 1. The columns contain, respectively, the pulsar name, the logarithm of the characteristic age (yr) of the pulsar, the signal-to-noise of the timing activity, the range $\log(f/f_{max})$ over which the slope was estimated, the spectral slope (i.e., the power-law index) and its 1σ formal uncertainty. The last two columns give an indication of any consistency of the power spectrum slopes (NPPSS) or the results of the time-domain analysis (NPTDA) presented in D'Alessandro *et al* (1995) with one or more of the simple noise processes described earlier. A "?" indicates that there appears to be no consistency with such a process.

In order to check the validity of the derived slopes, time sequences corresponding to PN, FN and SN were generated using the same sampling pattern, signal-to-noise, time span, etc. as the data for each pulsar. The CLEAN technique was then applied to these time sequences to obtain their power spectra. The spectral slopes obtained for the PN, FN and SN simulations were in the range – 1.7 to –2.3, –3.8 to –4.3

and –5.7 to –6.3 respectively, with typical 1σ uncertainties of ~ 0.2 in individual estimates.

3. Discussion

The spectral slopes obtained in the previous section provide a useful comparison with the results obtained from previous analyses of timing noise using time-domain methods (D'Alessandro *et al.* 1995). The spectrum for PSR B0835-41 has a slope of ~ –2, consistent with a pure PN process. Likewise, the spectra for PSRs B1240-64, B1323-58, B1323-62 and B1706-16 have slopes of ~ –4, consistent with a pure FN process. The timing noise spectra for PSRs B0736-40, B0740-28, B1558-50, B1737-30, B1737-39, B1742-30 and B1749-28 also have power-law slopes, in the range –3.3 to 5.4. Regardless of whether or not these slopes are consistent with those expected for a pure random walk process (i.e., –2, –4, –6), they cannot be interpreted in a straightforward manner because the results of the time-domain analyses did not show consistency with such noise processes. The remaining six pulsars have composite power spectra that are most easily described using a two-component slope model. The time-domain analyses performed on four of these pulsars were inconclusive, while for the other two, they showed rough consistency with FN.

Although a single power-law slope of –3.6 adequately modelled the power spectrum for PSR B1558-50, there is some evidence of structure in the spectrum not dissimilar to one of the models proposed by Alpar *et al.* (1986). Power-law fits over the logarithmic frequency ranges –2.0 to –0.8 and –0.7 to –0.45 yield slopes of ~ 3, while the slope over the range –0.8 to 0.7 is ~ –9. Translated into a power spectrum of fluctuations in $\dot{\nu}$, this spectrum is similar to the Alpar *et al.* "mixed event" model (shown in figure 3a of their paper), although the slopes are not exactly the same. The "step" or "knee" in the spectrum is marginally significant (~ 2σ) and occurs at $\log(f/f_{\max}) \simeq -0.75$. At this point in the spectrum, $f \approx 1/\tau$ where τ is the relaxation timescale of the response to the triggering events (Alpar *et al* 1986). For PSR B1558-50, this timescale is approximately 340 days.

Three pulsars in the present sample overlap with the JPL pulsar sample analysed by Boynton & Deeter (1986) for power spectrum investigations, namely PSRs B0736-40, B1706-16 and B1749-28. The spectral slopes obtained for these pulsars are in good agreement with the estimates obtained by Boynton & Deeter. Cheng (1987b) has interpreted the Boynton & Deeter spectra for the latter two pulsars in terms of his SN/PN magnetospheric model, but there is no evidence of two such components in the spectra derived from the Mt Pleasant data for these pulsars. However, the extent of the spectra is not very large in the present case and the white noise may have masked the high frequency component which, in the Boynton & Deeter spectra, becomes significant at relatively nigh frequencies.

The applicability of the various theoretical models to observations of pulsar timing noise has been discussed by D'Alessandro *et al* (1995), based on the results of time-domain analyses. The power spectra presented in this paper also enable some conclusions to be drawn in this regard. It is clear that the timing noise of some pulsars is well described by a single power-law spectrum while for others, a composite spectrum is a more appropriate description. The models proposed by Alpar *et al.*

(1986), Cheng (1987a,b) and Jones (1990) predict a range of slopes in $P_{\dot{\nu}}(f)$, ranging from +2 to –2. The main limitation of these models is the fact that they only predict *even* spectral slopes (with the possible exception of the model proposed by Jones). This is because they assume that *purely white noise* exists in the *r*-th derivative of the phase. However, the power spectral estimates obtained in the present work, as well as those obtained by Boynton & Deeter (1986), show that the timing noise of some pulsars has spectral power varying as odd powers of the fluctuation frequency. Both the Alpar *et al.* and Cheng models can account for any single, "even" power-law slopes in the observed $P_{\phi}(f)$, if the range of timescales or fluctuation frequencies spanned by the data is restricted to that part of a composite spectrum. For example, in D'Alessandro *et al.* (1995), the timing noise of eight pulsars was found to be consistent with a PN process (one of these, PSR B0835-41, was confirmed by the power spectrum analysis) which can be explained using the microglitch model proposed by Alpar *et al.* (1986) and Cheng (1987b), in the limit of $f\tau \ll 1$. On the other hand, the Jones model only predicts slopes ≥ 4 in $P_{\phi}(f)$. The data for three out of the seven pulsars with composite power spectra, namely PSRs B0940-55, B1358-63 and B1641-45, can be accommodated by the magnetospheric SN/PN model proposed by Cheng (1987b), i.e., a red/blue composite spectrum in $\dot{\nu}$.

4. Conclusions

Spectral analysis of the residual pulse arrival times of pulsars is a useful tool for testing the theoretical models that have been developed to explain the timing noise observed in these objects. Estimates of the power spectrum of the phase residuals for 18 southern pulsars were obtained using a technique based on the CLEAN algorithm. In general, the derived spectra are well-described by a single or double-component power-law model. The power spectrum for PSR B1558-50 contains a marginally significant step which, if interpreted in terms of the "mixed event" model proposed by Alpar *et al.* (1986), implies a relaxation timescale of $\tau \geq 300$ days in response to the triggering events.

Together with the time-domain results obtained recently (D'Alessandro *et al.* 1995), the present work enables a number of conclusions to be drawn regarding the applicability of the three main theoretical models of pulsar timing noise, namely, those proposed by Alpar *et al.* (1986), Cheng (1987a, b; 1989) and Jones (1990). None of the models proposed by Alpar *et al.* (1986) are, by themselves, able to account for the range of microactivity evident in the pulsars studied. In particular, none of the Alpar *et al.* models are able to explain the occurrence of positive-going jumps in ν. However, these models do support observations in a few cases, for example, where the timing activity is consistent with phase noise, and the step in the power spectrum for PSR B1558-50.

There is considerably more support for the Cheng magnetospheric model, which incorporates the ideas of Alpar *et al.*, and the Jones corotating vortex model. Both of these models are able to account for the wide range of microjump event signatures, as well as single and composite power-law spectra of the timing noise. However, neither model can explain *all* of the observations. The magnetospheric model does not easily explain the fact that the bulk of the timing activity in a number of pulsars is due to a *small* number of microjumps in ν and $\dot{\nu}$ and the corotating vortex model does not explain a spectrum, or a spectral component, of the $\dot{\nu}$ fluctuations which is "blue".

Both the magnetospheric and corotating vortex models provide promising bases for a better understanding of pulsar timing noise. However, they may need to be modified in the light of these observational results in order that predictions can be sharpened. This is particularly relevant in the case of the latter model, for which a full quantitative solution is yet to be published.

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