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Effect of Strong Magnetic Field on Cosmic Quark-Hadron Phase Transition and Baryon Inhomogeneity

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Abstract. The effect of intense magnetic field on the cosmic quark-hadron phase transition and also on the baryon number inhomogeneity has been investigated using phenomenological MIT bag model for the quark sector. For the sake of simplicity an ideal gas equation of state has been considered for the hadronic phase.

Key words: Cosmic quark-hadron phase transition—Baryon number inhomogeneity—equation of state—Landau levels—quark nuggets.

1. Introduction

In the past few years a lot of work has been done on the effect of strong magnetic field on various properties of dense astrophysical and cosmological matter. These works are mainly related to the emissions from magnetized neutron matter (Kaminker *et al*. 1991, 1992), electron-induced electromagnetic and weak processes (Shul'man 1991), and the Landau diamagnetism and Pauli paramagnetism of dense nuclear matter (Shul'man 1991; Fushiki *et al*. 1992). Very recently the effects of such intense magnetic field on the primordial nucleo-synthesis and also on the expansion rate of the universe micro-second after the Big Bang have been studied by Schramm *et al*. (Cheng *et al*. 1993, 1994) and Grasso & Rubinstein (1994). In recent years some interesting work has also been done on quantum electrodynamics in the presence of strong external magnetic field (Persson & Zeitlin 1994; Zeitlin 1994; Danielsson & Grasso 1995) and also on the stability of matter in the presence of strong magnetic field (Vshivtsev & Serebryakova 1994; Elliott *et al*. 1995). In a series of publications we have also reported some new results on the effect of strong magnetic field on first order quark-hadron phase transition at the core of a neutron star. In particular, we have studied the stability of strange quark matter, equation of state of such stable phase and also the nucleation of quark bubbles in metastable neutron matter in the presence of strong magnetic field. We have seen that strange quark matter becomes more stable in the presence of strong magnetic field and the equation of state of magnetized strange quark matter differs significantly from that of a non-magnetized one. The most interesting observation was that there cannot be any quark droplet nucleation in metastable neutron matter if the strength of the magnetic field is strong enough to make the quarks populate the Landau levels (Chakrabarty & Goyal 1994; Chakrabarty 1994, 1995a, b).

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In all these reported results, mentioned above, the possible effects of strong magnetic fields on cosmic QCD phase transition have not been considered. Initially the universe is at a high temperature and in the quark-gluon plasma phase. The net baryon number of the universe resides entirely in the primordial quark soup. Due to cosmic expansion the temperature of the quark soup decreases and near the critical temperature, T_c, a transition to colour singlet hadronic phase takes place. During phase separation the baryon number is transported to the hadronic phase mainly by the formation of nucleons which are the lightest baryons. The other possible hadrons are mostly light mesons, which do not carry the baryon number. Now, the quarks in the quark gluon plasma are almost massless and carry 1/3 baryon number, whereas the mass of the lightest baryon, the nucleon, is ~ 940 MeV. A simple thermodynamic calculation shows that the baryon number solubility of primordial quark soup is much higher than that of hadronic matter at the critical temperature T_c. This gives rise to inhomogeneity in the baryon number distribution. This particular aspect has also not been studied in the presence of strong magnetic field. The aim of this paper is to investigate the effect of strong magnetic field on both cosmic quark-hadron phase transition and baryon inhomogeneity.

In § 2 we give the basic thermodynamic formalism of quark matter in the presence of strong magnetic field. In sections 3 and 4 we study the effect of strong magnetic field on the cosmic QCD phase transition and on the baryon inhomogeneity. Section 5 contains the conclusion of this work.

2. Formalism

As is well known, the energy of a charged particle changes significantly in the quantum limit if the magnetic field strength is equal to or greater than some critical value $B_m^{(c)} = m_i^2 c^3/(q_i h)$ in Gauss; where m_i and q_i are respectively the mass and charge (absolute value) of the particle (e.g., $q_i = 2e/3$ for u-quark, $e/3$ for d and s quarks, here $e = |e|$ is the absolute value of electronic charge), h and c are respectively the reduced Planck constant and velocity of light, both of which along with the Boltzmann constant k_B are taken to be unity in our choice of units. For an electron of mass 0·5 MeV, the strength of this critical field as mentioned above is $B_m^{(c)(e)} = 4{\cdot}4 \times 10^{13}$ G, whereas for a light quark of current mass 5 MeV, this particular value becomes ~ $10^2 \times B_m^{(c)(e)}$, arise for s-quark of current mass 150 MeV, it is ~ 10^{20} G, which is too high to realize at the core of a neutron star. But the possibility of such high magnetic field cannot be discarded in the case of the early universe (Vachaspati 1991). The critical magnetic field as defined above is the typical strength at which the cyclotron lines begin to occur, and in this limit the cyclotron quantum is of the order of or greater than the corresponding rest energy. This is also equivalent to the requirement that the de-Broglie wavelength is of the order of or greater than the Larmor radius of the particle in the magnetic field.

To study the cosmic QCD transition in the early universe in the presence of a strong magnetic field, we have considered the conventional MIT bag model. For the sake of simplicity we assume that quarks move freely within the system. The current masses of both u and d-quarks are assumed to be extremely low (in our actual calculation we have taken the current mass for both of them to be 5 MeV, whereas for s-quark, the current mass is taken to be 150 MeV).

For a constant magnetic field along the z-axis ($\vec{B}_m$ $B_m = B_{m,(z)} = B_m$ = constant), the single particle energy eigen value is given by

$$\varepsilon_{k,n,s}^{(i)} = [k^2 + m_i^2 + q_i B_m (2n + s + 1)]^{1/2}, \tag{1}$$

where n = 0, 1, 2,...., being the principal quantum numbers for allowed Landau levels, $s = \pm 1$ refers to spin up (+) or down (–) states and k is the component of particle momentum along the direction of external magnetic field. Setting $2v = 2n + s + 1$, where v = 0, 1, 2,..., we can rewrite the single particle energy eigen value in the following form

$$\varepsilon_v^{(i)} = [k^2 + m_i^2 + q_i B_m 2v]^{1/2}. \tag{2}$$

Now it is very easy to show that the $v = 0$ state is singly degenerate while all other states with $v \neq 0$ are doubly degenerate.

The general expression for the thermodynamic potential of the system at temperature $T = \beta^{-1}$ is given by

$$\Omega = -T \ln Z = \sum_i \Omega_i V + BV, \tag{3}$$

where Z is the grant-partition function and the explicit form of the thermodynamic potential density for the ith species is given by

$$\Omega_i = -\frac{Tg_i}{(2\pi)^3} \int d^3 k \ln (1 + \exp (\beta (\mu_i - \varepsilon_i))), \tag{4}$$

where g_i is the degeneracy of the ith species (= 6 for a quark or an antiquark), B is the bag pressure and V is the volume occupied by the system. The sum in equation (3) is over u, d, s-quarks and their antiparticles. We assume that for antiparticles the chemical potential $\bar{\mu}_i = -\mu_i$ i.e. they are in chemical equilibrium with respect to annihilation processes.

Now let us consider the necessary changes to be made in equation (4) if a strong external magnetic field is present in the system. Here we shall investigate the magnetism arising from quantization of orbital motion of charged particles in the presence of a strong magnetic field. We know that if the magnetic field is along z-axis, the path of the charged particle will be a regular helix whose axis lies along the z-axis and whose projection on an x-y plane is a circle. If the magnetic field is uniform, both the linear velocity along the field direction and the angular velocity in the x-y plane will be constant, the latter arises from the constant Lorentz force experienced by the particle. Quantum mechanically the energy associated with the circular motion in the x-y plane is quantized in units of $2q_i B_m$. The energy associated with the linear motion along the z-axis is also quantized; but in view of the smallness of the energy intervals, they may be taken as continuous variables. We thus have equation (1) or (2) as single particle energy eigen value. Now, these magnetized energy levels are degenerate because they result from an almost continuous set of zero field levels. All those levels for which the values of the quantity $k_x^2 + k_y^2$ lie between $2q_i B_m v$ and $2q_i B_m (v + 1)$ now coalesce into a single level characterized by the quantum number v. The number of these levels is given by

$$\frac{S}{(2\pi)^2} \iint dk_x \, dk_y = \frac{S q_i B_m}{2\pi}, \tag{5}$$

here S is the area of the orbit in the x-y plane. This expression is independent of ν. Then, in the integral of the form $\int d^3 kf(k)$, we can replace $\iint dk_x\, dk_y$ by the expression given above, whereas the limit of k_z, which is a continuous variable, ranges from $-\infty$ to $+\infty$. Then we can rewrite equation (4) for the thermodynamic potential density in the presence of a strong magnetic field in the form

$$\Omega_i = -T\frac{q_i g_i B_m}{2\pi^2}\sum_{\nu=0}^{\infty}\int_0^{\infty} dk_z \ln\left(1+\exp(\beta(\mu_i-\varepsilon_\nu^{(i)}))\right). \qquad (6)$$

We have used equation (4) for s and $\bar{s}$ if $B_m < 10^{20}\,G$ and equation (6) for u, d-quarks and their anti-particles. Whereas for $B_m > 10^{20}\,G$, equation (6) has been used for all the constituents. In equation (4), $\varepsilon_i = (k^2 + m_i^2)^{1/2}$, whereas in equation (6), it is given by equation (2).

The expression for kinetic pressure of the system is given by

$$P = -\sum_i \Omega_i, \qquad (7)$$

and the number density of the ith species is given by

$$n_i = \left(\frac{\partial\Omega_i}{\partial\mu_i}\right)_T. \qquad (8)$$

3. Cosmic quark-hadron phase transition

Assuming for the sake of simplicity that only nucleons and pions are present in the hadronic phase and that they obey the free hadronic gas equation of state, the conditions to be satisfied by the coexisting phases are

$$\mu_p = \mu_d + 2\mu_u, \qquad (9)$$

$$\mu_n = \mu_u + 2\mu_d, \qquad (10)$$

$$\mu_\pi = 0, \qquad (11)$$

$$P_h = P_q, \qquad (12)$$

and

$$T_h = T_q = T_c. \qquad (13)$$

The chemical equilibrium conditions are also valid for anti-quarks and anti-hadrons i.e. we can write down equations similar to equations (9) and (10) for $\bar{q}$, $\bar{n}$ and $\bar{p}$ (where $q = u$ or d). We also assume that the condition of β-equilibrium is satisfied in both the quark-gluon and hadronic phases. For the anti-particles, the chemical potential $\bar{\mu}_i = -\mu_i$. The expressions for kinetic pressures are then obtained from equation (7) by replacing Ω_i either by equation (4) or (6), depending on whether $B_m < B_m^{(c)(i)}$ or $> B_m^{(c)(i)}$, respectively.

In Fig. 1 we have shown the phase diagram (n_B vs T diagram) for such a mixed phase of quark-gluon plasma and hadronic matter. The dotted curve is for $B_m = 0$, whereas the solid curve is for $B_m = 10^3 \times B_m^{(c)(e)}$. This figure shows that the phase diagram changes significantly in the presence of a strong magnetic field.

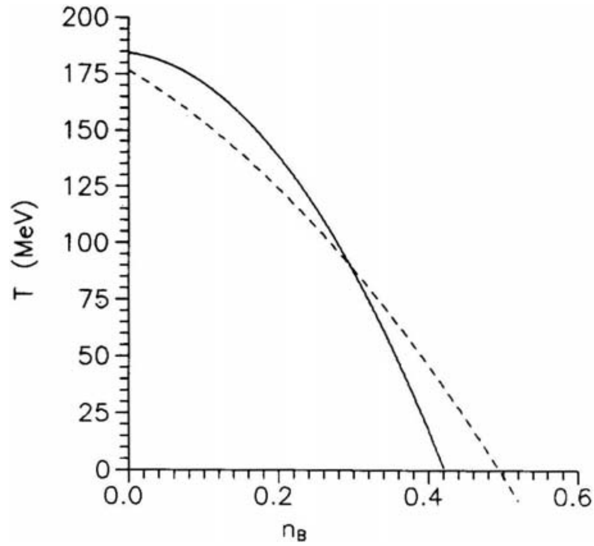

Figure 1. Phase diagram ($T - n_B$ plot) for a mixture of QGP and hadronic matter. The dotted curve is for $B_m = 0$ and the solid one is for $B_m = 10^3 \times B_m^{(c)(e)}$.

4. Baryon inhomogeneity

Before the quark-hadron phase transition in the early universe, the quark gluon plasma had almost an equal number of quarks and anti-quarks, since the baryon number of the universe is so small,

$$\frac{n_B}{n_\gamma} \sim 10^{-9}, \tag{14}$$

where n_B ad n_y are respectively the net baryon number and photon number densities in the early universe. This ratio is too small to affect the dynamics or the evolution of the early universe. Since the present universe is matter dominated, all the anti-baryons must have been annihilated with their counterparts.

Now in the quark gluon phase, the baryon number is carried by the quarks, which are almost massless (except s-quark, whose mass is ~ 150 MeV). On the other hand, in the hadronic sector, the lightest baryons are nucleons of mass ~ 940 MeV. Therefore the baryon number prefers to reside in the quark gluon phase during phase transition (Witten 1984), or in other words, the baryon number solubility of primordial quark soup is much higher than that of hadronic phase at T_c. We can estimate the ratio of baryon number densities in these two phases at T_c, given by

$$R = \frac{n_q^{(B)}}{n_h^{(B)}}, \tag{15}$$

A simple non-relativistic thermodynamic calculation (Witten 1984) shows that $R \sim \exp(M_h/T_c)$, where M_h is the hadronic mass $\gg T_c$. Therefore the ratio R is a large

Table 1.

$B_m/B_m^{(c)(e)}$	T_c(MeV)	$n_q^{(B)}/n_h^{(B)}$
0·0	100	455·4
	150	33·4
	180	14·7
	200	9·9
10^2	100	2732·69
	150	200·66
	180	88·49
	200	59·21
10^3	100	2783·01
	150	201·39
	180	89·03
	200	61·35
10^5	100	51908·2
	150	1695·85
	180	520·75
	200	285·51

number. It has been shown that $R \approx 6$ or 100 for T_c = 200 or 100 MeV respectively (Kurki-Suonio 1988, 1991; Fuller *et al*. 1988; Kapusta & Olive 1988).

Since the net baryon number of the universe is small, $\mu_B = 3\mu_q \to 0$. Then the ratio R is given by

$$R = \lim_{\mu_q \to 0,\, \mu_B \to 0} \frac{1}{3} \frac{(\partial\Omega_q/\partial\mu_q)_{T=T_c}}{(\partial\Omega_B/\partial\mu_B)_{T=T_c}}. \tag{16}$$

Using the explicit form of Ω_i ($i = q$ or B), it is very easy to show that in the limiting condition $n_B \to 0$, the ratio R reduces to 0/0 form. Then by L'Hospital's theorem, we have

$$R = \lim_{\mu_q \to 0,\, \mu_B \to 0} \frac{1}{3} \frac{(\partial^2\Omega_q/\partial\mu_q^2)_{T=T_c}}{(\partial^2\Omega_B/\partial\mu_B^2)_{T=T_c}}, \tag{17}$$

which is not in the 0/0 form.Using the explicit form of the second derivatives as written above we have calculated R for four different values of T_c (= 100, 150, 180 and 200 MeV) and for B_m = 0, $10^2 \times B_m^{(c)(e)}$ $10^3 \times B_m^{(c)(e)}$ and 10^5 $B_m^{(c)(e)}$. In Table 1 we have shown the variation of R with T_c and B_m. The variation of R with T_c has already been reported and the results presented here are consistent with those of old published data. The variation of R with B_m has not been done before. If $B_m > 10^2 \times B_m^{(c)(e)}$, R changes significantly from zero field values. If this is true for the early universe, this could possibly lead to a very high baryon number inhomogeneity, which could modify the picture at the end of the quark-hadron phase transition. It is also obvious that the presence of a strong magnetic field favours the formation of quark nuggets during the early universe quark-hadron phase transition, which is expected to be one of the natural candidates for baryonic dark matter.

5. Conclusions

We have seen that in the presence of a strong magnetic field greater than the corresponding critical field, the phase diagram for a mixture of quark gluon plasma and hadronic matter differs significantly from that for zero magnetic field case.

The other interesting observation is that the baryon number inhomogeneity in the early universe increases by a few orders of magnitude if the magnetic field strength becomes greater than $10^2 \times B_m^{(c)(e)}$.

If this theoretical prediction is correct, then it may have some effect on the primordial nuclear abundances and also on the formation of quark nuggets during the first order quark hadron phase transition in the early universe, which in turn affects the Big Bang nucleo-synthesis.

Here we have given a very simplified picture of the baryon number transport during the first order cosmic quark hadron phase transition in the presence of a strong magnetic field. Such calculations should only be used to indicate the order of magnitude of the baryon number penetrability and a rough comparison with zero magnetic field case.

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