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Lense-Thirring Precession of Radio Pulsars

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Abstract. It is shown that if neutron stars contain dynamically decoupled components, most plausibly discrete superfluid zones, then it is possible for the spin axes of these components o become slightly misaligned with respect to the crustal spin following a series of glitches. The crust will then undergo Lense-Thirring precession about the total angular moment with a period of $\sim 6 - 7.5P$ (assuming a crustal superfluid) and $\sim 3 - 6P$ (if the core superfluid is not tightly coupled to the crust). The precise precessional period is diagnostic of the mass distribution within each component. The implications of recent observational inferences concerning glitching pulsars are discussed. The conditions necessary for precession to be observable are analysed phenomenologically and a search of pulse-timing data for evidence of a Lense-Thirring modulation within the period range $\sim 3 - 8P$ is proposed.

1. Introduction

In this talk I will present a summary of a quite speculative possibility that Paolo Coppi and I have thought about over the past five years. To be specific, we suggest that neutron star interiors contain dynamical components which are quite weakly coupled to the crust and which can become misaligned following glitches. If this really can happen, (and we concede that most expert opinion is that it cannot), then the interior and the exterior undergo mutual Lense-Thirring precession which may be detectable as a tiny, periodic modulation of the pulsed signal. As the cost of attempting to measure this precession using a satellite orbiting the Earth may be five orders of magnitude more expensive than a pulsar experiment, I contend that a modest search for this effect in suitable pulsars is worth a try even if the odds are long. A fuller discussion of these ideas is contained in Blandford & Coppi (1994, in preparation).

Let me begin with a phenomenological approach to describing pulsar glitches that eschews microscopic theory. What we observe is that pulsars slow down in a more or less regular fashion, except that, occasionally, they undergo glitches - sudden increases in angular frequency. In young pulsars at least, these are also followed by a significant, though temporary, increase in the rate at which they slow down. Beyond this, the details vary in a rather confusing manner from pulsar to pulsar and glitch to glitch, although, as we have heard here from Andrew Lyne, some systematics are starting to emerge. Now pulsars are neutron stars with moments of inertia $I \sim 10^{45}$ g cm ^{2} and it seems highly unlikely that an external torque (which would have to be several orders of magnitude larger than the steady decelerating electromagnetic torque and always have the same sign) can be responsible for glitches. For this reason, internal changes in the angular momentum are now universally invoked to explain large glitches.

There appear to be two generic models that can account for observed glitches. I shall call them the "Spin-exchange" model and the "Starquake" model. In the simplest version of the spin-exchange model, the neutron star has two separate components each with a fixed moment of inertia. One component, which I call "crust", is rigidly attached to the external magnetic field which causes it to decelerate and its angular frequency, $\Omega(t)$, is what radio astronomers measure. The second component, which I name "superfluid" is weakly coupled to the crust so its angular frequency Ω_2 is somewhat larger than Ω. (It is not necessary that it be an actual superfluid, but this is by far the most likely possibility based upon our understanding of neutron star structure.) During a glitch, the coupling between these two components becomes very strong for a short while and there is a rapid transfer of spin angular momentum from the superfluid to the crust, which conesquently speeds up. After the glitch, the difference in angular velocity between the two components is reduced, along with the weak torque that couples them. The crust therefore decelerates more rapidly under the action of the electromagnetic torque until the lag in angular frequency recovers sufficiently to re-establish an equilibrium where both components decelerate at the same rate or another glitch takes place. During this whole process, the change in total angular momentum is the integral over time of the external torque and just what it would have been had there had been no glitch.

In the elementary starquake model, there are again crustal and superfluid components but this time the crustal moment of inertia is rot constant and during a glitch, it suddenly decreases causing the crust to start spinning a little faster. This then activates a crust-superfluid torque which causes the crust to decelerate more rapidly, simultaneously accelerating the superfluid until, once more, it shares a common rotation with the crust or another glitch occurs. Again the variation of the total angular momentum is unaffected, but the average angular velocity will increase following a glitch because the total moment of inertia decreases.

Now, for both of these models it has been generally assumed that when the angular velocities of the crust and one or more superfluid components differ, they do so only in magnitude, not in direction. However it would seem very strange if either the application of a sudden braking torque or the rearrangement of the superfluid were strictly axisymmetric with respect to the angular velocity and surely, this is not going to be the case. Therefore the assumption that all angular frequencies are parallel is really equivalent to assuming that, if we decompose the immediate post-glitch torque into three components, one aligning, one precessing and one decelerating, then the first does its job very quickly, making the second irrelevant and leaving us only the effects of the third torque to observe. It has never been clear to me why spin alignment has to be this rapid and so the question that I now address is "What happens if the post-glitch angular velocities become misaligned?"

2. Lense-Thirring Precession

Neutron stars are relativistic objects and when they rotate, they "drag" local inertial frames with them - the Lense-Thirring effect. In the post-Newtonian limit, this effect can be thought of as an additional non-Newtonian, gravitational torque acting in a flat space. It is convenient to make the gravitoelectric/gravitomagnetic decomposition of the post-Newtonian equations of motion (*e.g.*Thorne, Price & MacDonald 1986). A spinning ring of matter with mass current density ρv acts as a source for a "gravitational vector potential" β,

$$\nabla^2 \boldsymbol{\beta} = \frac{16\pi G}{c} \rho \mathbf{v}. \tag{1}$$

This vector potential is related in the usual manner to the gravitomagnetic field h= $\nabla \times \beta$. For simplicity, just confine attention to a spherically symmetric distribution of matter $\rho_1(r_1)$ spinning with uniform angular velocity Ω_1 about the z axis. (We can think of component. 1 as being the crust and label the superfluid with the subscript 2.) The vector potential at a point labelled using a radial coordinate r_2 and a polar angle θ_2 measured from the same z axis is given by

$$\beta_\phi(r_2, \theta_2) = -\frac{16\pi G}{3c} \Omega_1 \sin\theta_2 \int dr_1 r_1^3 \rho_1(r_1) \frac{\min[r_1, r_2]}{\max[r_1^2, r_2^2]} \tag{2}$$

Now add a second component, also labelled 2 and which also has a spherically symmetric mass distribution. These components may interpenetrate one another. The total torque that body 1 exerts upon it is obtained by taking a moment of the "Lorentz force"

$$\begin{aligned} \mathbf{G}_{12} &= \int d^3 r_2 \rho_2(r_2) \mathbf{r}_2 \times \left[\frac{\mathbf{v}_2(\mathbf{r}_2)}{c} \times \mathbf{h}(\mathbf{r}_2) \right] \\ &= \int d^3 r_2 \rho_2(r_2) [\mathbf{r}_2 \cdot \mathbf{h}(\mathbf{r}_2)] \frac{\mathbf{v}_1(\mathbf{r}_2)}{c}. \end{aligned} \tag{3}$$

After substituting for h and some further manipulation, this can be cast in the form

$$\mathbf{G}_{12} = K_{LT} \frac{G}{c^2} \frac{\mathbf{S}_2 \times \mathbf{S}_1}{R^3} \tag{4}$$

where R is the total stellar radius and K_{LT} is a dimensionless structure constant given by

$$K_{LT} = \frac{2R^3 \int_0^R dr_1 r_1^3 \rho_1(r_1) \int_0^R dr_2 r_2^3 \rho_2(r_2) \left[\frac{\min[r_1, r_2]}{\max[r_1^2, r_2^2]} \right]}{\int_0^R dr_1 r_1^4 \rho_1(r_1) \int_0^R dr_2 r_2^4 \rho_2(r_2)}. \tag{5}$$

Note that $G_{12} = -G_{21}$, consistent with Newton's third law and that both components undergo a mutual Lense-Thirring precession about the total angular momentum **J**, given by

$$\boldsymbol{\Omega}_{LT} = K_{LT} \frac{G}{c^2 R^3} \mathbf{J} \tag{6}$$

Note also that, for a given star, the predicted precessional period P_{LT} is a fixed multiple of the spin period P

Coppi and I have computed this structure constant for a variety of assumptions about the internal structure of the neutron star and a variety of equations of state. If we assume that component 2 is a crustal superfluid and the total stellar mass is 1.4 $M_\odot$ with radius in the interval 7 km< R < 16 km, then

$$5.9P \lesssim P_{LT} \lesssim 7.4P \tag{7}$$

where the variation is mostly attributable to the variation of the total mass distribution. Alternatively, if component 2 is a core superfluid, then

$$3.2P \lesssim P_{LT} \lesssim 6.1P \tag{7}$$

These two cases ought to be distinguishable.

Before continuing, I should list some uncertainties in the technical calculation, even granted its premise. Firstly, the post-Newtonian approximation is only marginally valid and a fully relativistic calculation may change the answer by as much as 30 percent if the equation of state is soft. Secondly, there is a genuinely Newtonian contribution to the precession in addition to the Lense-Thirring term. This is caused by the Newtonian gravitational attraction of the misaligned rotational bulges of the two components. This can actually be computed and the result is

$$\Omega_N = K_N \frac{T_{rot}}{|W|} \Omega_2 \tag{9}$$

where T_{rot} is the rotational kinetic energy and $|W|$ is the total gravitational potential energy. Typically $K_N \sim K_{LT} \sim 2$ and, to order of magnitude, $P_N / P_{LT} \sim (P/P_{min})^2$ where P_{min} is the shortest possible pulsar period. This contribution need only be considered further for millisecond pulsars and its small size emphasizes that we are truly in a regime where general relativistic effects are important. Thirdly, there is ample evidence that there are more than two components (*e.g.*Lyne, Graham-Smith & Pritchard 1992 and below). This multiplies the dynamical possibilities (*cf.*Hamilton & Sarazin 1982) but does not change the general principles.

3. The Interpretation of Glitching Pulsars

3.1 *Observations of Glitches*

Glitching pulsars have been carefully monitored ever since the first example was discovered in the Vela pulsar in 1969 (Radhakrishnan & Manchester 1969, Reichley & Downes 1969). For completeness, let me summarise the salient points of the observations which are discussed more comprehensively here by Lyne. The largest glitches involve relative frequency jumps $\Delta\Omega/\Omega \sim 4 \times 10^{-6}$ (Lyne 1987) and associated relative frequency derivative changes that can be as large as $\Delta\dot{\Omega}/\dot{\Omega} \sim 0.4$ (Chau *et al.*1993). Glitches seem to happen very quickly, in one instance, in less than two minutes (McCulloch *et al.*1990). As far as can be measured, post-glitch recoveries can be described by the superposition of simple exponential functions with time constants that vary from ~ 10 hr. to much longer than the inter-glitch interval. (Sometimes a quadratic fit is adopted for long recoveries, but for our purposes, it is simpler to replace this with an incomplete exponential recovery.) Up to five distinct components may be involved in a single glitch. The fraction of the relative frequency jump that is recovered after a glitch, known as Q, is quite small in old pulsars, though it can approach, or even exceed, unity in young pulsars such as the Crab pulsar. A recent discovery (Lyne, these proceedings) is that, although young pulsars provide the most spectacular examples of period changes, it is their middle-aged counterparts that are *relatively* most active. On statistical grounds, it appears that over a long time interval the accumulated period *increase* attributable to large glitches is roughly two percent in magnitude of the total period *decrease* associated with the external electromagnetic torque. In the case of the Vela pulsar, this translates to one glitch, with $\Delta\Omega/\Omega \sim 3 \times 10^{-6}$, every ~ 3 yr. which is roughly what has been observed over the past ~ 25 yr.

3.2 *The spin exchange model*

3.2.1 Dynamics

A natural phenomenological interpretation of the observed exponential recovery is that the relative frictional torque between two components varies in direct proportion to their relative angular velocity (*cf.*Michel, Bland-Hawthorn & Lyne 1990). Let us suppose that there are only two components, a crust and a superfluid, that are related in this manner. Let the crust have a moment of inertia that is a fraction f of the total stellar moment of inertia. The equations of motion for the two components can be cast in the form

$$f\dot{\Omega} = \dot{\Omega}_e + \frac{f(1-f)\omega}{\tau} \tag{10}$$

$$(1-f)\dot{\Omega}_2 = -\frac{f(1-f)\omega}{\tau}; \quad t > 0 \tag{11}$$

where we drop the subscript 1 for the crustal component and $\omega \equiv \Omega_2 - \Omega$ is the angular velocity lag. (In this section I assume that all spins are parallel.) Combining these two equations, we identify the constant $\dot{\Omega}_e < 0$ with the steady spin down rate when $\dot{\omega} = 0$. Equivalently, it is the steady electromagnetic torque divided by the total stellar moment of inertia. (This quantity is actually time-dependent but, for present purposes, it is not necessary to include this refinement.) The solutions of these differential equations for the observable $\Omega(t)$ and the unobserved $\omega(t)$ are

$$\Omega = \Omega_- + \Delta\Omega + \dot{\Omega}_e t - Q\Delta\Omega(1 - e^{-\frac{t}{\tau}}) \tag{12}$$

$$\omega = \frac{-\dot{\Omega}_e \tau}{f} - \frac{Q\Delta\Omega}{(1-f)} e^{-\frac{t}{\tau}} \tag{13}$$

where $\Delta\Omega$ is the observed frequency jump and

$$Q = \frac{(\dot{\Omega}_e - \dot{\Omega}_0)\tau}{\Delta\Omega}, \tag{14}$$

the frequency recovery, appears as a constant of integration. The definition of Q involves $\dot{\Omega}_e$, which is not directly observed in an individual glitch and Ω_0 the angular frequency derivative measured immediately after the glitch. It differs from the conventional definition when glitches do not recover completely (see below) but is equivalent to the normal definition when glitches do recover and $\dot{\Omega}_e \equiv \dot{\Omega}_-$. Now we also observe the crust angular frequency just prior to the glitch, Ω_- and if we assume that angular momentum is conserved during the glitch, with no change in the moments of inertia, then the unobserved angular velocity lags just before (ω_-) and immediately after (ω_0) the glitch are related by

$$\omega_- = \omega_0 + \frac{\Delta\Omega}{(1-f)} \tag{15}$$

We can use these equations to express the angular velocity lag observed before, immediately after the glitch and long after its recovery ω_+ (if this happens before another glitch), in terms of the fractional moment of inertia of the superfiuid component.

$$\omega_- = -\frac{\dot{\Omega}_e\tau}{f} + \frac{(1-Q)\Delta\Omega}{(1-f)} \tag{16}$$

$$\omega_0 = -\frac{\dot{\Omega}_e\tau}{f} - \frac{Q\Delta\Omega}{(1-f)} \tag{17}$$

$$\omega_+ = -\frac{\dot{\Omega}_e\tau}{f} \tag{18}$$

The relative decrease in the angular velocity lag between the two components over a fully-recovered glitch is then given by

$$\frac{\omega_- - \omega_+}{\omega_+} = \left(\frac{1-Q}{Q}\right)\left(\frac{f}{1-f}\right)\left(\frac{\Delta\dot{\Omega}}{\dot{\Omega}}\right) \tag{19}$$

3.2.2 Application to the Vela pulsar

Now let us apply this analysis to the observations of the Vela pulsar (Cordes, Downes & Krause-Polstorff 1988, Chau *et al.*1993). (A similar application has been made to the frequently glitching pulsar PSR 1737–30 by Michel *et al.* 1990.) Nine glitches have been observed with varying precision and we are due for a tenth. For illustration purposes, let me take the eighth glitch which occured on December 24 1988 (Flanagan 1990, McCulloch *et αl.*1990, Alpar Pines & Cheng 1990) as this has received most scrutiny and has had a long recovery. It also seems to be typical in its recovery. (It should be emphasised that different groups derive significantly different timing models when analysing the same glitch, at least partly due to the insidious influence of timing noise Cordes *et al.*1988, Alpar *et αl.*1992. However the qualitative conclusions that I shall draw are not influenced by these differences.)

Flanagan's model for the eighth glitch resolves the initial frequency jump of $\Delta\Omega/\Omega = 1.8 \times 10^{-6}$ into three superposed frequency jumps, each of which recovers exponentially with a separate time constant. If we try to apply the two-component, Spin-exchange model and assume complete recovery between glitches, then we should focus on the largest and slowest component, for which $\Delta\dot{\Omega}/\dot{\Omega} = 0.0029$, $Q = 0.018$, $\tau \equiv \tau_3 = 96$ d, and treat the other two jumps as perturbations. Adopting, the preglitch parameters of $\Omega_- = 70.4$ rad s^{-1}, $\dot{\Omega}_- = -9.8 \times 10^{-11}$ rad s^{-2}, Eq.(19) gives

$$\frac{\omega_- - \omega_+}{\omega_+} = 0.16\left(\frac{f}{1-f}\right) \tag{20}$$

Now Vela macroglitches recur, on average every 2.8 yr, and the present slowing down timescale is $-\Omega/\dot{\Omega} = 2.3 \times 10^4$ yr. If this glitching behaviour is to recur for a third of the slowing down timescale, say, then we require $(\omega_- - \omega_+)/\omega_+ \lesssim 4 \times 10^{-4}$ or $f \lesssim 2 \times 10^{-3}$. One physical way in which this condition may arise is if glitches only happen when ω exceeds a critical value that varies smoothly with Ω. However, the condition is really more general than this. Using Eq.(20), the crust, together with all of the star to which it is coupled, would have $f \lesssim 2\times 10^{-3}$, a very small fraction of the total stellar moment of inertia.

We can see why this is required in a quite direct manner. Immediately prior to the first observed glitch in 1969, when the star was (by hypothesis) dynamically relaxed, the frequency was $\Omega_1 = 70.43196$ rad s^{-1}. Just before glitch eight, when the star should also have been relaxed, $\Omega_8 = 70.37177$ rad s^{-1}, $\dot{\Omega}_8 = -9.79066 \times 10^{-11}$ rad s^{-1}, $\ddot{\Omega}_8 = 2.6 \times 10^{-21}$ rad s^{-2}. The interval between these two glitches is $t_8 - t_1 = 6.3 \times 10^8$ s. Now suppose, still by hypothesis, that the total moment of inertia of the star, $I = 10^{45}\, I_{45}$ g cm^2, does not change and the torque $N(t)$ is unaffected by the glitches. We can estimate the rotational impulse due to the external torque by Taylor expanding about glitch 8.

$$\int_{t_1}^{t_8} dtN = [\dot{\Omega}_8(t_8 - t_1) - 0.5\ddot{\Omega}_8(t_8 - t_1)^2]I = -6.17 \times 10^{43} I_{45}\text{g cm}^2\text{s}^{-1} \tag{21}$$

This ought to equal the decrease in the total stellar angular momentum. The actual decrease is given by

$$\Delta J = (\Omega_8 - \Omega_1)I = -6.02 \times 10^{43}\, I_{45}\text{g cm}^2\text{s}^{-1} \tag{22}$$

Even if we make a generous allowance for the uncertainty in the long term timing model, we find that the star has somehow recovered a fraction 0.021 ± 0.005 of the spin angular momentum extracted by the electromagnetic torque. Therefore, one of our assumptions must be wrong.

We can locate the probable source of the problem by noticing that $\ddot{\Omega}$ is still unexpectedly large just before glitches. This suggests that there is a longer time scale coupling than τ_3 which does not proceed to completion. This invalidates the assumption that we made in deriving Eq.(18) namely that $\dot{\omega} \to 0$ before the next glitch. So, let us now treat the 96 d recovery as a perturbation and remove it as well and try to model the incomplete recovery which we treat as exponential with time constant τ_2. Operationally we can estimate τ_2 by measuring the best value of $\ddot{\Omega}$ for the long term recovery from Chau. *et al.*(1993) and correcting for the expected rate of change of Ω_e, to obtain $\ddot{\Omega} \simeq 3 \times 10^{-21}$ rad s^{-3}. The recovery time is then given by differentiating Eq. (10), (11).

$$\tau_2 = \frac{\dot{\Omega}_e - \dot{\Omega}_0}{\ddot{\Omega}} \simeq 20\mathrm{yr} \tag{23}$$

where $\dot{\Omega}_e$ can be estimated from the total change of Ω over a long interval, *i.e.*

$$\dot{\Omega}_e \simeq \frac{\Omega_8 - \Omega_1}{t_8 - t_1} \simeq -9.63 \times 10^{-11}\mathrm{rad\ s}^{-2} \tag{24}$$

and $\dot{\Omega}_0 = -9.84 \times 10^{-11}$ rad s^{-2}, (Flanagan 1990)

We can now determine when the angular velocity lag will recover to its pre-glitch value using Eq.(13),(14), to obtain a time interval

$$t = \tau_2 \ln\left(\frac{\dot{\Omega}_e - \dot{\Omega}_0}{\dot{\Omega}_e - \dot{\Omega}_-}\right) \simeq 6\mathrm{yr} \tag{25}$$

This is about twice the actual time that elapsed, but considering the crudity of the assumptions is not discouraging. What this calculation does make clear is that, if the recovery is incomplete, then it is physically possible under the spin exchange model for glitches to recur over the whole lifetime of the pulsar. A global re-analysis of the whole Vela data set is probably necessary to see if this interpretation makes sense. In the limiting case that the superfluid decouples completely from the crust and $\tau_2 \to \infty$, then the lag will obviously recover in a time interval $\Delta\Omega/(\dot{\Omega}_e - \dot{\Omega}_0)$.

To what extent does adopting this model constrain the crustal moment of inertia? Unfortunately, not much. This can be seen directly from Eq.(10) which allows us to compute the crustal moment of inertia for different assumed values of ω_0. If the frictional torque is ignorably small, then $f \sim 0.98$. However, a difference as large as $\omega_0 = 0.7$ rad s^{-1} gives $f \sim 0.1$. In conclusion, unless we make some additional assumption about the dynamics, changes in the inferred crustal moment of inertia are indistinguishable from changes in the crust-superfiuid frictional torque under the spin-exchange model with incomplete recovery and we cannot deduce f directly from the observations, only an upper bound ~ 0.98 (*cf.*Alpar *et al.*1993).

3.2.3 Relaxation of secondary components

As an aside, we can go back and try to interpret the three small recoveries from the eighth Vela glitch on the assumption that each one is associated with the coupling of these additional components to the crust of additional superfluid components with a small fractional moment of inertia f_i, i= 3,4,5 to the crust; the coupling to superfluid component 2 is assumed, for simplicity, to be negligible. Now the dynamics is dominated by the exchange of spin between the crust and component 2. The perturbations associated with the interaction with component i is given by

$$\delta(\Delta\dot{\Omega})_i = -\frac{f_i}{f}\frac{\Delta\Omega}{\tau_i} \tag{26}$$

¿From this we infer that

$$\frac{f_i}{f} = Q_i \tag{27}$$

For component 3 with τ_3 = 96 d, I obtain $f_3/f \sim Q_3 \sim 0.02$ and find that the total angular frequency jump in the glitch is ~ 0.16 times the equilibrium angular frequency difference between component 3 and the crust. For component 4, with $\tau_4 \sim 4$ d, $f_4 \sim 0.004f$ and the crust changes during the glitch from trailing behind component 4 by ~ 3×10^{-5} rad s^{-1} to leading by ~ 10^{-4} rad s^{-1} For component 5, which recovers in ~ 10 hr, $f_5 \sim 0.005f$ and the frequency jump, is ~ 40 times the equilibrium lag. The combined moment of inertia of these three additional superfluid components is only ~ 3 percent of the total crustal moment of inertia. (It is also possible that there are only two additional components and one of the three recoveries involves a separate frictional interaction with component 2.)

3.2.4 Physical interpretation of the spin-exchange model

The most popular microphysical interpretation of the spin exchange model (*e.g.*Alpar *et al.*1992) involves a crustal superfluid weakly coupled to the remainder of the star. (The core neutron superfluid is believed on theoretical grounds to be strongly coupled to the superconducting protons which in turn are argued to be coupled magnetically to the crust *e.g.*Sauls 1989). Now the velocity of a superfluid is proportional to the gradient of its Bose-condensed wave function and it is therefore curl-free. This means that the vorticity associated with the observed angular velocity has to be concentrated in a series of quantised vortex lines of normal fluid, each of which has carries a single quantum of vorticity of magnitude $h/2m_n$ so that the number of vortex lines per unit area is $n = 4m_n\Omega/h$ (*e.g.*Feynman 1972). The typical separation of vortex lines in the case of the Vela pulsar is ~ $n^{-1/2} \sim 40\mu$. These vortex lines are believed to be be pinned onto lattice sites in the solid crust so that there has to be a potential flow of genuine superfluid around them. This flow exerts an outwardly directed Coriolis plus Magnus force acting on the vortex lines which, in turn acts upon the lattice of $f = -\rho_s(2\Omega + \xi) \times (\omega \times r)/n$ per unit length of vortex line, (where ρ_s is the superfluid density and ξ is the vorticity measured in the frame of the crust). This is equivalent to a force density acting upon the lattice of $F = -2\rho_s\,\Omega_2 \times (\omega \times r)$ averaging over many vortex lines if the fluid is uniformly rotating. However, the force is transmitted to the lattice primarily at lattice points where the stress can be far greater than the average value. When this stress becomes large enough, there is supposed to be a catastrophic failure and a large number of vortex lines move radially outward, despinning the superfluid and transferring angular momentum to the lattice.

There are various opinions concerning the strength of the pinning interaction and the nature of the failure. The interaction is variously characterized as essentially rigid (*e.g.*Ruderman 1993), of intermediate strength (*e.g.*Alpar *et al.*1993) and negligible (*e.g.*Jones 1991). (It is possible for each of these interpretations to be correct for some component of some pulsar.) Ruderman (1993) attributes glitches to mechanical failure of the solid crust when subjected to strains $\gtrsim 10^{-4}$. Following a glitch, the whole crust is forced bodily outward through $\delta r \sim R\Delta\Omega_2/\Omega \sim$ several meters, releasing a strain ~ $\delta r/R \sim 10^{-4}$, which will be re-established in time for the next glitch. There must be a compensatory upwelling near the rotational poles and subduction near the equator. In the limiting case when the superfiuid spins down only at glitches, $f \sim 0.98$. although some apparent recovery associated with an increase in the moment of inertia of the crust as it is re-stressed is permitted. This model is consistent with the reported statistical rate of glitches in older pulsars.

Alpar *et al.*(1992) argue that that there is a catastrophic unpinning of vortex lines from the crust, again over a large area, so that the vortex lines move outward through the crust. The kinematic consequences are broadly similar to those in Ruderman's model. In between glitches, the vortex lines creep steadily through at least some of the crust in response to the applied stress rather like dislocations which can be trapped on impurities. This creep is associated with the partial recovery and will happen far more readily at higher temperatures. This is qualitatively consistent with the observation that younger and presumably hotter pulsars like the Crab pulsar exhibit smaller, fractional glitching activity and greater post-glitch recoveries. For older pulsars, like PSR 0355+54 (Lyne 1987) the recovery is quite small.

An additional complication is associated with the interior magnetic field, that is also quantised and whose dynamical evolution has proved to be no less controversial than that of the vorticity. Bhattacharya *et al.*(1992) have argued that magnetic and vortex lines cannot intercommute. If magnetic flux is rigidly pinned to the outer crust and if vortexfield crossing is strictly forbidden then this would also require that the core and crust be tightly coupled; otherwise the magnetic field strength in the core will double in a time $\sim \omega_+^{-1}$ s. In addition, the core magnetic field must be expelled as the crust decelerates. Alternatively, a weak resistance to flux - vortex line crossing could be responsible for dynamical relaxation.

3.2.5 Precession of pinned superfluid

I have reported a divergence of published opinion as to whether or not the vortex lines associated with a superfluid component are pinned to the crustal lattice. If the pinning interaction is ignorable, then the fluid will behave purely classically and the vorticity will be Lie-transported by the fluid in the normal manner familiar from geophysical applications. However, there is a relativistic complication in that the wave function is defined relative to locally non-rotating inertial reference frames and these are dragged differentially by the spinning crust, leading to considerable stretching of the vortex lines. I suspect that the end result will be to damp out all but the component of vorticity parallel to the crust spin axis on a relatively short timescale. This needs further study. Obviously, if the vortex lines are loosely pinned, then there will be strong dissipation of both the parallel and the perpendicular components of ω. Observationally this appears not to be the case.

However, if pinning is strong, the vorticity will be Lie-transported by the crust. For the moment, let us ignore all relativistic effects including the Lense-Thirring precession. If we transform into the frame of the crust which rotates which rotates with a uniform angular velocity Ω with respect to an inertial frame. Let the velocity of the superfluid in this frame be u(r) and the residual vorticity in this frame (after subtracting off 2Ω from the vorticity measured in the inertial frame) be $\xi(r)$. The secondary flow can then be determined (under the Boussinesq approximation) by solving

$$\nabla \times u = \xi, \quad (29)$$
$$\nabla \cdot u = 0. \quad (29)$$

Now we can, in principle, solve the differential equation for the secondary flow u, given a particular vorticity source by introducing a vector potential w, where $u = \nabla \times w$ and a gauge condition $\nabla \cdot w = 0$, so that $\nabla_2 w = -\xi$. This could then be solved subject to boundary conditions such as vanishing of the radial velocity component on the spherical boundaries of the superfluid zone. It does not matter if ξ is not parallel to Ω. For the simple case of uniform rotation, $u = \xi \times r/2$. However, as we expect the vorticity to evolve in a quite irregular manner, the flow is likely to be more complicated. Given a stationary velocity field, we can then solve for the density variations. The normal equation of motion for fluid in a rotating frame must now be augmented with the force density that the lattice exerts on the fluid which cancels the Coriolis plus Magnus force and which allows the vorticity to be constant in the frame of the crust. This leaves

$$\nabla(P + \frac{1}{2}u^2) = \mathbf{g}\left(\frac{\delta\rho}{\bar{\rho}}\right) \quad (30)$$

where g is the gravitational plus centrifugal acceleration and $\bar{\rho}\ \nabla P$ is the true pressure gradient minus $\bar{\rho}$g (*e.g.*Hide 1978). By combining with the equation of state of the superfluid, it is possible to solve for the self-consistent density and pressure fluctuations in a given vorticity field. Finally, having derived this flow, it is principle possible to repeat the exercise of section 2 and obtain the slower, mutual, Lense-Thirring precession. I would make the conjecture that the precession angular frequency is most sensitive to the radial location of the superfluid and the overall stellar structure and relatively insensitive to the details of the vorticity distribution and consequently the estimates of section 2 will still be valid. However more detailed calculations are needed to study this. Furthermore, whatever the nature of the superfluid-crust coupling, and there are many intriguing possibilities including creep, electron scattering and Eckman pumping, (*e.g.*Bildsten & Epstein 1989, Epstein, Baym & Link 1993, Reisenegger 1993), I believe that it will diminish the perpendicular and parallel angular velocity differences from steady deceleration on similar timescales.

3.3 *The starquake model*

3.3.1 Dynamics

Now let us see if these glitches can be interpreted adopting the starquake model. As an abrupt spin up is observed, the crust must undergo the catastrophic change in its moment of inertia, $\Delta f = -f\Delta\Omega/\Omega$. Again, let us make a two component model (*cf.*Baym *et al.*1969). The post glitch relaxation will proceed according to Eq.(10), (11) but with different initial conditions. Adopting the same notation, for the completely relaxed, simple two component model, we obtain

$$\Omega = \Omega_- + \Delta\Omega + \dot{\Omega}_e t - (1-f)\Delta\Omega(1 - e^{-\frac{t}{\tau}}) \quad (31)$$

$$\omega \doteq \omega_- - \Delta\Omega e^{-\frac{t}{\tau}}; \qquad t > 0 \quad (32)$$

where $\omega_+ = \omega_- = -\dot{\Omega}_- \tau / f$. We see immediately that, unlike with the completely relaxed, spin-exchange model, the frequency difference between the two components does not have to decrease through the glitch and the recovery is $Q = 1 - f$. Note also that the equilibrium frequency difference ω_+ is indeterminate. When the coupling is strong, $(\omega_+ < \Delta\Omega)$, the crust will spin faster than the superfluid temporarily when it is weak the lag can be quite large.

For an incompletely relaxed starquake, if we continue to adopt a linear coupling and denote the interglitch interval by Δt, then we see that the preglitch. Angular frequency lag will adjust to an average value

$$\omega_+ = \omega_- = \frac{-N\tau}{f} - \frac{\Delta\Omega}{e^{\frac{\Delta t}{\tau}} - 1} \quad (33)$$

In the limiting case, $\tau >> \Delta t$, the superfluid will be almost completely decoupled from the crust and will follow a quite independent history.

3.3.2 Application to the Vela pulsar

Let me repeat the exercise of tentatively applying this model to the eighth Vela glitch. Again, I consider two subcases. If I adopt $= \tau_3 = 96$ d as the relaxation time and regard the longer timescale partial relaxation as a sort of crustal rebound, then the "crustal" fraction of the moment of inertia has to be $f \sim 0.98$. This requires the core to be tightly coupled to the crust, consistent with theoretical dogma. The pre-glitch angular frequency lag between the crust-core and what is presumably a crustal superfluid is $\omega_= - \Omega_- \tau_3 / f = 6 \times 10^{-4}$ rad s^{-1}. Furthermore, we can adapt the argument we have just given to consider the angular momentum budget over 20 yr to compare the initial spin with the final spin corrected by the angular momentum extracted electromagnetically to estimate that the permanent reduction in the crust-core moment of inertia is

$$\frac{\Delta f}{f} = -2.14 \times 10^{-5} \quad (34)$$

Assuming a partial recovery, I again adopt $\tau_2 \sim 4$ yr and $Q_2 \sim 0.41$ to deduce that $f \sim 0.6$ for the crust and $1 - f \sim 0.4$ for the superfluid. In addition, the pre-glitch angular velocity difference is now $\omega_- = 0.02$ rad s^{-1}, much larger than the observed frequency jumps.

Taking the long term view, I calculate the angular momentum just prior to the eighth glitch allowing for the angular velocity difference of the two components to be $S_7 = 7.096 \times 10^{44}\ I_{7,45}$g cm^2 s^{-1}. Just prior to the first glitch, it was $S_1 = 7.102 \times 10^{44}\ I_{1,45}$ g cm^2 s^{-1}. The rotational impulse will, again, be given by Eq.(21) and so, in this case, the fractional change in the crustal moment of inertia over the ~ 20 yr interval is

$$\frac{\Delta f}{f} = -2.4 \times 10^{-5} \quad (35)$$

similar to before. Therefore, in either case, the crustal moment of inertia appears to change at a fractional rate ~ 10^{-6} per year. If this has happened for a time ~ $\Omega/3\dot{\Omega}$ then there must have been a total fractional reduction ~ 10^{-2} in the crustal moment of inertia over the pulsar lifetime. The interpretation of the secondary jumps should be broadly similar to that described under the spin-exchange model.

3.3.3 Physical interpretation of the starquake model

It has long been appreciated that the frequency of giant glitches in the Vela pulsar precludes attributing the structural changes to intermittent relaxation of the rotational bulge as the star decelerates. This is because the moment of inertia of the crust of the rotational bulge can only diminish at a rate $f / f \sim R^3\Omega\dot{\Omega}/GM \sim -10^{-9}$ yr^{-1}, some three orders of magnitude smaller than required. On these grounds, the starquake interpretation has been generally rejected. However, this seems unwarranted as there are alternative explanations of mechanical disequilibrium. For example, we can suppose that there is a large magnetic field within the core. The composition of the inner crust remains a source of mystery (*e.g.*Baym 1993, Pethick & Ravenhall 1992), but the shear modulus there might be as large as ~ 10^{32} dyne cm^{-2}. The yield strain is equally uncertain, but adopting a value of ~ 0.01, we find that an anisotropic magnetic stress below the neutron star surface of ~ 10^{30} dyne cm^{-2} ~ 10^{-4} of the total pressure and therefore able to support a non-axisymmetric mass distribution giving $\Delta f/f \sim 10^{-4}$, enough for ~ 100 Vela glitches. If the magnetic field changes its configuration very few hundred years, then starquakes large enough to sustain even Vela glitches might result.

An alternative appeal to our ignorance of the actual conditions within neutron star cores is to posit that they have solid phases which can crack, independent of magnetic and rotational stresses *cf.*Pines *et al.*1972).

4. Observability of Lense-Thirring Precession

The purpose of the foregoing analysis of glitches was to determine the conditions under which a significant component of spin might become misaligned with the total angular momentum and undergo Lense-Thirring precession. On *phenomenological* grounds, I have argued that if a pulsar like Vela glitches according to the spin-exchange model, then a completely relaxed recurrent glitching behaviour requires the superfluid to possess the major fraction of the moment of inertia of the star, but that if the glitches do not heal to give a constant lag, then as much as 98 percent (in the case of Vela) of the moment of inertia is associated with the core. On the starquake model, complete recovery suggests a superfluid moment of inertia ~ 2 percent of the total, whereas partial recovery is consistent with a more even partition of the mass between the two phases.

By comparison, *theory* asserts quite confidently that there should be distinct superfluids in the core and the crust and slightly more hesistantly that the core superfluid is strongly coupled to the crust. Both of these statements are in need of observational test and I would argue that observations of glitching pulsars, while naturally interpretable in these terms, do not yet provide such a test. Theory also goes on to speculate about the nature of vortex line pinning, unpinning, and creeping and crust cracking but here the details remain controversial. The problems are genuinely hard and their laboratory analogues have proved difficult to analyse.

¿From a phenomenological perspective, all of these models allow the parallel component of angular velocity of the major superfluid components to be significantly different from that of the crust. On either the spin-exchange or the starquake model, I expect that the angular velocity jumps will in general not be parallel to the total angular momentum and that the linear, frictional coupling I have invoked will produce a torque that is antiparallel to the difference in the vector angular velocity just as effective in diminishing the perpendicular component of ω as the parallel component. (On theoretical grounds, I have speculated that the parallel component will dissipated very quickly if the vortex lines are unpinned.) Consequently, if glitches fully recover, the mean square precession angle for the crustal angular momentum can be estimated by $< \alpha^2 > \sim < (\Delta\Omega/\Omega)^2 >$, immediately after the glitch and this will diminish to zero on a timescale τ. Under these circumstances, Lense-Thirring precession will be extremely hard to observe.

However, when the glitches do not heal on the inter-glitch timescale, as the observations suggest may actually be the case, then α can build up. If it increases stochastically, then in a steady state.

$$< \alpha^2 >= R\tau \left\langle \left(\frac{\Delta\Omega}{\Omega} \right)^2 \right\rangle \tag{36}$$

where R is the glitch rate. Typically $10^{-5} \lesssim \alpha \lesssim 3 \times 10^{-4}$. However, glitching need not be stochastic. For example, under the starquake model, a large density anomaly might relax through a series of glitches that always displace the angular velocity away from J. In this case, even larger precessional angles might be possible. In the limiting case of complete decoupling, the precessional angle will reflect conditions soon after the birth of the star and is essentially inestimable.

How small a precessional angle are we likely to be able to observe? In the absence of a detailed theory of pulsar emission, it is not possible to answer this question. However, some indication is given by examining the average variation of the emission with longitude, the pulse profile. That such a profile exists and has long-term stability suggests very strongly that the permanent field geometry will also imprint itself on the latitudinal variation of the radio emission. If we just consider a pulsar precessing uniformly about J, and introduce Euler angles in the notation of Landau & Lifshitz (1969), then, to first order in α, we identify the observed angular frequency Ω with $\phi + \psi$ and the precessional frequency Ω_{LT} with ϕ. The observed pulsar latitude b and longitude l are then given by

$$b = b_0 + \alpha \sin \Omega_{LT} t \tag{37}$$

$$\ell = -\Omega t + \alpha \tan b_0 \cos \Omega_{LT} t \tag{38}$$

where b_0 is the mean latitude observed. Both are modulated at the precessional frequency.

Suppose that a single measurement of a pulse arrival time of a non-precessing pulsar can be made with a standard deviation of δt by recording pulses over a single interval $\Delta t >> P$ and comparing with a mean pulse profile. (We assume that we know the pulsar period and its derivative.) Now, suppose that we seek a precessional modulation with an unknown but stable period in a frequency band of width ~ 0.03Ω, corresponding to $7P \lesssim P_{LT} \lesssim 8P$. We can search for ~ $0.03\Omega\Delta t$ discrete precessional periods in a discrete block. Typically this might be several hundred to a thousand. As a consequence, we estimate that it will take at least a ~ 10σ detection of any individual period for it to have any statistical significance. One reasonable way to proceed, at least conceptually, is to gate the observations with trial precessional periods and to seek differences in arrival time or pulse shape. I would guess that the accuracy with which we can measure pulse longitude is comparable to the accuracy with which we can measure latitude in a precessing pulsar. (This would be true, to give one simple example, if there were a single sharp pulse for which the rate of change of longitude with respect to latitude.) If, finally, we observe for an interval t so that we have ~ (t / Δt) independent measurements of this period, we find that we should be able to measure a modulation with angular amplitude

$$\alpha \sim 10\Omega\delta t \left(\frac{\Delta t}{t}\right)^{\frac{1}{2}} . \qquad (39)$$

For illustration purposes, let us again turn to the Vela pulsar (though this may be too young to be the most promising candidate for the reasons outlined above). McCulloch *et al.*(1990) used Δt = 120 s to obtain an accuracy $\delta t = 50\mu s$ at 950 MHz. Flanagan (1990) achieved $\delta t \sim 15\mu s$ using Δt = 45 s at 1.6 and 2.3 GHz. I therefore estimate that it should be possible to measure $\alpha \sim 3 \times 10^{-4}$ with ~ 10 d of accumulated data using McCulloch *et al.*'s equipment and an accuracy $a \sim 10_{-4}$ d observing with Flanagan's system for a comparable time. Obviously the period immediately following a giant glitch would be most propitious. Comparable time scales are predicted for some of the older pulsars which are well timed. Although the reliability of these estimates is probably not high, they probably do demonstrate that a scrutiny of pulsar records for periodic modulation may be worthwhile. If it is detected, and we develop confidence in the view that neutron stars have masses close to 1.4 $M_{\odot}$, then it will provide a unique, quantitative diagnostic of the structure of cold nuclear matter.

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