J. Astrophys. Astr. (1996) **17**, 7–16

Application of 'CLEAN' in the Power Spectral Analysis of Non-Uniformly Sampled Pulsar Timing Data

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Received 1996 May 13; accepted 1996 May 30

Abstract. Spectral analysis of the residual pulsearrival times of pulsars is a useful tool in understanding the nature of the underlying processes that may be responsible for the timing noise observed from pulsars. Power spectra of pulsar timing residuals may be described by one or a combination of powerlaws. As these spectra are expected to be very steep, it is important to ensure a high dynamic range in the estimation of the spectrum. This is difficult in practice since one is, in general, dealing with timing measurements made at unevenly placed epochs. In this paper, we present a technique based on, 'CLEAN' to obtain high dynamic range spectra from unevenly sampled data. We compare the performance of this technique with other techniques including some that were used earlier for estimation of power spectra of pulsar timing residuals.

Key words: Pulsars: timing noise–power spectra– data processing: CLEAN algorithm.

1. Introduction

After allowing for the deterministic pulsar spindown ('pulsar braking') and any resolved period discontinuities (discrete events or 'glitches'), the timing residuals display irregularities in the pulsar rotation ('timing noise' or 'timing activity'), in excess of the estimated measurement uncertainties. It has been proposed that this variation is a result of the response of the neutron star to a 'noisy' torque (either magnetospheric or related to the moment of inertia), with an assumed simple power-law spectrum.

The 'random walks' seen in the timing residuals are thought to be the result of fluctuations in three observables – the pulse phase, ϕ ('phase noise', PN), frequency, ν ('frequency noise', FN) or frequency derivative, $\dot{\nu}$ ('slowing-down noise', SN). These processes have a 'red' power spectrum (i.e., excess power at low frequencies), and can be considered as a repeated integral of white noise. Since ϕ, ν and $\dot{\nu}$ are simply related by differentiation, the power density spectra are related by factors of f^2: $P_\nu(f) \sim f^{-2} P_\phi(f)$, $P_{\dot{\nu}}(f) \sim f^{-2} P_\nu(f)$. Deeter & Boynton (1982) use the terminology 'r^{th} order red noise', denoting a variable $x(t)$ which is the r-fold integral of white noise (i.e., the r^{th} time derivative, $x^r(t)$, reduces to white noise). Hence, the power spectrum of $x(t)$ obeys the law $P_x(f) \sim f^{-2r}$. The orders r =1, 2 and 3 correspond to phase, frequency and slowingdown noise respectively.

To investigate successfully all of the proposed noise processes over a frequency range of a decade, a dynamic range of at least six orders of magnitude must be attainable. Conventional Fourier transform (FT) techniques fail when they are used to estimate the spectral power density characteristic of red noise processes, particularly from a non-uniformly sampled time sequence.

A basic reason is that there is substantial power 'leakage' from the sidelobes of the equivalent power density estimators that can very easily mask any steep variations in the spectrum. While dealing with steep red spectra, simple FT techniques produce meaningless power spectra with a steepest power-law slope of ~–2.

The situation is further complicated due to the non-uniform sampling of the time series inevitably arising from practical astronomical observations. Interpolation of the data is inappropriate as the resulting 'jitter' introduces equivalent steps in. he time series that seriously affect the power spectrum estimation at the higher frequency end.

Hence, both of these issues need major consideration if one is to correctly recover red noise spectra from the timing data.

Detailed analyses of the noise in pulsar rotation have been undertaken by a number of workers. These are briefly reviewed in the context of the alternative method proposed in this paper.

1.1 *Analysis of timing noise: A review of techniques*

Boynton *et al.* (1972) were the first to publish work on timing noise, which was based on the first three years of the Crab pulsar timing data. Using standard Fourier techniques, they found that the timing residuals (after fitting a cubic and allowing for any glitches) had a power spectrum most closely resembling a frequency-jump noise model.

The difficulties involved in the estimation of power spectra using conventional techniques prompted workers to develop alternative (time domain) techniques to analyse the noise process, the methodology of which has been described by Groth (1975), Cordes (1980) and Cordes & Downs (1985).

Groth (1975) developed a new technique which accounted for effects such as non-uniform sampling and non-uniform data quality. The method consists of the expansion of the data in a set of orthonormal polynomials from which one can extract the slowdown parameters (and the degree to which they are contaminated by the noise) as well as a strength parameter for the noise process model which best fits the data. Hence, the method requires the input of an *assumed* model for the noise process, the result being a consistency check of the validity of the model. Application of this method to the Crab pulsar timing data also showed that the observed fluctuations in pulse phase was consistent with a random walk in the rotation frequency.

Cordes (1980) developed a method similar to that of Groth (1975), except that it uses the integrated variance rather than a decomposition of the variance into polynomial components. In the same way as Groth (1975), one assumes a model and tests for consistency with that model. However, Cordes points out that showing consistency of a random walk is a necessary but not sufficient condition for demonstrating that a random process is occurring in a pulsar's rotation. Cordes & Helfand (1980) applied this technique to the timing noise of 11 pulsars (from a sample of 50 pulsars). The results indicated that 2, 7 and 2 pulsars show a random walk in rotational phase, frequency and frequency derivative respectively.

Cordes & Downs (1985) analysed the pulse phase residuals and their derivatives in the time domain by examining the polynomial coefficients and residuals from polynomial fits made over a variety of data spans and time origins. The method was partly described in Cordes (1980) and used by Cordes & Helfand (1980), but it is augmented with a more sophisticated error analysis and through the study of structure functions of the phase. One of the arms of the structure function analysis is to determine whether the discrete events (found by Cordes & Downs for a number of pulsars) are the result of fluctuations in a random walk process, or other phenomena (either internal or external to the pulsar).

Other workers have approached the problem by obtaining estimates of the power spectra of the time series. Deeter & Boynton (1982) and Deeter (1984) have developed a general mathematical framework, specifically designed for non-uniformly sampled data, leading to a power density estimation technique which is valid for red powerlaw spectra. Their work is essentially an extension of the work of Groth (1975). The method uses orthonormal polynomials as power density estimators whose frequency response is such that leakage through the sidelobes of the transfer function is minimised (hence correct estimation of power density for processes which are "red"), while sacrificing frequency resolution to a certain extent. Their modest one octave frequency resolution is enough to identify features like the step in $P_{\dot{\nu}}(f)$ characteristic of a viscously-coupled crust-core model driven by white torque noise. A true power spectrum calculated in this manner allows power-law behaviour (over the available frequency range) to be tested directly rather than assuming this result as Groth (1975) and Cordes (1980) have done. This power spectrum technique has been applied to pulsar timing data by Boynton (1981), Boynton & Deeter (1986) and Deeter *et al.* (1989).

Power spectra are particularly useful for comparison with theoretical models of neutron star interiors, such as the vortex creep theory (Alpar *et al.* 1986). A flat spectrum is characteristic of rigid-body behaviour, whereas structure in the observed spectrum is characteristic of non-rigid-body behaviour. Alpar *et al.* (1986) have compared their theoretical predictions with the power spectrum in ν of 25 pulsars, obtained by Boynton & Deeter (1986). The results indicate that vortex unpinning is not the underlying cause of timing noise in the Crab and Vela pulsars, and possibly also unimportant for the other pulsars.

2. A technique for spectral estimation using ' CLEAN'

Non-uniform (or incomplete) sampling of a function limits the dynamic range of estimation of its Fourier components. This problem is routinely encountered while dealing with aperture synthesis data. There the sampling in the spatial frequency domain is incomplete and often nonuniform and one is interested in high dynamic range imaging. A technique called 'CLEAN' (Hogbom 1974) is commonly used to obtainhighdynamic rangeimages from the of ten patchysampling of visibilities. We see an almost directly analogous situation of this in the estimation of power spectra from non-uniformly sampled pulsar timing residuals. Considering this, we have attempted to investigate the possibility of using the basic 'CLEAN' algorithm for enhancing the dynamic range in the estimation of spectra from the timing residuals.

Let us assume $R(t)$ and $S(f)$ to be a Fourier transform pair where $R(t)$ is the true continuous time sequence of pulsar timing residuals and $S(f)$ is the spectrum of $R(t)$. The true power spectrum is obtained simply as $|S(f)|^2$. Let $\mathcal{X}(t)$ be the sampling function which is unity at the sampled epochs and zero elsewhere. The spectrum$S_D(f)$ of the sampled sequence $R'(t)$ [where $R'(t) = R(t).\ \mathcal{X}(t)$] can be written as

$$S_D(f) = \mathrm{S}(f) * X(f) \tag{1}$$

where $X(f)$ is a Fourier transform of$\mathcal{X}(t)$ and * denotes convolution. Our aim is to obtain an estimate of $S(f)$ given the estimates of SD and $X(f)$.

The 'dirty' spectrum S_D and the 'dirty' response function $X(f)$ are not available directly from observations. If the span of$\mathcal{X}(t)$ is T, then the 'dirty' response function can be estimated with a frequency resolution of $\Delta f = (1/T)$. However, it is desirable to oversample this response function and the 'dirty' spectrum by a factor of 2 or more to improve the performance of the deconvolution by 'CLEAN'. The extent in frequency (f_{max}) over which the spectral estimation may be performed is not unique in the case of a non-uniformly sampled time sequence. However, it can be argued that a more appropriate span corresponds to that implied by an average sampling rate for the time sequence, i.e., 0 to $N/(2T)$ where N is the number of time samples. The extent of the 'dirty' response function is then twice that of the spectrum to be 'CLEANed'. With this understanding, we can compute the functions $S_D(f)$ and $X(f)$ at discrete frequencies by Fourier transforming the measured time sequence of the pulsar timing residuals and the sampling function respectively.

This operation is performed by summing the spectral contributions from each of the sampled points of the time sequence (i.e., $S_D(f) = \Sigma_i R(t_i) \exp(2\pi i f t_i)$). In this way, gridding of the time samples is not required, avoiding the possible phase jitters due to quantized sampling intervals.

It should be noted that the complex spectra thus obtained are hermitian symmetric innature, unlike in the case of aperture synthesis data. The'CLEAN'algorithm to be used therefore needs the following minor modifications: (i) while searching for the maximum in the spectrum to be 'CLEANed', phases in the spectrum are ignored, i.e., the location of the maximum spectral amplitude is found. But the actual contribution at that location is considered including the phase; and (ii) the search for the maximum is made only over one half of the spectrum but subtraction of a scaled (by a complex quantity) version of the response function is performed over both halves of the spectrum after accounting for the hermitian symmetric contribution. This makes the algorithm somewhat faster and, more importantly, ensures the hermitian symmetry in the 'CLEANed' spectrum. Given that the extent of the 'dirty' response function is twice that of the so called 'dirty' spectrum, the contributions from hermitian symmetric partners always overlap over the entire span of the 'dirty' spectrum. Hence, low values of 'loopgain' are used to avoid possible instabilities that would otherwise occur particularly while 'CLEANing' features close to the 'zero-frequency'.

The resulting 'CLEANed' spectrum corresponds to a time sequence which is an interpolated and/or extrapolated version of the original non-uniformly sampled time sequence, while being consistent with the original time sequence at the epochs of measurement. As we are not looking for superresolution in the spectrum, the 'CLEANed' spectrum is to be restored to a resolution which is approximately the original resolution (i.e., $1/T$). In applications such as aperture synthesis imaging, the 'CLEANed' versions are restored to a desired resolution by convolving the 'CLEANed' components in the image with a Gaussian 'CLEAN' beam (i.e., without any sidelobes). In the present case too, spectral smoothing with a Gaussian would be satisfactory if the spectrum is to be viewed on a linear frequency scale. As mentioned in an earlier section, the spectra of pulsar timing residuals may more likely be of power-law nature, making restoring functions with long tails undesirable. Hence, we have used a half-a-cycle cosine bell as the restoring function, with a half-power width close to the original resolution. (It is worth noting that our use of the cosine bell for restoration avoids the possibility of any interchannel leakage particularly close to the 'zero-frequency' in the spectrum unlike when a Gaussian function is used.) After the 'CLEANed' complex spectra are restored to a desired resolution, the power spectra are computed in the usual way.

3. Simulations and results

To judge the performance of the technique described above, we have applied it to simulated time sequences corresponding to steep red spectra and we find the results very encouraging. The simulated time sequences were generated for 5 cases, namely, white noise, phase noise, frequency noise, slowing-down noise and a test sine wave. This was done first with uniform sampling. Figure 1 shows the set of simulated patterns in the first four cases. It is worth mentioning that these data simulated in the time domain do not, in general, obey periodic boundary conditions as is the case for real data. Also, P and P were fitted to these sequences and the corresponding 'second-order baseline' contribution was removed as would be done for real data. We added 0.1% white noise to the simulations of PN, FN and SN to

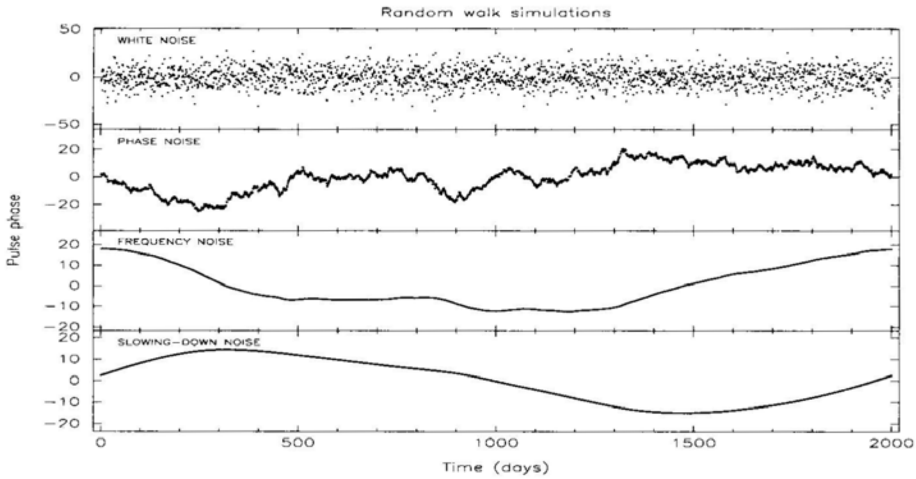

Figure 1. Uniformly sampled time sequences which simulate white noise (WN), phase noise (PN), frequency noise (FN) and slowingdown noise (SN) respectively, with 0.1% of white noise added to the sequences for PN, FN and SN.

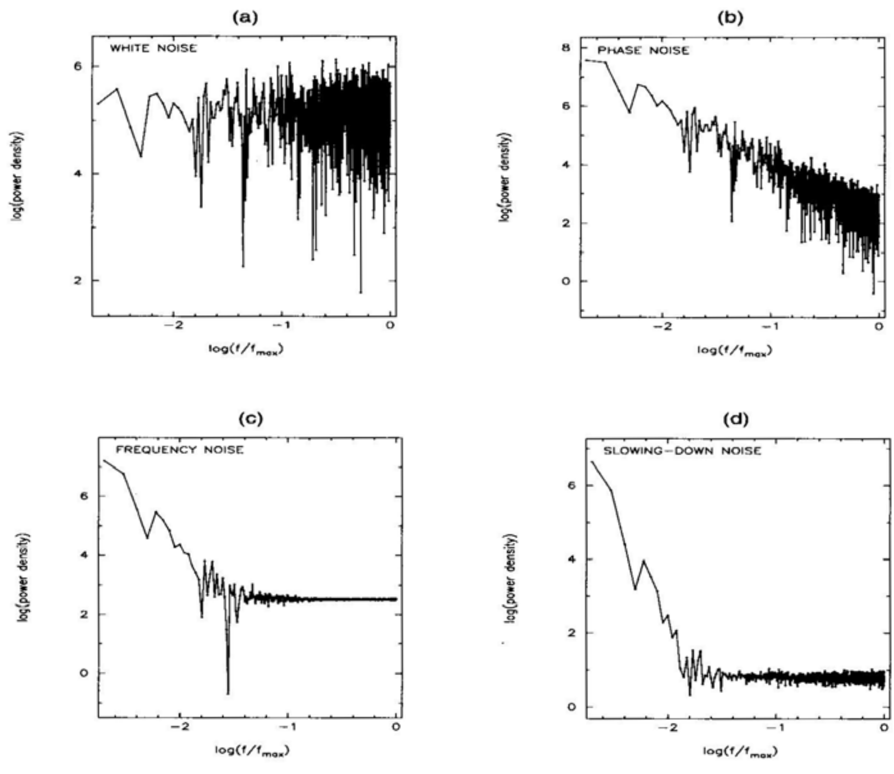

Figure 2. Power density spectra of the simulations shown in Fig. 1. The plots show log(power density) as a function of log (f/f_{max}) in the four cases.

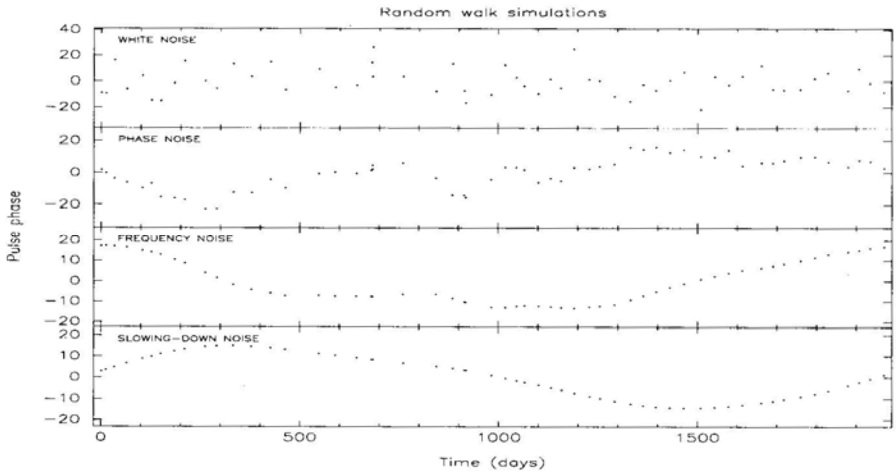

Figure 3. Simulations of white noise, phase noise, frequency noise and slowing-down noise, sampled according to the observation epochs for PSR 1641 – 45.

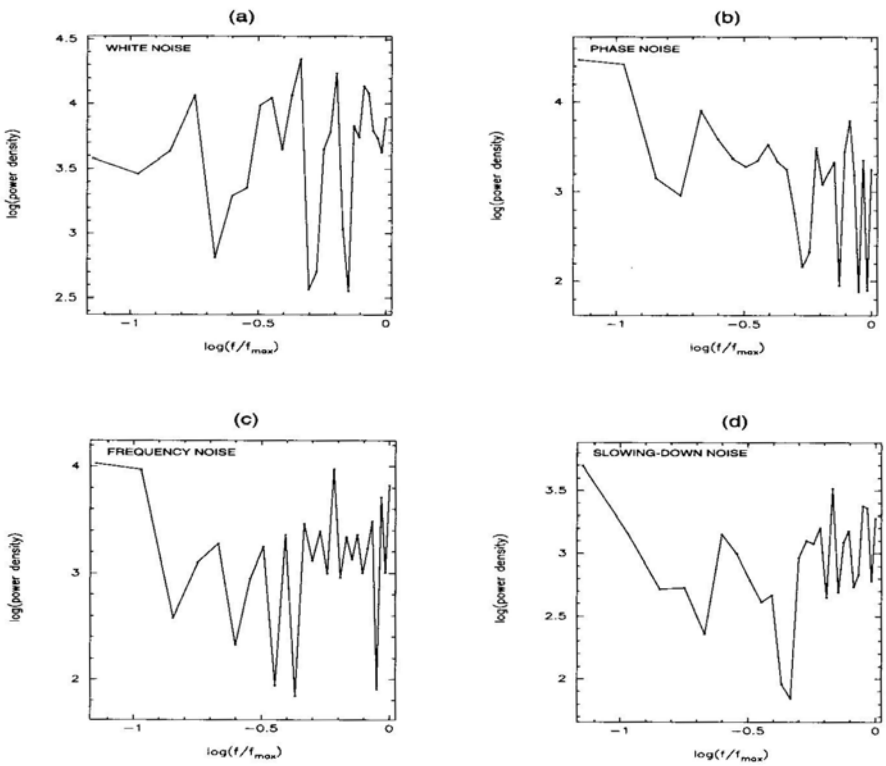

Figure 4. Power density spectra of the non-uniformly sampled time sequences shown in Fig. 3. The spectra were obtained using conventional discrete Fourier transform techniques.

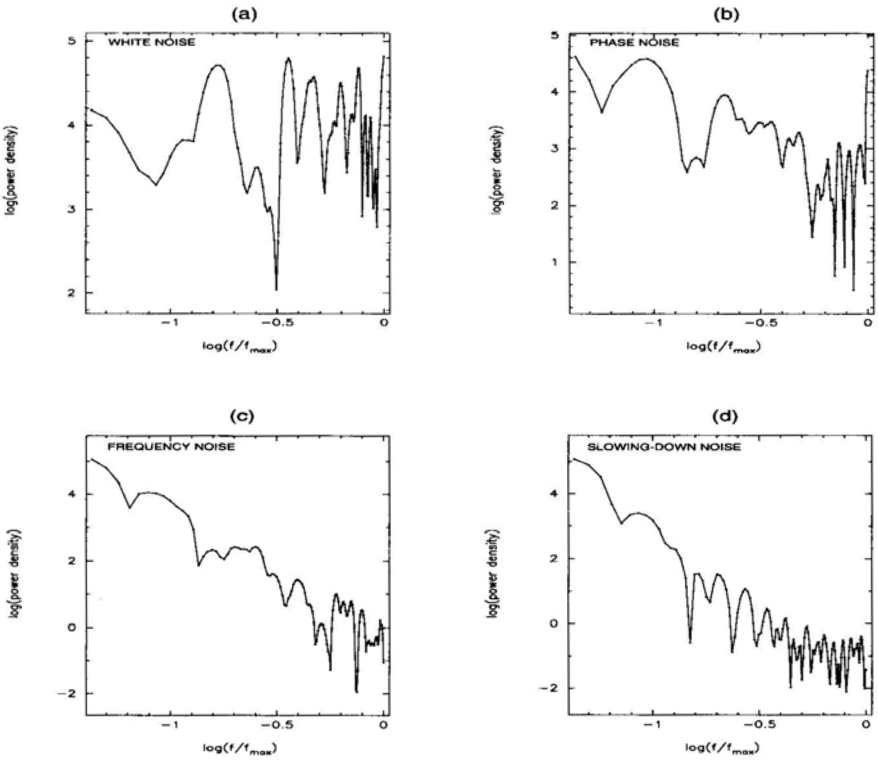

Figure 5. Power density spectra of the non-uniformly sampled time sequences shown in Fig. 3. The spectra were obtained using the technique based on ‘CLEAN’.

demonstrate the effect of even a small amount of measurement error on the high frequency end of the power spectrum. Our ‘CLEAN’ procedure was used on this set of simulated data and we confirmed that our procedure gives the expected output power spectra in the case of uniformly sampled time sequences. Figure 2 shows these power spectra. The non-uniformly sampled versions were obtained from the above simulated time sequences by using sampling functions that we encounter in practice. Figure 3 shows such versions when we used the sampling pattern that we have for PSR 164–145 from our observations at the Mt. Pleasant Observatory (D’Alessandro *et al.* 1993). Each of the ‘dirty’ spectra was ‘CLEANed’ down to the expected spectral contribution from the measurement error in the timing residuals. The procedure was seen to converge within typically a few hundred iterations when a loopgain of 0.1 was used.

We subjected these data to a number of different procedures, including the technique based on ‘CLEAN’, in order to evaluate their performance. The other procedures include: (i) interpolation of the time sequence atepochs spaced at regular intervals using a polynomial and then the use of a standard FFT on the interpolated data, (ii) use of the LombScargle periodogram method for non-uniformly sampled data (Press & Rybicki 1989), (iii) use of suitable-order harmonic fits to the time

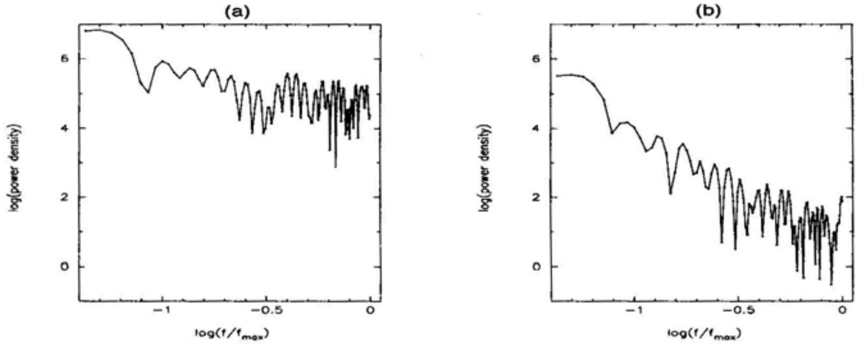

Figure 6. The power density spectrum estimated from the phase residuals for PSR 1641 – 45 using: (**a**) conventional discrete Fourier transform methods, and (**b**) the 'CLEAN' technique.

sequence giving the best fit estimates of the power spectrum (similar to the method used by Boynton *et al.* 1972), and some valiants of these. Of these other procedures, the third method was found to perform much better than the other two. However, even in this case, the slopes of the reconstructed red spectra were consistently lower than those expected.

Figures 4 and 5 show the power spectra before and after we apply the 'CLEAN' procedure respectively. The improvement in the dynamic range and the quality of reconstruction due to the 'CLEANing' is dramatic. However, the reconstruction of the spectra at the higher frequency end of the spectrum is comparatively poor.

It should be pointed out that the sampling function we have chosen in the present case, although free of any large gaps, has severe non-uniformity and should be treated as a situation close to the worst case of non-uniform sampling. If large gaps comparable to the total time span itself exist in the sampled data, it is more appropriate to use the portions of the time sequences that avoid such gaps.

In any case, realistic measurements would include measurement uncertainties that contribute a white noise component in the spectrum, masking the steep drops in the spectral power towards the higher frequency end of the red spectra. After including random measurement noise in our simulations, we find that even a modrate amount of the noise dominates the contributions at the high frequency end of the spectrum.

Hence, we consider the performance of our procedure as satisfactory, since it reconstructs the steep spectra very well over the more relevant (lower) frequency range and with a dynamic range exceeding 6 orders of magnitude.

4. Conclusions

In this paper, we have explored a suitably modified form of the 'CLEAN' technique foruse in power spectral analysis of pulsar timing residuals. This technique is shown to overcome the already noted problems of dynamic range limitations in obtaining reliable power spectra from non-uniformly sampled time sequences. Using this technique, we have obtained estimates of the power spectrum of the timing residuals in pulse phase for a number of southern pulsars. The complete results of this analysis will be published elsewhere.

Figure 6 shows a sample 'CLEANed' spectrum of the phase residuals for PSR 1641–45. Comparison with the 'dirty' spectrum clearly demonstrates the dynamic range improvement achieved by our technique in the spectral estimation of non-niformly sampled time sequences of pulsar timing residuals.

Acknowledgement

A. A. D. thanks A. P. Rao of NCRA, Pune for making the basic suggestion that led to the work reported in this paper. We would also like to thank R. Nityananda of RRI, Bangalore, for his critical comments on the manuscript and an anonymous referee for his suggestions to clarify some important points related to the implementation of this technique.

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