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Bipolar Jets in Planetary Nebulae: An Analytical Model

G. Pascoli *Department of Physics, Faculté des Sciences, 33, rue Saint-Leu, F-80039 Amiens Cedex, France*

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Abstract. The various and hitherto partially unsolved problems relative to the origin of bipolar jets or highly collimated fast outflows in planetary nebulae are reviewed within the framework of a stationary magnetohydrodynamic model.

In order to explain the observations of high polar velocities and the presence of polar blobs or knots in planetary nebulae, theoretical models are proposed taking into account both a large scale azimuthal magnetic field and an anisotropic turbulent velocity field.

The models predict equatorial-to-polar density ratios which are rather small, in the range 2 to 3. Conversely, the polar-to-equatorial velocity contrasts are higher, with typical values upto 10. Thus the *ad hoc* hypothesis implicit in the literature that the density contrast is varying in inverse ratio to the velocity one, does not seem well adapted to the bipolar jet phenomenon in planetary nebulae.

We point out, therefore, that the bipolar jets have to be considered as a transient aspect of a very complex phenomenon. The model can be applied to objects such as He 2–104 or Mz3, M2–9.

Key words: Planetary nebula—bipolar—magnetic field

1. Introduction

Many theoretical investigations have been devoted to answer the question of how planetary nebulae (PNe) with their conspicuous symmetries are formed. In spite of this, some facts remain enigmatic and this constitutes one of the most exciting problems under consideration in the post main sequence evolution. One of these facts is relative to the origin of bipolar jets in PNe. Bipolar outflows of polar knots are not characteristics of PNes alone but are rather ubiquitous phenomena. A wealth of observational data is now available on the bipolar outflows that are associated with young stellar objects and, generally, with star forming regions (Lada 1985; Snell 1987). The patchy structure of the bipolar jets closely connected with the so-called Herbig-Haro objects suggests that a high degree of instability is prevalent in these outflows (Reipurth 1991 and references therein). Various theoretical collimation mechanisms have been proposed to explain the origin of the high velocity outflows (Rozyczka & Tenorio-Tagle 1985; Shibata & Uchida 1986; Canto, Tenorio-Tagle & Rozyczka 1988; Kwan & Tademaru 1988; Blondin, Königl & Fryxell 1989; Raga & Canto 1989). In another context, the nucleus of active galaxies and quasars shows conspicuous radio-lobes located on both sides of a compact central object (Osterbrock 1983; Röser

& Meisenheimer 1986; Asseo & Sol 1987; Wielenbinski 1990). Plausibly, all these structures have a common origin; but this suggestion is as yet highly speculative.

Classification schemes and mechanisms for producing the bipolar outflows in PNe have been proposed in the framework of the interacting winds model (Balick 1987; Icke 1988; Icke, Preston & Balick 1989), considering binary stars (Soker & Livio 1989; Igumenshev, Tutukov & Shustov 1990) or more complex combinations (Morris 1987, 1990; Lutz *et al* 1989). The simultaneous study of both the morphology and velocity fields of proto-PNe and evolved PNe strongly suggests that bipolar PNe are well represented by a relatively dense equatorial torus composed of cool gas and dust. This primary structure is associated with two polar blobs moving at a higher velocity and directed along the major axis (OH 0739–14: Cohen *et al.* 1985; CRL 2688: Nguyen-Q-Rieu, Winnberg & Bujarrabal 1986; Bieging & Nguyen-Q-Rieu 1988; 19W32: Lopez 1987; OH 231.8 + 4.2: Morris *et al* 1987; Cnl-1: Bhatt 1989; IRAS 09371 + 1212: Morris & Reipurth 1990). Rapid polar motions are also found in PNe (M2-9: Walsh 1981; NGC 7026: Solf & Weinberger 1984; Cuesta, Phillips & Mampaso 1990; NGC 2392: Gieseking, Becker and Solf 1985; NGC 6751: Gieseking & Solf 1986; Hbl2: Miranda & Solf 1989; NGC 4406: Sahai *et al.* 1991). But the question of how such a peculiar morphology arises has never been completely elucidated.

Various mechanisms for producing a high-density torus have, however, been proposed. One of the first category of models is based on the ejection by a binary star (Fabian & Hansen 1979; Livio *et al.* 1979; Morris 1981, 1987, 1990; Livio & Soker 1988 and references therein; Kolesnik & Pilyugin 1986). One version of these models invokes an ejection by a wide binary – a possible progenitor for bipolar PNe. Unfortunately, an oblate configuration for the nebular gas is generated in this process and the density minimum will be located in the equatorial plane contrary to the observations (Pascoli 1986, 1990).

The principal reason for this result is the capture of a part of the gas by the secondary in the orbital plane (Kolesnik & Pilyugin 1986). However, because the system tends towards synchronisation the rotation rate of the red giant ejecting the gas can considerably increase. Pilyugin (1987) concludes that this process can contribute to a preferential ejection of the matter towards the equatorial plane. In fact, even with this subsidiary process, it is not clear whether the density will be more concentrated towards the equatorial plane. To be really effective, this process would require the rotational velocity of the red giant to be greater than both the orbital velocity and the escape velocity at the surface of this star. This condition could be, but not always, realized. Furthermore, the velocity of ejection is a decreasing function of the latitude in this model, perhaps contrary to observations.

Another subset of models considers the case of close binaries (Livio *et al.* 1979; Livio & Soker 1988). A non-negligible (~ 15–20%) fraction of PNe is expected to be formed in close binaries in which one component fills its Roche lobe after the exhaustion of hydrogen or helium to its center (Iben & Tutukov 1989). Bipolar PNe are created from binary systems by strong tidal interactions between an orbiting main sequence star or a white dwarf and the expanding red giant envelope. As the orbit shrinks, the compact secondary becomes immersed in a common distended envelope (Meyer & Meyer-Hofmeister 1979; Livio & Soker 1988; Bond & Livio 1990). Livio and Soker argue that a large density contrast in the outflow between the equatorial and polar directions is expected in the case of giants, giving rise to a butterfly nebula. On the other hand, the density contrast is expected to be mild in the case of very evolved AGB supergiants resulting in an elliptical nebula. This raises an interesting question: Are the expansion velocity and the morphology of bipolar PNe closely connected? Indeed, the ejection velocity at the surface of a very extended object such as a high evolved AGB star is low and conversely the ejection velocity is relatively high in the case of a less evolved object. A rapid examination of a catalogue of expansion velocities (Sabbadin 1984; Weinberger 1989) seems to support such a conjecture (see also Pascoli 1990).

Nevertheless, a serious difficulty with the close binary star model is that not all PNe possess binary nuclei. Iben & Tutukov (1989) have estimated the fraction of PNe with close binary systems to be of the order of 15–20%. On the other hand, Zuckerman & Aller (1986) have pointed out that ~ 50% of all PNe display a bipolar morphology and an additional ~ 30% display an elliptical symmetry. To explain this discrepancy, rotation of a single star has been invoked (Calvet & Peimbert 1983) but Pascoli (1986, 1987) and Zuckerman & Gatley (1988) have argued that this is likely to be unimportant.

Another very different and attractive mechanism capable of generating a density contrast is the magnetic field. Pascoli (1987) has argued that bipolar PNe can be shaped by an adequately azimuthal magnetic field deeply generated within the core of a red giant, and subsequently uplifted to the stellar surface by magnetic buoyancy (Pascoli 1992). VLA observations of 1612 MHz and 1665 MHz OH MASERS in circumstellar envelopes associated with cool giant stars support this view (Bowers, Johnston & de Vegt 1989; Nedoluha & Bowers 1992). At the radius of the maser shells (~ 10^4 AU) the magnetic field is estimated to be ~ 0.1 to 10 milliGauss. From this result Nedoluha & Bowers (1992) estimate magnetic field strengths ~ 100 Gauss to 1 kGauss at the stellar surface. These values are in good agreement with previous estimates by Barvainis, McIntosh & Predmore (1987) for several red giant stars based on observations of SiO masers. There is also evidence for magnetic fields in the envelopes of true protoplanetary-nebulae (Hu Jing-Yao 1992) and similar conclusions can be derived for the photospheric magnetic field strengths. These various authors also suggest that the large magnetic field strengths observed for the cool red giant stars may influence both the geometry and initiation of aspherical mass-loss. In Pascoli's model (1987), the bipolar appearance is envisaged as due to a magnetic striction in the equatorial plane accompanied by a deflection of matter towards higher latitudes. The equatorial-to-polar density ratios are rather small and do not exceed 2–3. Accordingly, the theoretical maps nicely reproduce the observational data (Pascoli 1987). Nevertheless, none of the models considered above really explain the high velocity polar outflows. Further enquiries are thus needed.

In an alternative approach, Balick (1987); Balick & Preston (1987), Balick, Preston & Icke (1987), Icke (1988) and Icke, Preston & Balick (1989) have attempted to explain the origin of high velocity polar knots in an interacting-winds-model (Kwok, Purton & FitzGerald 1978; Okorokov *et al.* 1985; Volk & Kwok 1985) taking into account the axisymmetry of the slow wind. The basic principle of this model is that the high velocity polar knots are formed from the collimation of the fast wind by a prolate inner shock (Balick, Preston & Icke 1987; Icke 1988). The reality of such an inner shock has been occasionally questioned. Unfortunately since this shock is much smaller than the outer one it is very difficult to observe directly. However, the recent detection of X-rays from planetary nebulae (Kreysing *et al.* 1992) may be strong evidence for the existence of this inner shock. Using two-dimensional hydrodynamics, Soker & Livio (1989) have investigated the problem of shaping the bipolar PNe by interacting winds. These authors show that a bow shock appears between the inner shock and the outer one, which roughly seems to correspond to the observed polars caps. The calculations of Igumenshev, Tukukov & Shustov (1990) lead to a similar conclusion. However, under the assumption that the magnetic field inhibits heat conduction and that radiative cooling of the shocked fast wind is inefficient, Soker (1990) finds no real focusing of shocked fast wind stream lines towards the polar axis, contrary to Icke's investigations (1988). Instead, Soker suggests that two jets from the binary central star are formed during the time between the end of the slow wind and the beginning of the fast wind.

Before ending this short review, we briefly mention a distinct scenario that has been suggested by Morris (1987, 1990) and which has been applied to He 2–104 by Lutz *et al.* (1989). In this model, a binary star creates a bipolar outflow. The wind of the red giant is partially captured into an accretion disk around the much smaller secondary. Such an accretion disk can collimate a second wind into polar jets. Unfortunately, Icke (1988) concludes that accretion disks do not produce very good collimation of a wind and further enquiries based on hydrocalculations are needed to confirm Morris' scenario. In Pascoli's hydromagnetic model (1987, 1992) high velocity polar jets are missing but a polar concentration of matter produced by a magnetic deflection is present. However, whether this model can really account for the large difference in velocity between the equatorial and polar outflows remains to be demonstrated (Morris 1987). The purpose of the present paper which is a direct continuation of the magnetohydrodynamic model (Pascoli 1987) is to investigate this question further. The following section is devoted to the formal development of the dynamics of a stationary bipolar jet in the framework of this model.

2. Formation of bipolar jets in a stationary hydromagnetic model

The basic equations under the hypothesis of axisymmetry $\left(\frac{\partial}{\partial \phi} \equiv 0\right)$ and absence of rotation can be found in Pascoli (1987, 1992). Spherical coordinates r, θ, ϕ are used. Here, we assume the various functions–the radial and tangential velocities v_r and v_θ, the magnetic field components H_r, H_θ, H_ϕ and the density ρ – to be represented by the following relationship:

$$f(r,\theta,\phi,t) = \bar{f}(r,\theta) + f'(r,\theta,\phi,t) \tag{1}$$

where $\bar{f}$ represents the averaged function with regard to the time and f' the fluctuating part ($\bar{f'} = 0$). Furthermore, we shall impose the subsidiary condition of axisymmetry

$$\bar{f}'^2 \equiv \bar{f}'^2(r,\theta). \tag{2}$$

In other words, the root mean square of the fluctuating part is independent of the azimuthal angle ϕ . Let us now consider two distinct physical quantities f and g. We shall assume that

$$\overline{(\bar{f}+f')(\bar{g}+g')} = \bar{f}\bar{g} + \bar{f}'^2\delta(f,g)$$

where (3)

$$\delta(f,g) = 1 \text{ if } f \equiv g \text{ and } 0 \text{ if } f \not\equiv g.$$

Physically, this expression means that two distinct quantities, for instance v_r and v_ϕ are uncorrelated with regard to the time.

In a steady state both $\bar{f}$ and $\bar{f}'^2$ are independent of time. The equations in Pascoli (1987, 1992) become:

$$\bar{\rho}\left[\bar{v}_r\frac{\partial\bar{v}_r}{\partial r}+\frac{\bar{v}_\theta}{r}\left(-\bar{v}_\theta+\frac{\partial\bar{v}_r}{\partial\theta}\right)\right]=-\frac{\bar{H}}{4\pi r}\frac{\partial}{\partial r}(\bar{H}r)-\frac{1}{8\pi}\frac{\partial}{\partial r}(-\overline{H_r'^2}+\overline{H_\theta'^2}+\overline{H_\phi'^2})$$
$$+\frac{1}{4\pi r}(2\overline{H_r'^2}-\overline{H_\theta'^2}-\overline{H_\phi'^2})+\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\pi_{rr})-\frac{(\pi_{\theta\theta}+\pi_{\phi\phi})}{r} \tag{4}$$

$$\bar{\rho}\left[\bar{v}_r\frac{\partial\bar{v}_\theta}{\partial r}+\frac{\bar{v}_\theta}{r}\left(\bar{v}_r+\frac{\partial\bar{v}_\theta}{\partial\theta}\right)\right]=-\frac{\bar{H}}{4\pi r\cos\theta}\frac{\partial}{\partial\theta}(\bar{H}\cos\theta)-\frac{1}{8\pi r}\frac{\partial}{\partial\theta}(\overline{H_r'^2}-\overline{H_\theta'^2}+\overline{H_\phi'^2})$$
$$+\frac{1}{4\pi r}(\overline{H_\phi'^2}-\overline{H_\theta'^2})\tan\theta+\frac{1}{r\cos\theta}\frac{\partial}{\partial\theta}(\pi_{\theta\theta}\cos\theta)+\frac{\pi_{\phi\phi}\tan\theta}{r} \tag{5}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2\bar{v}_r)+\frac{1}{r\cos\theta}\frac{\partial}{\partial\theta}(\cos\theta\bar{\rho}\bar{v}_\theta)=0 \tag{6}$$

$$\frac{1}{r}\left[\frac{\partial}{\partial r}(\bar{v}_r\bar{H}r)+\frac{\partial(\bar{v}_\theta\bar{H})}{\partial\theta}\right]=0 \tag{7}$$

The large scale frozenin magnetic field is assumed to be entirely azimuthal ($\bar{H}_r\equiv 0$, $\bar{H}_\theta\equiv 0$, $\bar{H}_\phi\neq 0$) and its unique component $\bar{H}_\phi$ will be denoted by $\bar{H}$. Taking into account the definitions adopted for the averaged physical quantities, the turbulent stress tensor takes the following form:

$$\bar{\pi}=-\bar{\rho}\begin{bmatrix}\overline{v_r'^2} & 0 & 0\\ 0 & \overline{v_\theta'^2} & 0\\ 0 & 0 & \overline{v_\phi'^2}\end{bmatrix} \tag{8}$$

With $(\overline{v_r'^2})$, $(\overline{v_r'^2})$ ~ the Alfvén velocity: $v_A=\left(\frac{\bar{H}^2}{4\pi\bar{\rho}}\right)^{1/2}$, $\bar{v}_\phi\equiv 0$ and, $(\overline{v_\phi'^2})^{1/2}$ ~ the, thermal velocity: $v_{th}=\left(\frac{KT}{m_H}\right)^{1/2}$ ~ 1 km/s $\ll v_A$ ~ 20–30 km/s if T~100K (refer Pascoli 1987).

Within the regions where turbulence is assumed to be low (the equatorial plane), we can reasonably consider that $(\overline{H'^2}_{r,\theta,\phi})^{1/2}\sim 0$. On the other hand, at higher latitudes the magnetic field is negligible. Hence, the terms corresponding to the fluctuating part of the magnetic field will be omitted in the following.

The component $\pi_{\phi\phi}=-\bar{\rho}\overline{v'^2_\phi}$ is assumed to be small compared to π_{rr} and $\pi_{\phi\phi}$, except in the polar regions ($\theta>80°$) where we shall assume that $\pi_{\phi\phi}\sim\pi_{\theta\theta}$ to avoid a polar divergence in velocity. At a sufficient large distance from the central star $r>r_1\sim 10^{14}$ cm, the stellar gravitation and the gradient of thermal pressure (T~100K) are negligible compared to both magnetic and turbulent stresses. Accordingly, these former terms are simply ignored in the equations (4)–(7). Finally, averaged viscous stresses are negligible compared to Reynolds stresses.

An important point concerns the timescale to dissipate the turbulent energy and the magnetic field. Magnetic dissipation is very low (Pascoli, Leclercq & Poulain 1992), but the timescale to dissipate the turbulent energy is poorly known, nevertheless it is likely to be greater than the dynamical timescale of the outflow (perhaps $\gg 10^4$ years).

In a steady state, the tangential velocities can be important in the vicinity of the star and of the order of the radial velocities (Pascoli 1987). The main consequence of this is that the matter is deflected by the magnetic stresses towards the poles. This process is similar to a pinch effect instability. At greater distances from the star $r \gtrsim r_1$, a new tangential equilibrium takes place where $\bar{v}_\theta \sim 0$ and $\bar{v}_r \equiv \bar{v}_r(\theta)$ is approximately independent of r.

Pascoli (1987, footnote p. 195) has concluded that the equatorial magnetic stresses ultimately lead to a polar singularity of the type $\rho \sim \cos^{-2}\theta$, $H \sim \cos^{-1}\theta$ if polar anisotropic turbulent stresses are not taken into account to compensate them. With the help of a semiempirical model and with an adequate form for the turbulence (considered here as a free parameter), we shall show that this necessarily leads to the development of polar velocities much greater than the equatorial velocities. In other words, we make the hypothesis that turbulence is inhibited in the vicinity of the equatorial plane by the magnetic field stresses and, conversely, turbulence can freely develop at higher latitudes where the magnetic field is very low. Such a hypothesis could possibly explain the appearance of high velocity bipolar jets in our model.

Two important physical quantities which appear in the problem are the mass loss $\dot{\mathcal{M}}$ and the magnetic field flux ϕ. Both these quantities are assumed to be constant.

First let us consider the motion of a toroidal tube of force from a point (r_0, θ_0) at the stellar surface to another point (r, θ) within the nebular gas (Fig. 1). The equations relating them are:

$$\bar{H}(r,\theta) r \, \mathrm{d}r \, \mathrm{d}\theta = \bar{H}(\theta_0) r_0 \, \mathrm{d}r_0 \, \mathrm{d}\theta_0 \tag{9}$$

$$\bar{\rho}(r,\theta) r^2 \cos\theta \, \mathrm{d}r \, \mathrm{d}\theta = \bar{\rho}(\theta_0) r_0^2 \cos\theta_0 \, \mathrm{d}r \, \mathrm{d}\theta_0. \tag{10}$$

Thus

$$\bar{H}(r,\theta) = \bar{H}(\theta_0) \frac{\bar{\rho}(r,\theta)}{\bar{\rho}(\theta_0)} \frac{r}{r_0} \frac{\cos\theta}{\cos\theta_0}. \tag{11}$$

Pascoli (1987) assumes that $\bar{\rho}(\theta_0) \equiv \rho_0$ and $\bar{H}(\theta_0) = H_0 \cos\theta_0$, where ρ_0 and H_0 are constant. More generally, one can always choose another θ-dependence for $\bar{\rho}(\theta_0)$ and

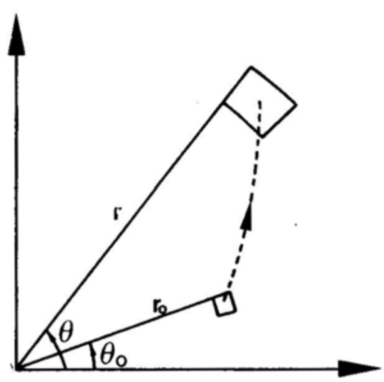


Figure 1. Motion of a meridional cross section of an axisymmetric flux tube.

$\bar{H}(\theta_0)$. For instance, if $\bar{\rho}(\theta_0) = \rho_0 \chi(\theta_0)$ and $\bar{H}(\theta_0) = H_0 \cos\theta_0 \chi(\theta_0)$ where $\chi(\theta_0)$ is any (non singular) function of θ_0 ,(11) becomes:

$$\bar{H}(r,\theta) = H_0 \frac{\bar{\rho}(r,\theta)}{\rho_0} \frac{r}{r_0} \cos\theta \tag{12}$$

As will be noticed, this expression is now independent of θ_0. In other words, (12) can be rewritten in the form:

$$\bar{H}(r,\theta) = H_1 \frac{\bar{\rho}(r,\theta)}{\rho_1} \frac{r}{r_1} \cos\theta \tag{13}$$

H_1 and ρ_1 represent the magnetic field and density values in the equatorial plane at $r = r_1$ respectively.

In this case, the matter can undergo a latitudinal reshaping ($\bar{v}_\theta \neq 0$) which is independent of the boundary conditions at the stellar surface. This latitude reshaping is essentially operative from the stellar surface to a distance $r_1 \sim 10^{14}$cm. In this region the solution is no longer analytical (Pascoli 1987). A latitudinal equilibrium progressively takes place when $r \gtrsim r_t$ with $\bar{v}_\theta \sim 0$ and v_r slightly dependent on r.

Assuming that a steady state defined by $\bar{v}_\theta \sim 0$ and $\bar{v}_r \equiv \bar{v}_r(\theta)$ is approximately realized at $r \gtrsim r1$. We have:

$$\rho(r,\theta) = \rho_j \left(\frac{r_j}{r}\right)^2 F(\theta) \tag{14}$$

where $F(\theta)$ is a function only dependent on θ. Here r_j designates a given value of r much greater than r_1, for instance $r_j \sim 10^{16} - 3.10^{17}$cm.

It follows immediately from (13) that:

$$\bar{H}(r,\theta) = H_j \left(\frac{r_j}{r}\right) F(\theta) \cos\theta \tag{15}$$

To be precise, this statement constitutes a first approximation to the problem because the tangential equilibrium is not rigorously realized. A very slow outflow, not considered here, towards higher latitudes nevertheless persists. $F(\theta)$ is undetermined but this function cannot be arbitrarily fixed. The principal reason is that we have assumed that both the mass loss $\dot{\mathcal{M}}$ and magnetic flux ϕ are constant.

The mass loss $\dot{\mathcal{M}}$ and the magnetic flux $\dot{\phi}$ through a sphere of radius $r_j \gg r_1$ is given by:

$$\dot{\mathcal{M}} = 4\pi \rho_j r_j^2 v_j I[F] \tag{16}$$

$$\dot{\phi} = 2 H_j r_j v_j I[F] \tag{17}$$

Where the functional $I[F]$ is defined by the following integral expression:

$$I[F] = \frac{1}{2} \int_{-\pi/2}^{\pi/2} \cos\theta F(\theta) u(\theta) \mathrm{d}\theta \tag{18}$$

By virtue of both the mass and flux conservation laws, we have not labelled $\dot{\mathcal{M}}$ and ϕ . In these expressions, we have put $\bar{v}_r(\theta) = \bar{v}_j\, u(\theta)$ where $u(\theta)$ is a dimensionless function of θ.

Following Pascoli (1987), Balick, Preston & Icke (1987), it may be assumed that

$u(\theta) = \frac{1}{F(\theta)}$. With this simple hypothesis, we have $\dot{\mathcal{M}} = 4\pi r_1^2 \rho_1 v_1$ and $\dot{\phi} = 2r_1 H_1 v_1$. If $\dot{\mathcal{M}}$ and $\dot{\phi}$ are given, the values of ρ_1 and v_1 are uniquely determined at $r = r_1$ and conversely. Let us choose $r_1 \sim 10^{14}$ cm, $v_1 \sim 10^6$cm/s, $\rho_1 \sim 1.67.10^{-14}$ g.cm^{-3} ($n_1 \sim 10^{10}$cm^{-3}) and $H_1 = 0.5$ Gauss. This gives respectively $\dot{\mathcal{M}} \sim 3.10^{-5}$ M. / year and $\dot{\phi} \sim 3.10^{28}$ Maxwells/decade. (For the sun we have $\dot{\mathcal{M}} \sim 3.10^{-13}$ M. / year and $\phi \sim 3.10^{24}$ Maxwells/decade, Priest 1987). If the ejection of matter is triggered by the magnetic field (Pascoli 1992), it turns out that these results can possibly be sketched by quite a realistic law such as:

$$\dot{\mathcal{M}} \propto (\dot{\phi})^2 \quad \text{or} \quad \dot{\mathcal{M}} \propto \overline{H^2}/8\pi$$

In the present work, we do not suppose that $u(\theta) = \left(\frac{1}{F(\theta)}\right)$ but both $u(\theta)$ and $F(\theta)$ are considered to be independent functions. Unknown quantities of the problem are the radial and tangential turbulent velocities $v'_r(r, \theta, t)$ and $v'_\theta(r, \theta, t)$. For simplicity, we have omitted here the azimuthal coordinate ϕ. Our aim here is to self-consistently determine $u(\theta)$ and $F(\theta)$ and the magnitude of both $(\overline{v'^2_r})^{1/2}$ and $(\overline{v'^2_\theta})^{1/2}$. A few observational constraints have to be imposed: $f(\pi/2)/f(0) \lesssim 3$ (Pascoli 1987), the polar velocities $\bar{v}_r(\pi/2) \lesssim 100$ km/s and $\bar{v}_r(\pi/2)/\bar{v}_r(0) \lesssim 10$. Furthermore, the mass loss and magnetic flux calculated by the equations (16) and (17) have to be in good agreement with the standard values given above. Any combination of $F(\theta)$, $u(\theta)$, $v'_r(r, \theta, t)$, $v'_\theta(r, \theta, t)$ which does not satisfy these various constraints is prohibited.

We shall use a semiempirical model where the turbulent velocity components are given by:

$$v'_r(r,\theta,t) = v_j h(r) m(\theta) \cos \Omega t \tag{19}$$

$$v'_\theta(r,\theta,t) = v_j k(r) g(\theta) \sin \Omega t \tag{20}$$

where h, k are functions of r, and l,m are functions of θ ; v_j is a free parameter analogous to a velocity. Ifthe period of averaging is larger than the period of fluctuations $\sim \frac{2\pi}{\Omega}$, , one can easily verify that $\overline{v'_r v'_\theta} = 0$.

How does the turbulence of this peculiar type appear? We can suggest that rings of matter lying initially in the vicinity of the equatorial plane are pulled out towards the higher latitudes by magnetic instabilities (for instance the Parker instability). This process can feed largescale meridional motions towards the poles. Finally axisymmetric turbulence is triggered where aleatory motions are mainly funneled towards the regions of "smaller impediment" i.e. the polar directions, and $(\overline{v_r'^2})^{1/2} \gtrsim (\overline{v_\theta'^2})^{1/2}$ (the convective cells are found elongated along the polar axis as we shall see below).

For the nonzero components of the turbulent stress tensor $\bar{\bar{\pi}}$, with (19) and (20) we have then:

$$\pi_{rr} = -\tfrac{1}{2}\rho_j v_j^2 \left(\frac{r_j}{r}\right)^2 h^2(r) F(\theta) m^2(\theta) \tag{21}$$

$$\pi_{\theta\theta} = -\tfrac{1}{2}\rho_{jj}^2 \left(\frac{r_j}{r}\right)^2 k^2(r) F(\theta) g^2(\theta) \tag{22}$$

On the other hand, if $\pi_{\phi\phi}$ is assumed to be proportional to $\pi_{\theta\theta}$, we can write:

$$\pi_{\phi\phi} = \chi(\theta)\pi_{\theta\theta} \tag{23}$$

where $\chi(\theta)$ is $\ll 1$ except within the polar regions ($\theta \gtrsim 85°$) where $\chi(\theta) \simeq 1$. Substituting (21),(22) and (23) in equations (4) and (5), and assuming ($\bar{v}_\theta \sim 0$, $\bar{v}_r$ is weakly dependent on $r\left(\frac{\partial \bar{v}_r}{\partial r} \sim 0\right)$ and the fluctuating part of the magnetic field is negligible we have:

$$m^2 \frac{\mathrm{d}h^2}{\mathrm{d}r} = g^2(1+\chi)\frac{k^2}{r} \tag{24}$$

$$\frac{H_j^2}{4\pi_j v_j^2} F\cos\theta \frac{\mathrm{d}}{\mathrm{d}\theta}[F\cos^2\theta] = -\frac{k^2}{2}\frac{\mathrm{d}}{\mathrm{d}\theta}[Fg^2\cos\theta] - \tfrac{1}{2}k^2\chi F g^2 \sin\theta \tag{25}$$

Assuming m(θ) = $g(\theta)$, we can immediately deduce from (25) that $k(r)$ = 1. Equation (24) then gives by simple integration:

$$h(r) = \left(1 + [1+\chi(\theta)]\ln\frac{r}{r_1}\right)^{1/2} \tag{26}$$

Finally, from (25) putting $R_j = \frac{H_j^2}{4\pi\rho_j v_j^2} \sim 1$ we arrive at the following relationship between the function $F(\theta)$ and $g(\theta)$:

$$g(\theta) = \left[2e^{-\int[\cdot\cdot]\mathrm{d}\theta}\left|\int_0^\theta \mathrm{d}\theta e^{\int[\cdot\cdot]\mathrm{d}\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}[F(\theta)\cos^2\theta]\right|\right]^{1/2} \tag{27}$$

where

$$\int[\cdot\cdot]\mathrm{d}\theta = \ln|F(\theta)\cos\theta| + \int \mathrm{d}\theta\,\chi(\theta)\tan\theta \tag{28}$$

Finally, we derive from this set of results that:

$$\overline{v_r'^2} = \frac{v_j^2}{2}\left(1 + [1+\chi(\theta)]\ln\frac{r}{r_1}\right)g^2(\theta) \tag{29}$$

$$\overline{v_\theta'^2} = \frac{v_j^2}{2}g^2(\theta) \tag{30}$$

Instead of the very restrictive condition viz., $\bar{v}_r(\theta) \propto \frac{1}{F(\theta)}$ considered earlier by Pascoli (1987); and Balick, Preston & Icke (1987) we shall use the following more adequate expression:

$$\bar{v}_r = (\overline{v_r'^2} + v_A^2)^{1/2} \tag{31}$$

Where $v_A = \left(\frac{\overline{H}^2}{4\pi\overline{\rho}}\right)^{1/2}$ characterizes the Alfven velocity and $\overline{v_r'^2}$ is defined by (29) Expressed in dimensionless from,(30) gives:

$$\bar{v}_r = v_j u \tag{32}$$

where:

$$u = \left(\frac{1}{2}\left(1 + [1+\chi(\theta)]\ln\frac{r}{r_1}\right)g^2(\theta) + F(\theta)\cos^2\theta\right)^{1/2} \tag{33}$$

v_j represents the equatorial velocity at r_j $\chi(\theta)$ is chosen to avoid a singularity at $\theta = 90°$, for instance

$$\chi(\theta) = e^{-(180/5\pi)^2[(\pi/2)-\theta]^2} \tag{34}$$

One can note that the dimensionless function u varies logarithmically with r. Even though this increases smoothly, it does not appear completely consistent with the trial solution where u is assumed to be only dependent on θ.

As a matter of fact, if we substitute equations (14), (29), (30) and (31) in equation (4), it is easy to see that the three terms $\frac{\rho}{2}\frac{\partial v_r^2}{\partial r}$, $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\pi_{rr}\right)$ and $\frac{\pi_{\theta\theta}}{r}$ nevertheless present a similar comportment. We can thus expect that this comportment is somewhat intuitively correct inspite of some internal inconsistencies.

More generally, if one supposes that u is dependent on both r and θ, let $u \equiv u(r, \theta)$, still assuming $v_\theta \sim 0$, $F(\theta)$ has to be replaced by $1/u(r, \theta)$ in equations (14) – (15) *et seq.*, but the system (24) – (25) unfortunately becomes insoluble in this case and $k(r)$ remains undetermined.

The kinetic energy of turbulence is drawn from the kinetic energy of the mean flow and conversely. A very ravelled situation can appear and this fact is basically responsible for the difficulties encountered above.

In order to preserve the characteristics of the bipolar structure along the outflow (to be precise the equatorial disk structure) we must ensure that the stream lines are quasi-radial at distances $r \gtrsim r_1$ (Pascoli 1987). Otherwise, the matter would be strongly collimated around the polar axis into a very dense column of gas by the magnetic stresses. Assuming the simple situation where the functions $\bar{v}_r$ and $\bar{v}_\theta$ are independent of r and θ, we have for a stream line originating from the vicinity of the equatorial plane:

$$\theta \sim \frac{\bar{v}_\theta}{\bar{v}_r}\ln\frac{r}{r_1} + \theta_1$$

with $\theta_1 \sim 5°$ and $\theta < 10°$ for $r \sim 0.1$ pc, we find $\bar{v}_\theta < 10^{-2}\,\bar{v}_r \ll \bar{v}_r$. The hypothesis $\bar{v}_\theta \sim 0$ is thus fully justified.

3. Discussion of the results

The analytic considerations presented in this paper emphasize the importance of both the magnetic and turbulent stresses in determining the velocity field in PNe. A quick examination of the relations (29) and (30) shows that the turbulent cells are very likely aspherical in the polar region when $r \gg r_1$. These cells are radially elongated: for $r \sim 3.10^{17}$ cm, $\left(\frac{\overline{v_r'^2}}{\overline{v_\theta'^2}}\right)^{1/2} \sim 3$. Assuming a Kolmogoroff Law – a rough approximation for anisotropic turbulence-between the turbulent velocity components and the corresponding cell dimensions l_r and l_θ, (where l_r and l_θ correspond to the radial and tangential dimensions, respectively), we have $\frac{l_r}{l_\theta} \sim 27$. Such cells would then appear similar to filamentary structures directed towards the polar directions. It should be mentioned that polar vortices were also found by Soker & Livio (1989), although in a very different context. These vortices could also be powered by an interacting winds mechanism as argued by these authors. The mean linear diameter of these cells

$l \sim (l_r\ l_\theta)^{1/2}$ can be calibrated by assuming that the size of the turbulent cells in the red giant atmosphere $\sim \frac{1}{10} r_* \sim 3.10^{12}$cm (Schwarzchild 1975) and that the Kolmogoroff Law is roughly valid from the stellar surface up to the nebula. Along this sequence, even though the medium is far from homogeneous $\rho \propto r^{-2}$, this law can provide a good idea of the cell size in the nebular gas. A simple calculation gives thus 1~ 10^{-3} pc for $r \sim 3.10^{17}$cm.

It is perhaps conceivable that these elongated structures ~3.10^{15}cm in size may be identified with those observed in NGC 7293 or other objects, even though it is a little premature to do this (as a matter of fact, another interpretation prevailing at the present time, is in terms of a Rayleigh-Taylor instability in a thin shell of compressed gas).

Elongated cells could be probably identified by polarization measurements. Nevertheless, the polarization vectors would be directed along the polar axis (by alignment of aspherical grains) and conversely the toroidal magnetic field polarizes the light parallel to the equatorial plane. Under low resolution measurements such features will therefore appear globally unpolarized or slightly polarized. Furthermore, the centro-symmetric reflection pattern of polarization produced by dust scattering can strongly affect this simple interpretation (Aspin *et al.* 1989). Four configurations for F(θ) have been considered (Fig. 2). The corresponding functions $g(\theta)$ for the turbulent velocity field are given by equation (27) and Fig. 3.

Figure 4 gives the normalized representation $u(\theta)$ in a meridian plane of velocity fields for the model (l)(4). By way of comparison, Figs. 4 and 6 give results when $\chi(\theta) \equiv 1$, that is when the isotropy for turbulence is realized between the θ- and ϕ-directions. $\dot{\mathcal{M}}$ and ϕ are assumed to be constant in our models i.e. independent of r. The conditions (16) and (17) have thus to be ensured at the very end of the calculations.

In model (1), $F(\theta)$ decreases monotonously towards zero at the pole (θ = 90°). In this model, polar blobs do not exist and polar velocities are unrealistically large (see Table 1). In models (2)–(4) a polar concentration of matter is apparent and the

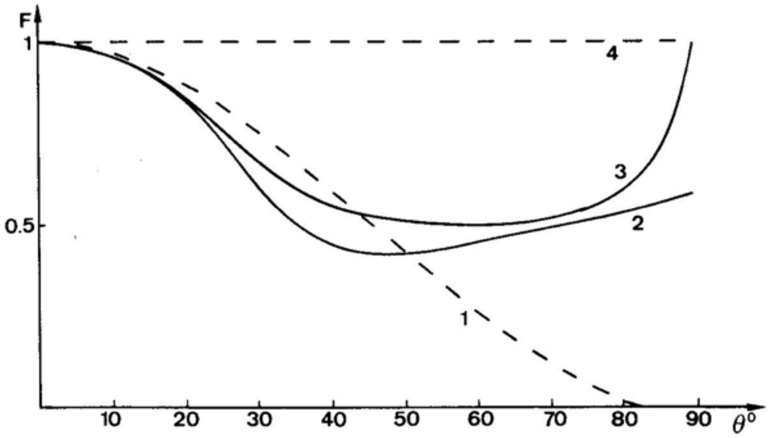


Figure 2 Density profiles as normalized functions of latitude for the four models: $F_1(\theta) = \cos^2\theta$; $F_2(\theta) = 0.580 - 1.599\cos^2\theta + 9.891\cos^4\theta\ 29.280\cos^6\theta + 36.930\cos^8\theta - 15.522\cos^{10}\theta$; $F_3(\theta) = 1 - 4.85\cos\theta + 21.249\cos^2\theta - 44.950\cos^3\theta + 43.019\cos^4\theta - 14.468\cos^5\theta$; $F_4(\theta) = 1$.

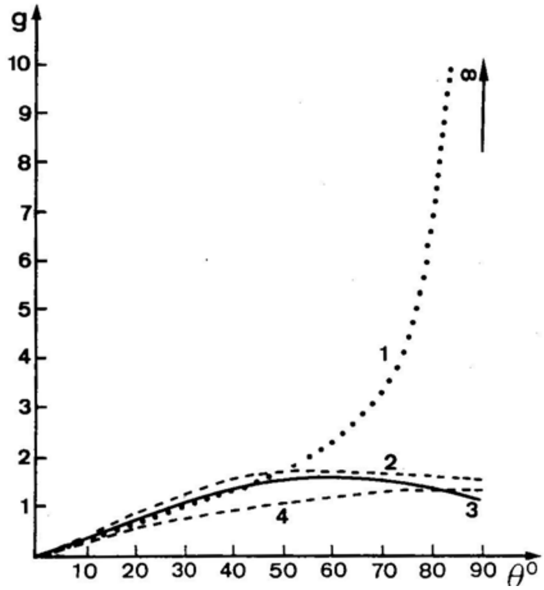


Figure 3. θ–dependence of both the radial and orthoradial turbulent velocity components for the four models presented (see text for the definitions).

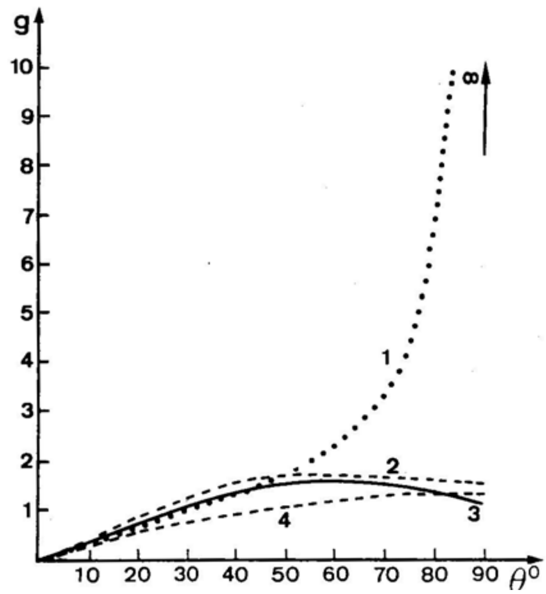


Figure 4. The same as Fig. 3 but for $\chi(\theta) \equiv 1$ ($\pi_{\theta\theta} \equiv \pi_{\varphi\varphi}$).

polartoequatorial velocity ratio is ~ 10, which is quite a realistic value for bipolar jets (Morris 1987, 1990).

Development of both a polar concentration of matter and high but realistic polar velocities are thus concomitant phenomena in our models. The density contrast –the equatorialtopolar density ratio does not exceed 2.5 in model (2) and 2 in model (3).

Table 1. Polar to equatorial velocity ratios, normalized mass loss $I\ [f\ ]$ and equatorial velocity values v_j for the models (l)–(4).

		$\chi(\theta) = e^{-(180/5\pi)^2[(\pi/2)-\theta]^2}$				$\chi(\theta) = 1$			
		(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
r_1	v_{pol}/v_{eq}	∞	3.5	3.3	3.9	∞	1.1	0.8	1
	$I[f]$	∞	0.9	0.9	1.2	∞	0.8	0.8	1.1
	v_1(km/s)	15	15	15	15	15	15	15	15
r_j	v_{pol}/v_{eq}	∞	10.4	9.9	8.3	∞	4.4	3.5	4.1
	$I[f]$	∞	1.8	1.8	2.2	∞	2.0	2.0	2.8
	v_j(km/s)	—	7.5	7.6	8.2	—	6.1	6.2	5.8

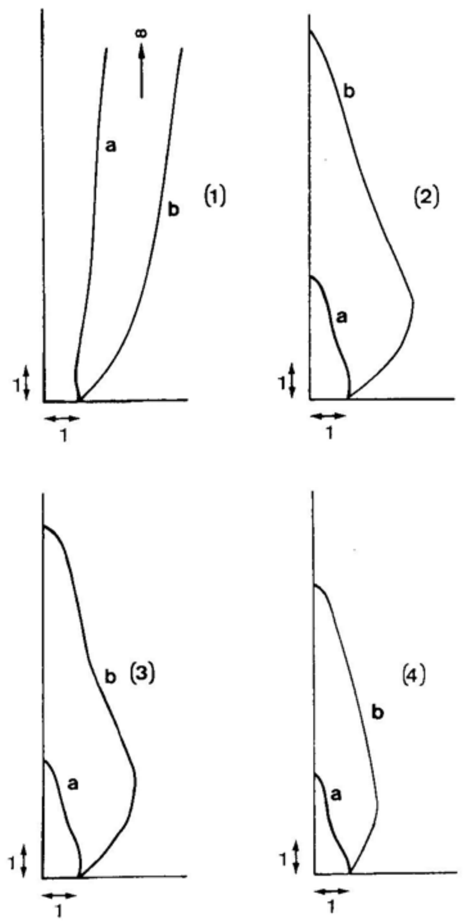


Figure 5. Normalized representation in a meridian plane of the velocity fields for the models (l)–(4). The shapes labelled (a) and (b) are given for $r_1 = 10^{14}$ cm and $r_j = 3.10^{17}$ cm, respectively. The unit length correspond to a velocity of 15 km/s for the case a, but is model-dependent for the case b (see table 1).

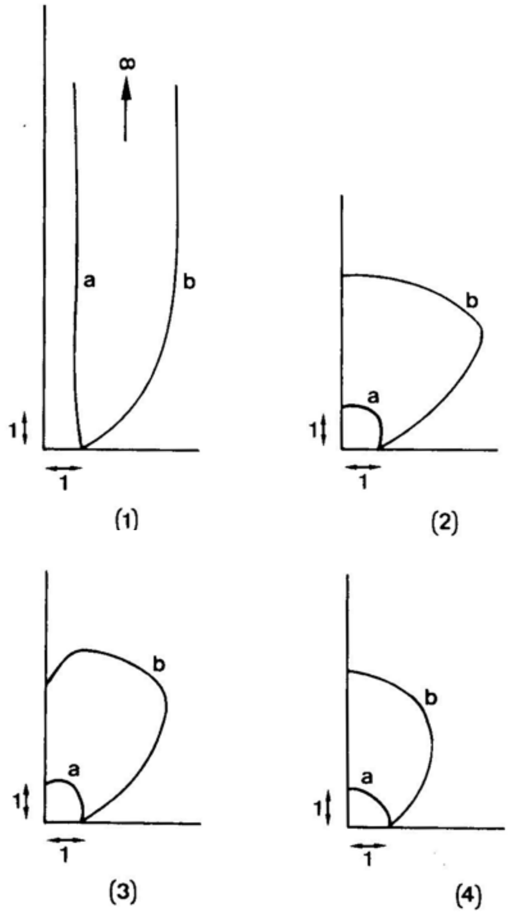
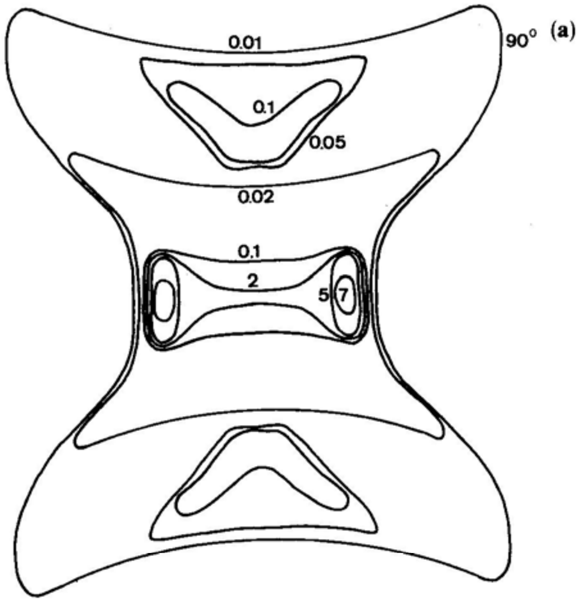


Figure 6 The same as fig. 5 but for $x(q) \equiv 1$ ($\pi_{\theta\theta} \equiv \pi_{\phi\phi}$).

An interesting case is the southern Crabe He 2–104 (Schwarz 1990; Lutz *et al.* 1989; Igumenshev, Tutukov & Shustov 1990). In order to compare with the results of these authors, we must derive isophotal maps from our models. We shall use a procedure previously presented by Pascoli (1987, 1990); Pascoli, Leclercq & Poulain 1992). The nebula is assumed to be isothermal and ionization bounded. The relative thickness of the ionized envelope is roughly of the order of 0.2 (Pascoli 1990). The $H\alpha$ contours were produced by assuming that the emissivity is simply proportional to N_e^2 where N_e designates the electron density.

For models (2) and (3), two polar blobs are apparent on each side of a central bright rectangle (Figs. 5 and 6). Although the density contrast is small and does not exceed 2.5 for model (2) and 2 for model (3), the relative intensity varies enormously from one point to another with a maximum of central intensity ratio as high as 100 when the nebula is seen pole-on. Thus, ionization effects play an essential role in the apparent structure of Proto-PNe and PNe. On the other hand, the polar velocities deduced from models (2) and (3) are of the order of l00 km/s (Table 1). These values are in good agreement with those observed in He 2104 and in other similar objects (Bowers 1991; Corradi & Schwarz 1992).



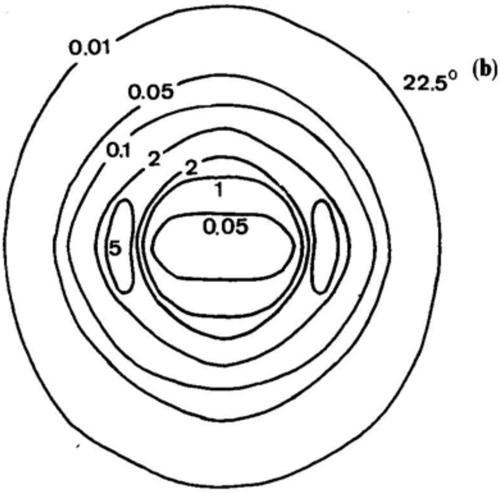


Figure 7(a) and (b). Theoretical isophotal maps for model (2). The angle between the polar axis and the line of sight is indicated at the right top on each map.

One may describe the present models as follows: equatorial magnetic “bottle” funnels the turbulent outflow towards the polar directions into high velocity bipolar jets (a kind of “kettle effect”). Even though this process is especially operative near the star for distances $r \lesssim r_1 \sim 10^{14}$cm, it continues likewise at larger distances because of the long range nature of both magnetic and turbulent stresses. This process differs fundamentally from the other scenarios proposed in the literature (Morris 1987,1990; Icke 1988). In other models the outflow is funnelled directly towards the poles, near the binary system (Morris 1987, 1990) or near the inner shock in the interacting winds model (Icke 1988).

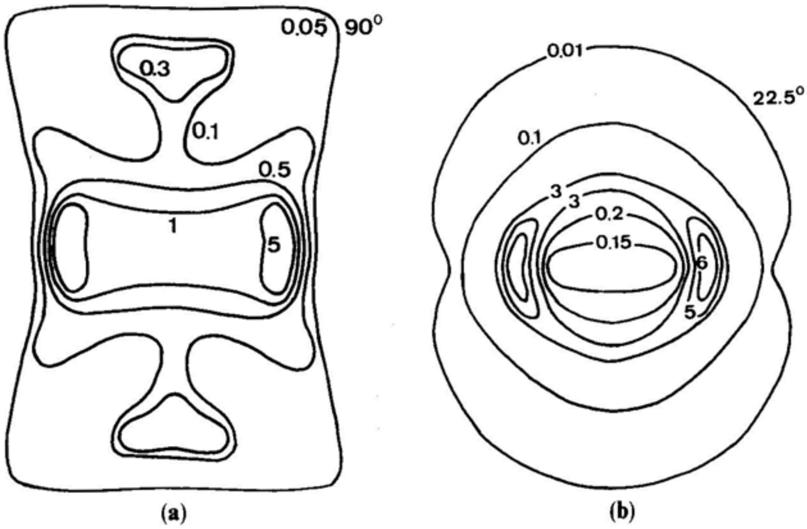


Figure 8(a) and (b). The same as Fig. 7 but for model (3).

4. Conclusions

Following earlier considerations (Pascoli 1987), we have developed here a simple analytical model of bipolar jets in PNe. The model takes into account both an azimuthal magnetic field and turbulent stresses, the latter is assumed to be not inhibited in polar regions. Drastic approximations had to be made in order to solve the problem by analytical methods: the turbulent velocity components are considered as free functions to be self-consistently determined assuming that $\bar{v}_\theta \sim 0$.

Improvements would consist of a non-analytical description of the dynamical structure. However, this procedure will need a detailed treatment of turbulence, a difficult task at the present time owing to the fact that the kinetic and turbulent motions are coupled in a highly intricate manner. Inspite of these simplifications, our calculations succeed in representing most of the bipolar jet characteristics as observed in proto-PNe and PNe. The model gives a simple expression relating the velocity field and the latitudinal distribution in density (equations 28–33).

Our main conclusions are the following:

- The equatorial-to-polar density ratio does not exceed 3. Polar blobs, symmetrically arranged on both sides of an equatorial dense disk, are due to a pinch effect which collimates the matter towards the poles.
- The polar velocities are ~ 100 km/s if we assume that the equatorial velocities are ~ 10 km/s. The matter is slightly (logarithmically) accelerated along the polar axis by turbulent stresses.
- The polar turbulent cells which appear are flattened along the polar directions with a length-to-width ratio ~ 27and amean size ~10^{-3}pc ata distance $r \sim 0.1$ pc.

Even though the present scenario has to be considered as a basic working hypothesis, it can provide an interesting alternative in cases where, for example, the close binary model (Morris 1987, 1990) is not applicable.

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