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Magnetospheric Models of Pulsars–Outstanding Questions

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Abstract. The difficulties of the pulsar magnetosphere problem are illustrated by two models for the axisymmetric magnetic rotator: (a) a classical model, in which the return current problem is linked with angular momentum dissipation' through incoherent gamma radiation beyond the light-cylinder; and (b) a quantum model, in which electron-positron pairs are produced near the star, and spin-down is primarily through the transport of angular momentum by an e^+-e^- wind. The first model has some pedagogic value, but it makes the embarrassing prediction that all the spin-down energy is emitted as gamma-radiation. In the second model a predicted small but significant emission in gamma- and X-rays is again linked with the return current problem, but the bulk of the energy emission is through the wind. The presence of both a high-γ primary electron beam and a moderate- γ pair plasma is as required by most models of coherent microwave emission. Problems persist in ensuring that the macroscopic conditions on the electric field are locally consistent with the microphysics. The most promising picture is of a spontaneous local hydromagnetic instability that yields a large effective resistivity. Some remarks are made on future extension of the theory to oblique geometry. For agreement with observations of the Crab nebula, the predicted dominance in the wind of the Poynting flux must be reversed in the far magnetosphere.

1. Introduction

The "outstanding question" of magnetospheric theory is that in spite of a great deal of effort, we still do not have an acceptable model, even for the superficially simplest case of the axisymmetric "non-pulsar", with the magnetic and rotation vectors aligned. Recall that the rate of energy loss from a rotating neutron star, inferred from the observed normal steady increase in the period, is orders of magnitude greater than that carried off by the pulsed emission in the radio waves, even after making generous allowance for beaming. Thus the radio noise is seen to be a diagnostic of much more powerful energy loss processes, which a complete magnetospheric theory should elucidate. In trying to understand how a rotating magnetized neutron star comes to terms with its environment by settling into a state of steady spin-down, the theorist is responding to a challenge: it is just intolerable that such a well-defined problem in both classical or quantum physics should remain unanswered. A complete answer would include the location of the sources of the pulsed emission in radio, optical, u-v, gamma and X-radiation as well as in energetic particles; but as Don Melrose has pointed out, such a deductive approach to the problem of the emission mechanisms is not very promising.

Let me begin by going back to the early days of pulsar theory. The first model for the magnetosphere predated the actual discovery of pulsars: in a prophetic paper, Pacini (1967) adapted the solution constructed earlier by A.J.Deutsch for the electromagnetic field of an obliquely rotating star *in vacuo* with its magnetic dipole moment p inclined to the rotation axis k by the angle χ. Far from the star the dominant terms are those forming the wave field emitted by the perpendicular dipolar component $p\sin\chi$, rotating with the angular frequency $\alpha = 2\pi/P_1$ with P_1 the period in seconds. The energy transport per second is

$$(2p^2\alpha^4/3c^3)\sin^2\chi = (B_s^2 R^6\alpha^4/6c^3)\sin^2\chi, \qquad (1)$$

where R is the stellar radius and B_8 the polar field strength. If χ is assumed not very small, then the equating of (1) to the inferred energy loss $-I\dot{\alpha}\alpha = 4\pi^2 I\dot{P}_1/P_1^3$ from a star of moment of inertia I with the observed period increase P_1 yields the canonical value of $B_8 \approx 10^{12}$G (Gold 1969). A corollary is that the energy loss vanishes as $\chi \to 0$: the aligned model *in vacuo* is dead.

The enormous gravitational field of the neutron star ensures that for any reasonable temperature, the thermal scale height is so small that a thermally-supported atmosphere would exponentiate down to virtually zero density within a few centimetres, so apparently justifying treatment of the surroundings as not just a dynamical but also an electrodynamical vacuum. The axisymmetric model would then be like a dynamo on open circuit, with the rotation of the highly conducting neutron star crust generating enormous surface potential differences (of the order of $10^{17}/P_1$ Volts), but with no opportunity for currents to flow. The essence of the Goldreich-Julian ("GJ") critique (1969) is that the system will in fact build up its own conducting leads. The axisymmetric vacuum magnetosphere model, subject to finiteness at infinity and continuity at the star's surface of B_r and E_θ, has an external quadrupole electric field **E** with a component $\mathbf{E}_\parallel$ along **B** of the same order at the star's surface as the internal field $E = -(\alpha k \times r) \times \mathbf{B}/c$. The consequent normal discontinuity in **E** then requires a surface charge density, which is subject to very large outward-acting unbalanced electrical forces. Thus a charged magnetosphere is spontaneously built up, and if there is continuous current flow into and out of the star, the aligned model is no longer dead but suffers a steady loss of rotational energy and associated angular momentum. The argument applies also to the oblique case (Cohen and Toton 1971; Mestel 1971). In both the aligned and the oblique cases, the magnetospheric charge-current field acts as an extra source of the electromagnetic field, with the modification to the magnetic field becoming important as the light-cylinder is approached. The formidable theoretical problem is to describe in detail the mutually interacting electromagnetic and particle fields.

2. An illustrative classical model

It is instructive to begin by studying the problem within the limitations of classical physics. It is then difficult to understand why a ”live” pulsar – i.e. one that is spinning down – does not manifest itself essentially as an efficient machine for the generation of gamma rays. The steps in the argument are as follows. The basic poloidal magnetic field is described by the flux function $P(\varpi, z)$:

$$\mathbf{B}_p = -\nabla \times (P\mathbf{t}/\varpi) = -\nabla P \times \mathbf{t}/\varpi, \tag{2}$$

where t is the unit azimuthal (toroidal) vector and (ϖ, ϕ, z) are cylindrical polar coordinates. For simplicity, P is for the moment given the dipolar form over the star’s surface, so that if the star’s surroundings were a strict electrodynamic vacuum,

$$P = -B_s R^3 \sin^2\theta / 2r, \tag{3}$$

where (r, θ, ϕ) are spherical polar coordinates, and B_s is the polar field strength at the star’s radius R. There will be a significant deviation from (3) as the 1-c is approached (*cf.* below). Nevertheless, we adopt (3) as a rough approximation: then the last field line to reach the 1-c defines the ”polar cap” $0 < \theta < \theta_0$, with

$$\sin^2\theta_0 = (\alpha R/c) = 2 \times 10^{-4} R_6/\mathrm{P}_1, \tag{4}$$

where $R_6 = R/10^6$ and P_1 is the rotation period in seconds. The rigidly rotating pulsar crust is well approximated as a classical perfect conductor, with

$$-\nabla\phi = \mathbf{E} = -\alpha\varpi\mathbf{t} \times \mathbf{B}/c = \nabla(\alpha P/c), \tag{5}$$

so that the electric potential $\phi = -\alpha P/c +$ constant. From (3) and (4), the potential variation over the polar cap is

$$\Delta\phi = (\alpha B_s R^2/2c)(\alpha R/c) \simeq 6 \times 10^{12}(B_{12}R_6^3)/\mathrm{P}_1^2 \text{ Volts.} \tag{6}$$

(Over a whole quadrant, the potential variation is $\alpha B_s R^2/2c = 3 \times 10^{16}(B_{12} R_6^2/\mathrm{P}_1)$ Volts.) The field line defined by θ_0 is the separatrix between those field lines that close within the 1-c and those that cross it. Charges in the closed domain can corotate with the star without contradicting special relativity, and will acquire only modest energies. By contrast, charges emitted from the polar caps will flow across the 1-c and can acquire γ-values up to $\gamma_c \simeq \Delta\phi/mc^2 = 1.25 \times 10^7 B_{12}R_6^3/P_1^2$; and particles of this energy will emit powerful gamma radiation.

Let us look at the problem more fully. In a steady state the curl-free electric field is conveniently written as

$$\mathbf{E} = -\alpha\varpi\mathbf{t} \times \mathbf{B}_p/c - \nabla\psi \tag{7}$$

so that the electric potential is

$$\phi = -\alpha P/c + \psi. \tag{8}$$

As noted, within the rigidly rotating, perfectly conducting stellar crust, the "non-corotational" part ψ of the potential is a constant. The simplest alternative to the untenable vacuum hypothesis for the surroundings is the original GJ proposal, by which charges originating in the star distribute themselves so as to short out the component $E_{\|}$, whence by (7), $\mathbf{B}.\ \nabla\psi = 0$. If there are no vacuum gaps between the star and the point considered, the constant value of ψ with in the star is then propagated into the magnetosphere: the vacuum condition of zero charge-current density is replaced by the corotational electric field condition $\mathbf{E} = -\alpha\varpi t \times \mathbf{B} / c$, B/c, maintained by the charge density

$$\rho_e = \nabla.\mathbf{E}/4\pi = -(\alpha/2\pi c)[B_z - \varpi(\nabla \times \mathbf{B})_\phi/2]. \tag{9}$$

Thus the GJ hypothesis converts the surroundings of the star into something like a classical perfect conductor. Much of the discussion of the different models in the literature centres around the domains of validity of the condition $\mathbf{E}.\mathbf{B} = 0$ and the consequences of its breakdown.

Prima facie, a classical magnetosphere built up and maintained by electrical forces rather than thermal pressure ought to be *charge-separated*, consisting of electrons where (9) yields $\rho_e < 0$ and ions where $\rho_e > 0$. Provided $E < B$, a charge subject to the orthogonal $\mathbf{E}$ and $\mathbf{B}$ fields performs the drift $c(\mathbf{E} \times \mathbf{B})/B^2$. If there were no poloidal current $\mathbf{j}_p$, the magnetic field remains purely poloidal, and this drift reduces to the corotation velocity $\alpha\varpi t$ for all charges within the light-cylinder ("l-c"). The simplest of such electrodynamically dead models (with $\mathbf{j}_p$ zero) has the corotating GJ charge density filling the whole domain within the l-c, and a vacuum beyond. The Ampere equation then yields for P within the l-c (*cf.* Michel 1991; Mestel and Wang 1979; Mestel and Pryce 1992)

$$\nabla^2 P[1 - (\alpha\varpi/c)^2] = 2\partial P/\varpi\partial\varpi. \tag{10}$$

The solution of (10), subject to a dipole singularity at the origin and with $B_z = 0$ at the l-c, illustrates how the magnetospheric currents distort the field significantly from the vacuum dipolar form as the l-c is approached. However, the discontinue-ities in E_ϖ and B_z across the l-c imply a surface charge-current distribution with corresponding unbalanced Maxwell stresses: like the original vacuum model, this dead model is again dynamically unacceptable (Mestel and Wang 1982). (Ax-isymmetric dead models have been constructed numerically by Krause-Polstorff and Michel (1985), satisfying the condition $\mathbf{E.B} = 0$ within regions containing charge, but with large vacuum gaps. The models all have a large positive charge, and it is questionable whether they would in practice avoid conversion into live models following either discharging by the interstellar medium or electron-positron pair production under the strong $E_{\|}$ fields in the gaps.)

As stated, it is the flow of poloidal current along the field lines crossing the l-c that is responsible for the spin-down of the star. We consider just the aligned rather than the counter-aligned case, so that the electric field is of the sign to draw out electrons rather than ions; the steady outflow of electrons from each polar cap forms therefore a *convection current*, with the poloidal component j_p defined by the stream function S:

$$\mathbf{j}_p = \rho_e \mathbf{v}_p = -\nabla S \times \mathbf{t}/\varpi \tag{11}$$

with $\rho_e = \nabla.\mathbf{E}/4\pi$. Rather than making the GJ *ansatz* $\mathbf{E.B} = 0$, it is preferable to retain inertial (and later dissipative) terms in the dynamical equations (though gravity is justifiably ignored); the validity of the GJ approximation in any domain can then be assessed. Thus for a cold, dissipation-free gas, the energy integral is

$$-e\phi + \gamma mc^2 = -e\phi^*(S) \tag{12}$$

and the angular momentum integral is

$$eP/c + \gamma m\Omega\varpi^2 = eP^*(S)/c, \tag{13}$$

where Ω is the local angular velocity. Equation (13) shows that electrons do not move strictly along the poloidal field but suffer an "inertial" drift: to increase its angular momentum $\gamma m\Omega\varpi^2$, an electron must have a component of poloidal velocity normal to $\mathbf{B}_p$ in order that the magnetic field exert the required torque. Together with the definition (8) of ψ, (12) and (13) combine into

$$\Gamma \equiv \{\gamma[1 - (\Omega\varpi/c)(\alpha\varpi/c)] - e\psi/mc^2\} = \Gamma(S). \tag{14}$$

The relativistic slingshot term $\gamma(\Omega\varpi / c)$ $(\alpha\varpi / c)$ is a consequence of the inertial drift, which takes the electrons to points with a higher corotational potential — $\alpha P / c$. The actual deviation from strict flow along $\mathbf{B}_p$ is of order $\gamma\Omega /(eB_p / mc)$ which remains very small until the gas becomes highly relativistic.

The current (11) generates by Ampere's law a toroidal field component $\mathbf{B_t} \equiv B_\phi \mathrm{t}$ given by

$$B_\phi = -4\pi S/c\varpi. \tag{15}$$

As in standard stellar wind theory, the sign of $B\phi$ ensures that the lines of $\mathbf{B}_p + \mathbf{B}_t$ are swept back with respect to the rotation axis $\mathbf{k}$, so the outflowing gas sub-rotates, allowing smooth sub-relativistic flow which can penetrate the l-c. The condition that γ remain finite at the l-c fixes the velocity v^* with which electrons leave the star, typically at $\simeq c/2$. In this respect the model is similar to that in the classical GJ paper (1969). The crucial difference is that there is no surrounding collar containing outstreaming ions. The difficulty with the GJ model (pointed out by the authors) is that the positively charged ions would have to flow through a sea of electrons to ensure that the net charge density satisfies the GJ requirement (9). In order that the present model should not break down through a continual charging up of the star, the electron stream must cross field lines in some domain and return to the star.

The precise behaviour of the flow beyond the 1c depends on the detailed structure of the magnetic field lines, which strictly can only be determined simultaneously with the density-velocity fields of the particles. However, with virtually any plausible guess for the field structure, one finds that over much of the domain the quasi-mind flow breaks down somewhere beyond the l-c, with γ becoming formally infinite when the electric field $\mathbf{E} = -\alpha\varpi\hat{t} \times \mathbf{B}_p/c$ becomes equal in magnitude to the strength $(B_p^2 + B_t^2)^{1/2}$ of the total magnetic field, and simultaneously $\alpha\Omega\varpi^2/c^2 = 1$. In fact, when γ approaches the value $\gamma_c = (eB/mc)/\alpha$ (the ratio of the local non-relativistic gyro-frequency ω_g to .the stellar rotation), the inertial drift terms are no longer small, and simultaneously the non-corotational potential ψ, given by (14), becomes comparable wih the corotational part $-eP/c$. Near the l-c, $\gamma_c \simeq 2.6 \times 10^7(B^*/10^{12})(R/10^6)^3/P_1^2$, where the rough vacuum dipolar approximation has been used for the field strength. However, even in the slow pulsars with $P_1 \simeq 1$, particles with γ-values near γ_c will be emitting powerful incoherent gamma-radiation, strongly beamed in the direction of motion. If V is the radiated power per particle, the associated linear momentum loss is $(\mathcal{P}/c^2)\mathbf{v}$, as is seen by making a Lorentz transformation from the particle rest frame to the inertial frame. The energy, angular momentum and Γ-integrals (12), (13) and (14) are now replaced by

$$\mathbf{v}.\nabla(\gamma mc^2 - e\phi) = -\mathcal{P}, \tag{16}$$

$$\mathbf{v}.\nabla(\gamma m\Omega\varpi^2 + eP/c) = -(\Omega\varpi^2/c^2)\mathcal{P}, \tag{17}$$

$$\mathbf{v}.\nabla\left\{\gamma\left[1 - \frac{\Omega\varpi}{c}\frac{\alpha\varpi}{c}\right] - \frac{e\psi}{mc^2}\right\} = -\frac{\mathcal{P}}{mc^2}\left[1 - \frac{\Omega\varpi}{c}\frac{\alpha\varpi}{c}\right]. \tag{18}$$

The first term on the right of (18) represents the direct loss of energy by radiation. The associated angular momentum loss in (17) implies a dissipative drift to field lines of higher corotational potential ($-\alpha P/c$), and so to a *gain* in energy, analogous to the inertial drift that yields the slingshot term in (14) and (18), In the present problem it is the radiation beyond the l-c of photons with high angular momentum that enables the electrons to pick up the energy available on field lines of higher potential, due ultimately to the rotation of the magnetic star. Clearly, this requires that the gain in energy due to the dissipative drift exceed the energy radiated. One can notionally integrate the equation (18) along a trajectory followed by an electron between emission from and return to the star (recall that Ψ is uniform over the star). Since $\mathcal{P}$ is essentially positive, it is clear that to avoid the nonsensical prediction of $\gamma < 1$ for a returning particle, $[1 - (\Omega\varpi/c)(\alpha\varpi/c)]$ must be negative in the strongly dissipative regions: the dissipation must occur primarily beyond the l-c. This links up with a comment made by Gold (1980) and Holloway (1977). Since it is the star's rotational energy $I\alpha^2/2$ and the associated angular momentum $I\alpha$ that is powering the overall pulsar activity, on balance: (energy loss rate) = α (angular momentum loss rate). Photons emitted within the l-c, even if they are perfectly beamed in the toroidal direction, have too small a lever arm, requiring a compensating photon emission well beyond the l-c. The equation (18) shows there is an automatic "Gold-Holloway" condition not only for the system as a whole but for each trajectory.

Once particles begin to radiate, then a good approximation to the equation of motion ignores inertia and balances the Lorentz force against radiation drag – the "Stokes-Aristotle" approximation (Mestel *et al.* 1985):

$$-e(\mathbf{E} + \mathbf{v} \times \mathbf{B}/c) - (\mathcal{P}/c^2)\mathbf{v} = 0. \tag{19}$$

As long as $E < B$, the poloidal velocity v_p remains more nearly parallel to $\mathbf{B}_p$,

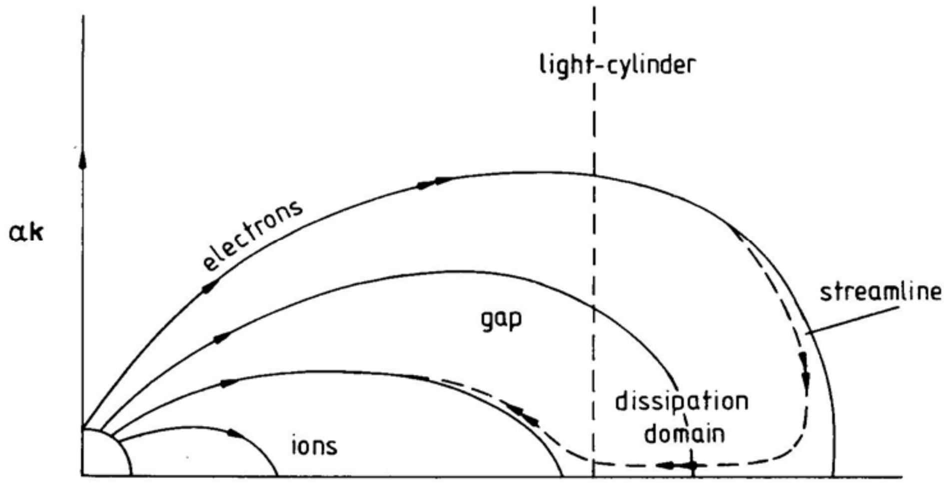

Figure 1: Classical model – schematic diagram

and in fact $\mathcal{P} \ll ec\mathbf{B}$: the dissipation term is small compared with the Lorentz force, and so the perfect mhd condition $\mathbf{E} + \mathbf{v} \times \mathbf{B}/c = 0$ remains a good approximation, with $E_{||} \ll E_{\perp}$. Once $E > B$, the flow more nearly follows $\mathbf{E}$. Strong trans-field flow under the essentially quadrupolar E-field can be expected close to the equator, where $\mathbf{B}_t$ goes to zero by symmetry. The emitting electrons are kept at an energy $\simeq 7 \times 10^{12} / P_1^{1/4}$ eV, and the photons emitted are gamma-rays of energy $\simeq 5 \times 10^7 / P_1^{7/4}$ eV; the total power is of the same order as that from a vacuum oblique rotator, given by (1) with $\chi = O(1)$.

Some very approximate analytical and numerical work on this classical model (Fig. 1) is described in Fitzpatrick and Mestel I,II (1988). Besides the circulating electrons, there is a domain with non-circulating corotating electrons and a domain of non-circulating but sub-rotating ions, separated by a wedge-shaped gap, which is approximated as a vacuum and so has strong **E.B** components. (The returning electron current is forced to flow partly through the ion and gap domains, so these descriptions are only approximate). To keep the model working the whole system must have zero net charge, so that the electric field well beyond the l-c has the quadrupolar $1/r^4$ dependence. Since E must exceed B in the equatorial domain where the returning electrons have dominantly trans-$\mathbf{B}_p$ motions, the toroidal magnetospheric currents are required to yield a dipole moment equal and opposite to that on the star.

A crucial point of the model is the link-up of the spontaneous loss of energy and angular momentum with the return current requirement, for which dissipation is essential. In spite of some clear differences, brought out above, the theory is quite similar to that in a classical "centrifugal wind" problem. In particular, it is the *perpendicular* component $\mathbf{E}_\perp$ that energizes the particles, through the sling-shot effect; but as long as inertial and dissipative drifts are small, the gas velocity is well described by the familiar mhd integrals, with the associated value of the non-corotational potential ψ given by the Γ-integral (14). The chain of causation is: $E_\perp \rightarrow \gamma \rightarrow E_{\parallel}$.

This classical model illustrates how dissipation-free flow spontaneously establishes domains in which acceleration to high γ-values occurs, leading inevitably to dissipation. The trouble with the model is that it proves too much: for although observations are now showing that gamma-ray emission from some pulsars is a significant fraction (up to 10%) of the power, the bulk of the energy loss is in other modes. One can imagine a coherent radiation process occurring, with the γ-values kept much lower and the total power being again emitted as radiation but at much lower frequencies. However, with the overwhelming evidence that e.g. the Crab pulsar loses much of its spin-down energy in the form of a relativistic particle wind, it is best to look on the classical model as illustrative, with some of its features persisting into the more realistic picture described below.

3. Models with pair production

A crucial step in Section 2 is the requirement that the quasi-mhd flow extend through the l-c, which limits the speed with which the electrons leave the polar caps typically to $\simeq$ c/2. If the emission speeds v^* are somewhat higher, the particles will achieve high γ-values well within the l-c. Continuity of flow that is nearly parallel to $\mathbf{B}_p$ requires $\rho_e v_p / B_p$ = constant. The energy-angular momentum integral (14) can be read to show that the GJ approximation - which constructs ρ_e with neglect of ψ – remains excellent up to high γ-values: $\rho_e \propto -aB_z / 2\pi c$, so $v_p \,\alpha\, B/p_e \,\alpha\, B/B_z$, and this increases outwards for field lines that curve away from the axis, as do all the lines, in the aligned case and most lines in moderately oblique cases. Thus if v^* is close to c, v_p is predicted to reach c on a surface Si quite near to the star. Beyond Si the flow will still follow the field lines closely, but now $v_p \simeq$ c, and continuity enforces $\rho_e \propto B$ instead of $\rho_e \propto B_z$. Thus the limitation imposed by special relativity shows that the neglect of $\nabla^2\psi$ in the construction of $\nabla.\mathbf{E}$ ceases to be valid near and beyond S_l. Equation (9) (with $(\nabla \times \mathbf{B})\phi$ still negligible) is replaced by

$$\nabla^2\psi \simeq (2\alpha B_z/c)[-1 + (B/B_z)/(B/B_z)_l], \tag{20}$$

so that beyond S_l, ψ is found to increase outwards in order to satisfy the Poisson-Maxwell equation. The integral (14) must now be read in reverse: because the geometry of the flow causes B/B_z and so also ψ to increase, there is simultaneously an increase in γ which is rapid, again because of the smallness of the ratio α/w_g.

Closer inspection shows that even within S_l the $\nabla^2\psi$ term is not strictly negligible. The GJ approximation holds in the mean, but superposed on the monotonic flow are stationary spatial oscillations in ψ , γ and v_p of scale $(\alpha/\omega_g)^{1/2}$ (c / α), the plasma wavelength corresponding to the local GJ density (Mestel, *et al.* 1985, Appendix B; Shibata 1991). As the gas flows across S_l, the solution with small-scale oscillations in ψ and γ is replaced by one with rapid monotonic growth.

It should be noted that the model is different from those in the literature which appeal to a vacuum gap at the stellar surface. For some years it was thought that in the case of counter-aligned rotation and magnetic axes, the spontaneous emission of ions under the outward pointing vacuum electric field would be inhibited because of a quantum-mechanical work function. The 10^{12} Volt potential difference which builds up at the poles would then lead to spasmodic breakdown of the gap through electron-positron production (Ruderman and Sutherland 1975). Subsequent computations shed doubt on the previous work function estimates, and it is now generally assumed that the classical boundary condition **E.B** = 0 is always a good approximation.

The present treatment of the aligned case shows that the curvature of the field lines away from the axis necessarily leads to "over-filling" of the flux tubes, in the sense that the relativistic limitation on v forces the density up beyond the GJ value, and it is the consequent $E_{||}$ which causes monotonic acceleration of the electrons to high γ-values. The discussion is fully consistent with that by Fawley et *al.* (1977), who show that no solutions exist for field lines curving away from the axis if the boundary condition $\mathbf{B}.\nabla\psi = 0$ is imposed not only at the star but also at the end of the acceleration domain.[1] A difference does arise in terminology: in the present treatment this upper boundary condition and so also the phrase "unfavourably curved" are inappropriate. The position and shape of the surface S_1 is not prescribable locally but must emerge from a global solution.

Beyond *S1,* particles are accelerated rapidly to $\gamma \simeq 10^7$, at which value there is copious emission of gamma-rays of energy $h\nu \simeq 3\gamma^3\hbar c/2R_c$, where R_c is the radius of curvature of the field-streamline. The crucial new factor, noted first by Goldreich in a conference report and by Sturrock (1971), is that hard gamma-rays moving across a very strong magnetic field convert spontaneously into electron-positron ("e^+-e^-") pairs. The newly-created particles are themselves accelerated by the electric field until they radiate gamma-rays which again create pairs. This carcade is ultimately self-limiting, since the dense e^+-e^- plasma accompanying the high-γ primary electron current will short out the accelerating $E_{||}$ field. The basic picture has been applied to different models in many papers (Ruderman and Sutherland 1975; Arons and Scharlemann 1979; Arons 1981; Daugherty and Harding 1982; Jones 1980; Shibata 1991; Mestel and Shibata 1994; among others). Thus the originally pure high-γ electron gas is converted into a plasma consisting of primary electrons plus a high-density sea of pairs. The multiplicity $\boldsymbol{M}$ (the number of pairs per primary electron) is estimated to be from a few hundred for the slower pulsars to $\sim 10^4$ for the Crab pulsar. The pairs are estimated to have $\gamma \sim 10^2$ and the primary electrons $\sim 10^6$. The process will cut off if the potential difference across the polar cap is inadequate to produce primary electrons of high enough γ. This gives an upper limit of $P_1 \sim 1.5$. If the e^+-e^- cascade is essential for the generation of the coherent radio emission from pulsars, then as argued by Sturrock, Ruderman and Sutherland and others, there is a clear physical reason

[1]Beskin (1990) and Muslimov and Tsygan (1990, 1992) have shown that this conclusion no longer holds when general relativistic corrections are included. This result is of interest but in my view does not change radically the physics of pulsars, as it is the assumed outer boundary condition that needs to be changed.

for radio pulsars' shutting off as their periods lengthen beyond ≈ Is. As found earlier by Ruderman and Sutherland (1975), to get a cut-off line that is consistent with observation, the magnetic field has to possess higher multipole components so that the radius of curvature near the star is smaller and pair production correspondingly more efficient.

Beyond the pair production region $E_{||}$ is again small, and as long as inertial drifts are small the flow is again quasi-mhd, with

$$\mathbf{v} = \kappa\mathbf{B} + \tilde{\alpha}\varpi\mathbf{t}, \qquad \tilde{\alpha} = \alpha - c d\psi/dP, \tag{21}$$

$$\mathbf{E} = -\alpha\varpi\mathbf{t} \times \mathbf{B}/c - \nabla\psi = -\tilde{\alpha}\varpi\mathbf{t} \times \mathbf{B}/c, \tag{22}$$

and

$$j_p/B_p = \{j_p/B_p\}_{S_l} = -(\alpha/2\pi c)\{B_z/B_p\}_l. \tag{23}$$

It should be emphasized that the non-corotational potential $\psi(P)$ and the associated isorotation function $\tilde{\alpha}$ (P) are fixed by both the physics of the acceleration-plus-pair-production domain and the shape of S_l, and so are strictly determined as part of the global solution. It is anticipated that $\tilde{\alpha} < \alpha$ - subrotation of the field lines in the post-cascade plasma.

In the limit in which S_l is supposed. to be at the pulsar surface, then the model would look as if there were a discontinuity in ψ at the star, as in the Lebedev model (Beskin *et al.* 1993), which however postulates a vacuum gap (a double layer) at the star's surface. We recall that the Ruderman-Sutherland counter-aligned "anti-pulsar" model produced e^+-e^- pairs through the spontaneous breakdown of such a gap, but that model was based on what at that time seemed to be a correct estimate of the work-function for ion emission from the star. I agree with Don Melrose that without such a work function there is no clear reason why such a double layer should form or survive.

The Lebedev group write (in my notation)

$$\psi = \psi_0 + (\alpha/c)\beta P, \qquad \tilde{\alpha} = \alpha(1-\beta). \tag{24}$$

They assume also

$$\varpi B_\phi = i_0(\alpha/c)P, \qquad j_p = -i_0(\alpha/4\pi)B_p, \tag{25}$$

with the return current concentrated into a sheet along the separatrix between the domains with respectively open and closed field lines. They then use the balance between the $B\phi$ and ψ contributions to the Maxwell stresses, applied near the cusp on the separatrix, to derive the relation

$$\beta = 1 - (1 - .4i_0^2)^{1/2}. \tag{26}$$

But as pointed out by Lyubarskii (1992), this suppresses any contribution to the Maxwell stresses of a discontinuity in the poloidal field across the separatrix field line: there is in fact no automatic relation that requires a poloidal current to be associated with a potential jump at the star's surface

The flow described by (21) passes smoothly through the l-c, but –in partial analogy to the charge-separated flow in the classical model of Section 2 –

breakdown in the flow will occur when $E \rightarrow (B_p^2 + B_t^2)^{1/2}$, and simultaneously $\tilde{\alpha}\Omega\varpi^2/c^2 \rightarrow 1$ and $\gamma \rightarrow \infty$ through the slingshot term. The location of the points of breakdown is again to some extent model-dependent. As long as γ is moderate the inertial terms are small and so the poloidal field flux function P satisfies the *relativistic force-free* equation

$$\nabla^2 P(1 - \tilde{\alpha}^2 x^2) - 2\partial P/x\partial x + SdS/dP - \tilde{\alpha}(d\tilde{\alpha}/dP)x^2(\nabla P)^2 = 0, \qquad (27)$$

where P, S, $\tilde{\alpha}$, $x = \alpha\varpi / c$ are appropriate non-dimensional forms, with $\chi = 1$ representing the light-cylinder. If $\tilde{\alpha} = 1$ and $S = 0$, (27) reduces to (10) for the corotating magnetosphere. The two equations are similar in having each a singular surface on which the coefficient of $\nabla^2 P$ vanishes; it is therefore instructive to study first the simpler (10), even though corotation of the gas cannot hold beyond the l-c. Equation (10) is conveniently solved by Fourier transformation in *z.* Extrapolation across the l-c, *with no discontinuities in P or* $\partial P / \partial x$ yields a solution which blows up at $x = 2$. This is because only one of the two solutions of the Fourier transform equation is finite at $x = 1$, and without a discontinuous gradient at $x = 1$ one cannot extend the solution to infinity. One expects and finds similar behaviour with the realistic equation (27). In rapid rotators, α may be approximated by unity. Numerical integration with simple $S(P)$ functions shows that solutions of (27), smoothly continuous at $x = 1$, again diverge, typically when $x \simeq 1.4$, very close, to the point where the breakdown in the flow occurs, as discussed above.

The next generalization must take account of inertia. For even high γ-values the inertial drifts are small, and the flow is still well approximated by the quasi-mhd equations. The generalization of (27) now has the singular surface $(1 - \tilde{\alpha}^2 x^2 - \boldsymbol{M}_A^2) = 0$, where $\boldsymbol{M}_A$ is an Alfvenic Mach Number. It is perhaps possible that there now exist solutions which are both smoothly continuous and everywhere regular: conceivably the inertial term $\boldsymbol{M}_A^2$ could allow the singular surface to adjust its shape so as to yield such a solution. However, in the preliminary studies, it is postulated rather that there has to be a domain of *dissipative flow.* The tentative model is as follows (Fig, 2). The dissipation domain is taken to be a thin cylindrical layer, located at $x = \tilde{x}$, a little beyond the l-c. The reason for the choice of a cylindrical dissipation domain is in order that the convenient technique of Fourier transformation in z can again be applied, at least in the first iterative stages. Out to $x = \tilde{x}$ the field is taken to be corotating with the star, so that in non-dimensional form, $\mathbf{E} = -x\mathrm{t} \times \mathbf{B}$. (As noted above, the degree of isosubrotation associated with the relativistic acceleration beyond S_l is in fact quite modest in a rapid rotator - $\tilde{\alpha} \simeq 1$). Beyond $x = \tilde{x}$, $E = -\beta(P)\, x\mathrm{t} \times \mathbf{B}$, where $\beta(P)$ is in fact approximated as a constant. Unless $\beta = 1$, there is a discontinuity in E_x at $\tilde{x}$, linked with the potential difference across the layer, which is in reality of finite thickness and of postulated high effective resistivity. Provided $\beta < 1$ – isosubrotation for $x > \tilde{x}$ – then the potential difference drives currents in the correct sense - inwards near the poles, outwards near the equator. For $x < \tilde{x}$, P is approximated by the solution of the force-free equation (27), with $S = -2P(1 - P/2\bar{P})$, where $\bar{P}$ is the field line along which j_p changes sign. For $x > \tilde{x}$, P satisfies the dissipation-free, quasi-mhd equation with inertial terms retained. Jump conditions at $\tilde{x}$ include radiation of energy and angular momentum, and continuity of the (x, z) component of the total stress tensor.

The first numerical studies have adopted $\tilde{x} = 1.1$ as a standard value. The results yield β close to the value .9, implying a modest departure from corotation beyond the layer. The radiation loss in the layer is < 10% of the total energy flux. About 10% of the poloidal current closes in the layer. For the Crab pulsar, the wind has $\gamma \simeq 10^6$, carrying typically about .2 of the total energy flux, the rest being carried by the Poynting flux. About 2/3 of the magnetic flux crossing $x = \tilde{x}$ is open, and one third goes into an equatorial disc. It is suggested that the disc becomes periodically unstable, so that the gas flowing from the layer into the disc ultimately joins the wind.

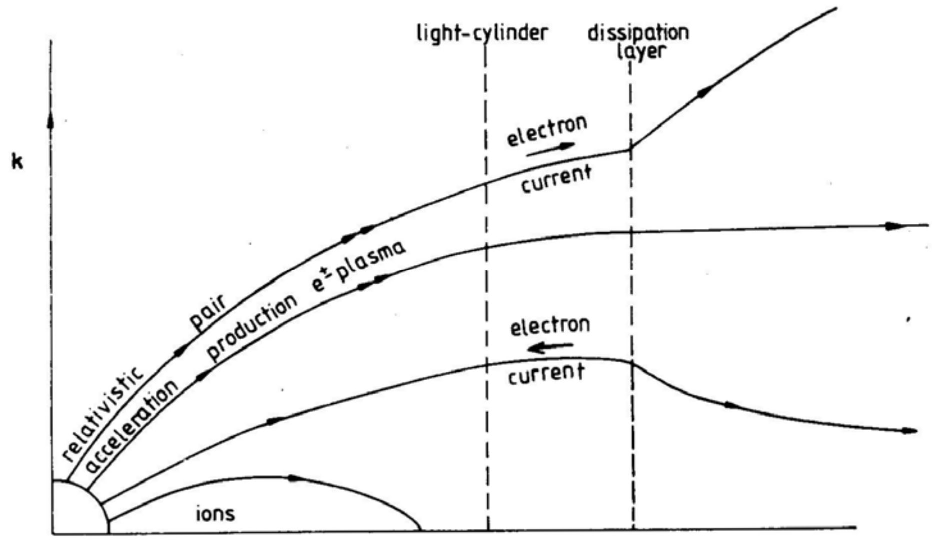

Figure 2: Model with pair production — schematic diagram

These numerical estimates emerge from approximate solution of the macroscopic equations with the appropriate boundary conditions. Just as in the classical problem, the same relativistic slingshot will force up the γ-values of the primary electrons from the value $\simeq 10^6$ pertaining at the end of the pair-production region, and again when γ reaches 10^7, strongly beamed incoherent gamma-radiation will occur. Within the layer, the parallel component of the electric force will maintain the γ-values at the level where the poloidal radiation drag force on the primary electrons is in balance with the electric force: again, the strong γ-dependence of incoherent radiative damping enables the primary particles to act as self-adjusting rheostats. The significant fraction of the power from a number of pulsars coming out as gamma-rays could have its origin as in this model. (However, other processes for gamma-ray emission have been proposed. It is likely that the a viable model will include a *vacuum gap* region, in which again **E.B** differs markedly from zero (e.g. Fitzpatrick and Mestel I, II 1988). Cheng *et al.* (1986) have pointed out that e^+-e^- generation will occur in such a gap, with the created particles being accelerated in opposite directions to highly relativistic energies and so again emitting gamma-rays, especially by inverse-Compton scattering on infra-red photons).

The e^+-e^- pairs are also energized by the slingshot effect, the energy source being as usual the potential variation across the polar caps. The pair electrons will be further accelerated through the layer by $\mathbf{E}_{||}$, though their motion may be inertially- rather than dissipatively-limited; but the pair positrons are *de*celerated by the $\mathbf{E}||$ component. Recall that the existence of a dissipation layer – with a potential variation along each field line segment as it crosses the layer – has emerged from attempts to model the macroscopic structure of the magnetosphere. In a normal plasma problem, the $\boldsymbol{E}_{||}$ -component is determined locally from the balance of electric force against Ohmic resistance, due to the mutual scattering that inhibits the relative motion of the oppositely-signed charges that makes up the $j_{||}$. A *sufficiently high* resistivity would then determine a layer thickness well below a l-c radius. The ordinary micro-resistivity is far too small to be important. The highly supersonic relative motion of the e^+-e^- gases is subject to the two-stream instability. One can estimate an effective ”anomalous” resistivity by replacing the binary collision frequency by the linear instability growth rate; however, this also does not appear to be large enough.

In the absence of any scattering, a positron could pass through the layer if it enters with a sufficiently high γ because of the slingshot effect. However, the total energy available is limited by the potential variation across the polar cap. Requiring a steady stream of positrons to cross against the adverse potential difference sets an upper limit on the multiplicity: $\boldsymbol{M} < 1/(1 - \beta)$. Thus if one were able to construct models with $\beta \approx .99$ rather than .9, a modest value $\boldsymbol{M} \approx 100$ might be acceptable within this no-scattering picture. However, a much more attractive alternative is that the system developes a more powerful effective resistivity, so that the outstreaming pairs are kept coupled as they cross the dissipative domain. It is significant that in the constructed model force-free fields, the toroidal component becomes large compared with the poloidal as the surface of breakdown is approached. This suggests that the poloidal currents are limited by a consequent hydromagnetic instability, which would yield a large effective macroresistivity (*cf.* Sect. 5, Mestel and Weiss 1987, Lyubarskii 1991, preprint).

Breakdown in the flow beyond the 1-c is found also by Beskin *et al.* (1993), even though they have postulated a potential jump at the star’s surface. They argue that the electrons and positrons flow across the field lines in their boundary layer, the electrons equatorwards and the positrons polewards, with both species ultimately returning to the star. They suggest that the energy loss to infinity is via mhd waves, rather than through the escape to infinity of the e^+-e^- gas, as assumed by most other workers (e.g. Kennel et *al.* 1983, Shibata 1991, Camenzind 1993). In the classical model of Sect. 2, the return of electrons to the star (after dissipative drift across the field lines) is mandatory to avoid continual charging up of the star; and in any case the GJ number-density is so low that flow to infinity would in general require γ-values so large as to yield far too much radiation loss.

By contrast, in a model with pair production, once the bulk of the plasma has crossed the domain with finite $E_{||}$ it can then flow to infinity, dragging the field with it, provided $2Mn_{GJ}\gamma c^2 \gg B^2/8\pi$; so as long as the multiplicity M is large, only moderate γ-values are required. Charge balance may again be maintained by the return to the star of electrons at the (GJ) emission rate. Alternatively, pairs may be produced in outer magnetospheric gaps (Cheng *et al.* 1986), but a preferential loss of positrons may occur (Shibata, private communication).

4. Non-axisymmetric models

Most of the analysis in the literature has been carried out for the aligned rotator, the "non-pulsar", though Beskin *et al.* (1993) do apply their ideas to the oblique rotator. Any rigidly rotating, non-axisymmetric system that is "steady in the rotating frame" can be analysed using the operator-equivalence $\partial/\partial t = -\alpha\partial/\partial\Phi$, applied to scalars and e.g. to cylindrical and spherical polar scalar components of vectors, where Φ is the azimuthal angle about the rotation axis. One readily arrives at the generalization of the relativistic force-free equation (27) with $\tilde{\alpha} = 1$ (Endean 1974; Mestel 1973; Mestel *et al.* 1976):

$$\nabla \times \tilde{\mathbf{B}} = \kappa \mathbf{B}, \tag{28}$$

where

$$\tilde{\mathbf{B}} = [B_x(1 - x^2), B_\Phi, B_z(1 - x^2)]. \tag{29}$$

The corotating G-J charge density is responsible for the replacement of $\mathbf{B}$ by $\tilde{\mathbf{B}}$, and $\kappa\mathbf{B}$ represents the currents along $\mathbf{B}$. Results similar to the axisymmetric case can be expected, such as the inevitable breakdown of a continuous force-free model on a surface beyond the l-c. For example, in the case of the perpendicular rotator, asymptotic analysis applied to the simplest case, with $\kappa = 0$ - the analogue of (10) - shows that a continuous solution would again blow up at $x = 2$; and with $\kappa \neq 0$, one expects the breakdown surface to be somewhat nearer to the l-c.

In the aligned case, it was noted that quasi-mhd flow breaks down beyond the l-c, at points close to where the breakdown in (27) occurs. As pointed out by Kahn (1971), in the oblique problem one must distinguish between field lines that point forward with respect to the sense of rotation from those that point backwards. Particles moving out along forward-pointing lines will approach speed c near the l-c, and so are forced to deviate from the field line through an inertial or dissipative drift just before the l-c. Particles moving out along backward-pointing have no difficulty in passing through the l-c, but in general can again be expected to approach $\gamma = \infty$ some way beyond, and so will again drift across field lines. There are thus some qualitative similarities between the aligned and oblique cases. Construction of a reasonably accurate global model will clearly be more difficult than for the aligned case (even for the perpendicular rotator), but the experience gained from the axisymmetric problem should be valuable. One again anticipates a domain in which; the field is nearly force-free, beginning at the star and terminating near the l-c, and again separated by a dissipative domain from a non-force-free but dissipation-free domain, extending to infinity.

5. The pulsar wind

Most workers picture the spin-down energy and angular momentum of an aligned or nearly aligned pulsar as being carried off by a magnetohydrodynamic wind. Michel (1970) and Goldreich and Julian (1970) pioneered the study of cold relativistic winds. Near the star, the transport is essentially all by the Poynting vector and the Maxwell stresses. A difficulty with the simple wind theory is that it predicts at infinity a dominant toroidal field component, and so also a Poynting flux that is still at least comparable with the particle energy flux. That this sets a difficulty emerged in early work by Rees and Gunn (1974). Their model for the Crab nebula combined a strong wave with a relativistic pulsar wind. The wind is decelerated by a strong shock at about 3×10^{17}cm from the pulsar. To make the post-shock speed decrease with radial distance, so as to match the observed 2000km/sec expansion speed of the outer Crab nebula, one needs rather that the ratio $\sigma \equiv$ (Poynting flux/kinetic energy flux)$\ll 1$ upstream -if instead $\sigma \gg 1$, the downstream flow is also Poynting dominated. Kennel and Coroniti (1984) came to a similar conclusion for their e^+-e^- wind: they require $\sigma_\infty = 3 \times 10^{-3}$, as compared with the value $\simeq 10^{4-5}$ yielded by most models. (The tentative Mestel-Shibata model referred to above has $\sigma \simeq 10$ after the dissipation layer, smaller but still too large).

The Poynting flux must be converted into kinetic energy but in the wind zone, well beyond the l-c, otherwise synchrotron losses would predict far too much X-ray and γ-ray emission near to the star. Lyubarskii and others suggest that the excess toroidal energy may ultimately be dissipated as a consequence of mhd instabilities that may set in when $|B_\Phi / B_p| \gg 1$, especially the kink instability (*cf.* the work of Tayler, Wright and others on instabilities of stellar magnetic fields). This is very plausible, though one has to extrapolate the linear instability theory into the unexplored non-linear domain. The oblique rotator offers other opportunities. Coroniti (1990) points out that the wind emitted by an oblique rotator will be magnetically "striped", with B of alternating sign. He therefore argues for the systematic mutual annihilation of opposite polarity stripes as the wind flows out, estimating that the initial Poynting flux is destroyed within about 8×10^6 l-c radii. It was noted in Mestel *et al.* 1976 that the extrapolation of the GJ *ansatz* **E.B** =0 to infinity led to unacceptable conclusions in the oblique problem. More recently, Melatos and Melrose (1994) have argued that it is in fact the breakdown in the ability of the charges to short out the displacement current that enforces efficient dissipation of the large toroidal fields.

6. Conclusion

The construction of fully self-consistent electromagnetic and particle fields remains still a formidable task for the future. It is noteworthy that although there are differences on the presence or absence of a potential gap at the star's surface, there is agreement on the appearance beyond the l-c of a domain of breakdown of the force-free, quasi-mhd approximations. Further effort should be devoted to elucidate the physics of this domain, in particular to estimate its effective resistivity. Concentration on the axisymmetric model should not be decried, as success there is likely to be a useful guide to the oblique models; however, the time seems ripe for a return to a parallel study of quasi-steady, force-free models.

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