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Stochastic Stellar Orbits in a Pair of Interacting Galaxies

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Abstract. A dynamical model composed of a disk galaxy with an elliptic companion, moving in a circular orbit, is used in order to study the stellar orbits in a binary galaxy. Using the Poincare surface of section we study the evolution of the stochastic regions in the primary galaxy considering the mass of the companion or the value of the Jacobi's integral as a parameter. Our numerical calculations suggest that the regions of stochasticity increase, as the mass of the companion or the value of the Jacobi's integral increase. An interesting observation is that only direct orbits become stochastic.

Key words: interacting galaxies—surface of section—stochastic orbits

1. Introduction

In many problems in stellar dynamics it is usual to consider galaxies without a nearby important companion. Schwarzschild (1981) calls these galaxies unperturbed galaxies. What in reality happens is that galaxies often appear in pairs, small groups or clusters. This galactic sociality suggests that they interact with their environment. Among these interactions gravitationally-induced collision and mergers of galaxies are possibly the dominant mechanism for their evolution (Schweizer 1986).

A well-known system of interacting galaxies is the Galaxy together with the Large and Small Magellanic Clouds. Information on the dynamical evolution of this triple system can be found in many interesting papers (see Kerr 1957; Avner & King 1967; Fujimoto & Sofue 1976 and references therein).

On the other hand it is worth mentioning the interesting work of Miller & Smith (1980) on the properties of individual encounters between pairs of interacting galaxies. Miller (1986) also discusses a spiral galaxy falling into a galaxy cluster and small galaxies in the potential field of larger galaxies.

Signs of galactic interactions come also from the study of elliptical galaxies with dust lanes. It is believed that the dust lanes are the result of a capture of a gas-rich system by the elliptical galaxy. This idea is strongly supported by recent observations (Bertola & Bettoni 1988) showing that the angular momenta of gaseous and stellar components are antiparallel. This fact supports the acquisition hypothesis and indicates that the dust lanes are a second event in the history of these galaxies.

In an earlier paper (Caranicolas & Vozikis 1988) we have studied the evolution of the families of periodic orbits in a pair of interacting galaxies. The main result of the above work was that the disappearance of the Lagrange points was followed by the disappearance of the majority of the periodic orbits. This suggests that a change in the topology of the potential affects drastically the behaviour of the periodic orbits.

The purpose of the present paper is to study the regular and stochastic motion in the primary galaxy under the influence of the companion. In fact in the absence of the companion all orbits in the primary galaxy are quasi-periodic while, when the companion is present, stochastic regions appear in the Poincare surface of section. We shall try to connect the degree of stochasticity with the value of the mass of the companion or the value of the Jacobi's integral. Our model is described in section 2. The main results of this work are given in section 3 while a discussion and the conclusions of this research are presented in section 4.

2. Description of the model

The primary galaxy in our model is described by Bottlinger force law

$$F_g = \frac{-ar}{1+br^3}, \tag{1}$$

where r is the distance to the centre while a, b are adjustable parameters. The companion galaxy is described by a homogeneous spheroid with a potential law

$$V_c = -\frac{GM_c}{r_c}\left[1+\frac{1}{10}\left[\frac{\varepsilon}{r_c}\right]^2+\frac{9}{280}\left[\frac{\varepsilon}{r_c}\right]^4+\frac{5}{335}\left[\frac{\varepsilon}{r_c}\right]^6+\cdots\right], \tag{2}$$

where M_c is the mass of the companion, r_c is the distance to its centre while $\varepsilon^2 = \alpha^2 - \gamma^2$; α, γ are the two semiaxes of the spheroid. Further information on Bottlinger's model and homogeneous spheroids can be found in Perek (1962).

The companion galaxy moves in a circular orbit of radius R about the primary lying on the plane of the disk, with constant angular velocity $\Omega_p > 0$ given by

$$\Omega_p = \left[\frac{a}{1+bR^3}\right]^{1/2} \tag{3}$$

In our study we use a rotating system with the origin at the center of the primary galaxy which rotates clockwise at the angular velocity of the companion Ω_p. The position of the companion in this system is fixed at $x_c = R$, $y_c = 0$. The total potential V_T responsible for the motion of a star, with a mass $m = 1$, in the disk of the primary galaxy is

$$V_T = V_g(r) + V_c(s), \tag{4}$$

where

$$V_g = -\frac{\alpha}{b^{2/3}}\left[\frac{1}{6}\ln\left(1+\frac{3b^{1/3}r}{b^{2/3}r^2-b^{1/3}r+1}\right)+\frac{1}{\sqrt{3}}\arctan\left(\frac{\sqrt{3}}{2b^{1/3}r-1}\right)\right], \tag{5}$$

with

$$r^2 = x^2 + y^2,\ s^2 = (x-R)^2 + y^2 .$$

The equations of motion are

$$\begin{aligned}\ddot{x} &= -2\Omega_p\dot{y} + \Omega_p^2 x + F_g x/r + F_c(x-R)/s \\ \ddot{y} &= 2\Omega_p\dot{x} + \Omega_p^2 y + F_g y/r + F_c y/s\end{aligned} \tag{6}$$

where $F_g = -\,\mathrm{d}V_g/\mathrm{d}r$, $F_c = -\,\mathrm{d}V_c/\mathrm{d}s$ and the dot indicates derivative with respect to the time. There is one exact integral of motion, the Jacobi's integral, given by the relation

$$J = \tfrac{1}{2}(\dot{x}^2 + \dot{y}^2) + V_T - \tfrac{1}{2}\Omega_p^2 r^2 = h, \tag{7}$$

where h is the numerical value of J, The units of length, mass and time are 20 kpc, $1.8 \times 10^{11}\, M_\odot$ and 0.99×10^8 yr respectively. In this system of units the velocity unit is equal to 197 km/sec while the constant of gravity G is equal to unity.

For large values of r the force given by equation (1) must be that of a point mass. This gives the mass of the primary galaxy as a function of a, b which is

$$M_g = \frac{a}{Gb}. \tag{8}$$

For all the numerical experiments, in the adopted system of units, we take $a = 9.1$, $b = 5.14$ so that $M_g = 1.77 = 3.2 \times 10^{11} M_\odot$. The values of α, γ are 0.15 and 0.075 respectively so that $\varepsilon = 0.06495$. Furthermore, from the series in equation (2) we take only the first four terms.

3. Regular and stochastic orbits

Figure 1 shows the contours of constant effective potential $V_{\mathrm{eff}} = h$, where

$$V_{\mathrm{eff}} = V_T - \tfrac{1}{2}\Omega_p^2 r^2. \tag{9}$$

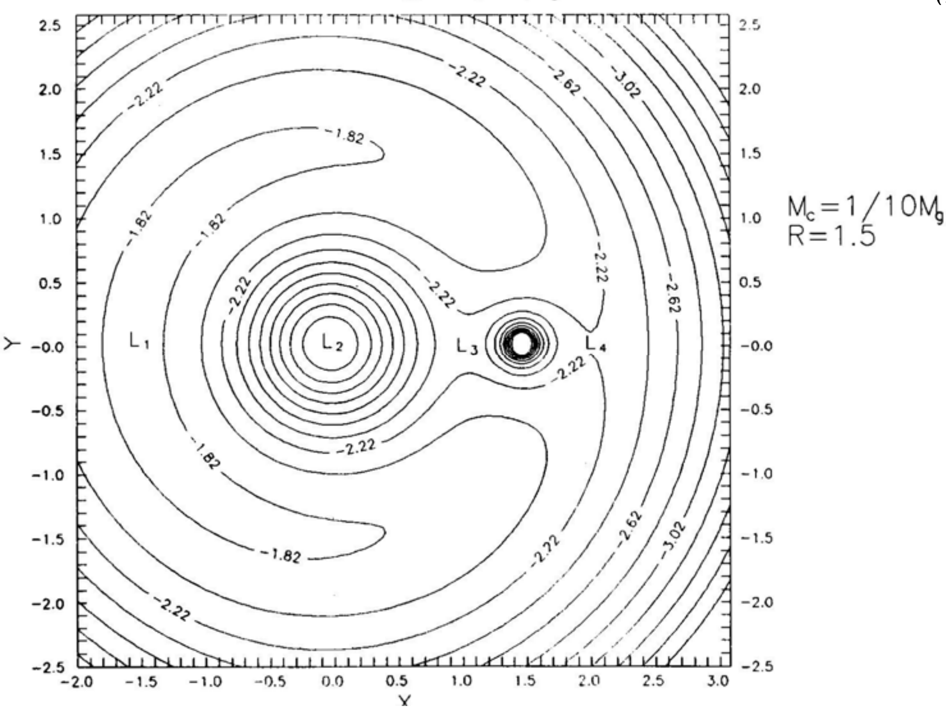

Figure 1. Contours of constant effective potential V_{eff} when $M_c = 0.1\, M_g$, $R = 1.5$.

The values of M_c, R are 0.1 M_g, and 1.5 respectively. The four points L_1, L_2, L_3, L_4 are the Lagrange equilibrium points at which both $\partial V_{eff}/\partial x$, $\partial V_{eff}/\partial y$ vanish. L_1 is a maximum, L_2 is a minimum while L_3, L_4 are saddle points of V_{eff}. The curves in Fig. 1 are often called the zero velocity curves.

In order to study the regular and stochastic motion we use the $(x, \dot{x})$ $(y = 0, \dot{y} > 0)$ Poincare surface of section. In the absence of the companion all orbits in the primary galaxy are regular. Of course this is natural for an integrable system, where all orbits are quasi-periodic. Fig. 2 shows the x, $\dot{x}$ phase space portrait when $R = 1.5$, $M_c = 0.1\ M_g$, $h = -3$. As one can see there are only well-defined invariant curves and the motion is regular. The two stable invariants points D and R represent the direct (i.e. in the same direction as the rotation) and retrograde 1:1 periodic orbits respectively. The outermost curve is the limiting curve defined by $J(x, \dot{x}) = h$.

In the phase space portrait shown in Fig. 3 the values of M_c, R are the same as in Fig. 2 but the value of h is now equal to – 2.3. We can see that the two periodic points D and R are still stable while two new stable periodic points P, Q have appeared. These belong to a stable periodic orbit shown in Fig. 4 together with the direct and retrograde 1:1 periodic orbits. But the most important characteristic is that almost half of the surface of section appears stochastic. It is interesting to notice that all stochastic orbits are direct orbits. This can be easily found if we compare Fig. 3 with Fig. 5, where the shaded area represents the region of starting positions, on the surface of section, of all orbits that are direct.

In order to see whether the companion affects the retrograde orbits we began gradually increasing the value of the mass of the companion up, to 0.5 M_g, taking the appropriate value of the Jacobi's integral so that the corresponding zero velocity curves

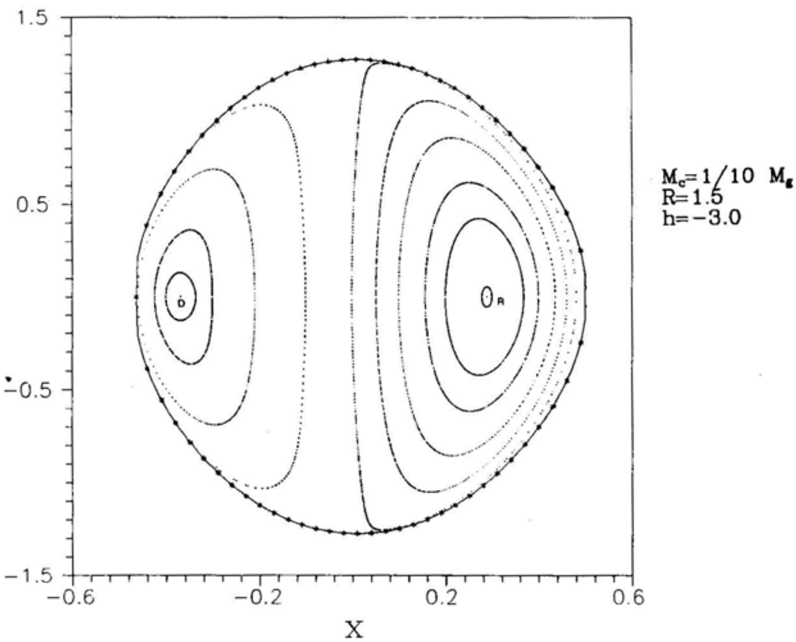

Figure 2. The x–$\dot{x}$ surface of section when $M_c = 0.1\ M_g$, $R = 1.5$, $h = -3.0$.

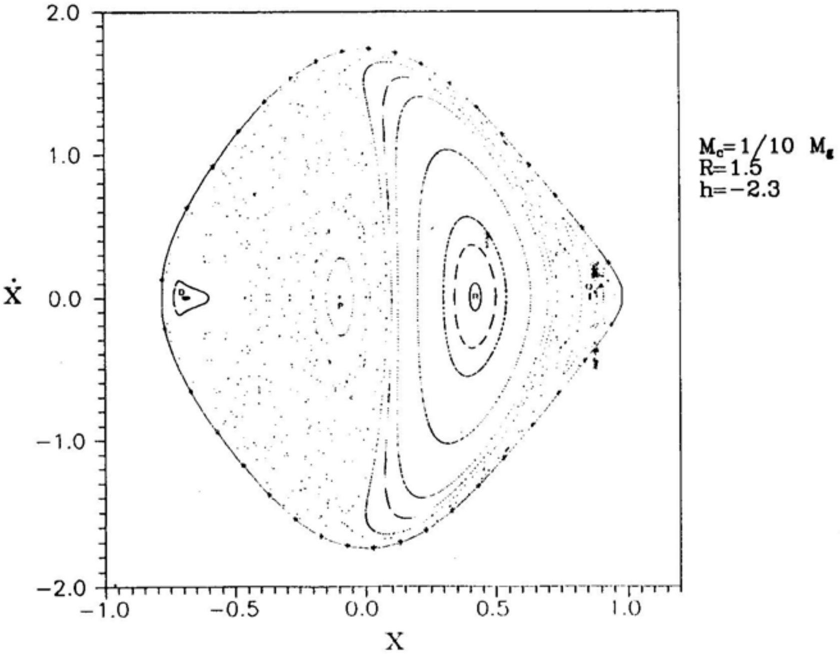

Figure 3. Same as Fig. 2 when $h = -2.3$.

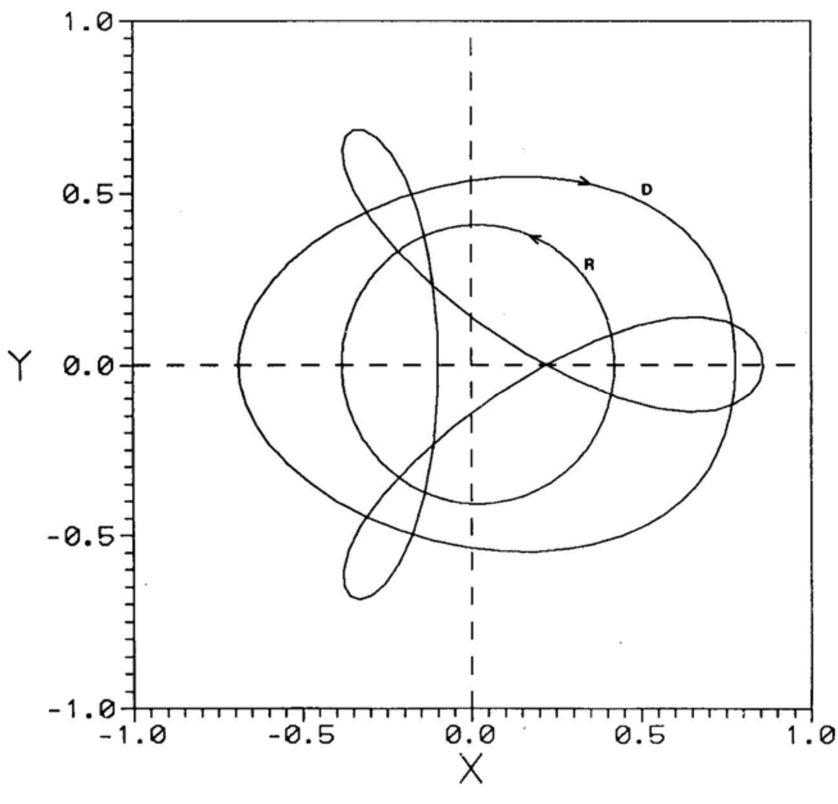

Figure 4. The direct and retrograde 1:1 periodic orbits together with a loop orbit. The values of the parameters are those of Fig. 3 while further details are given in the text.

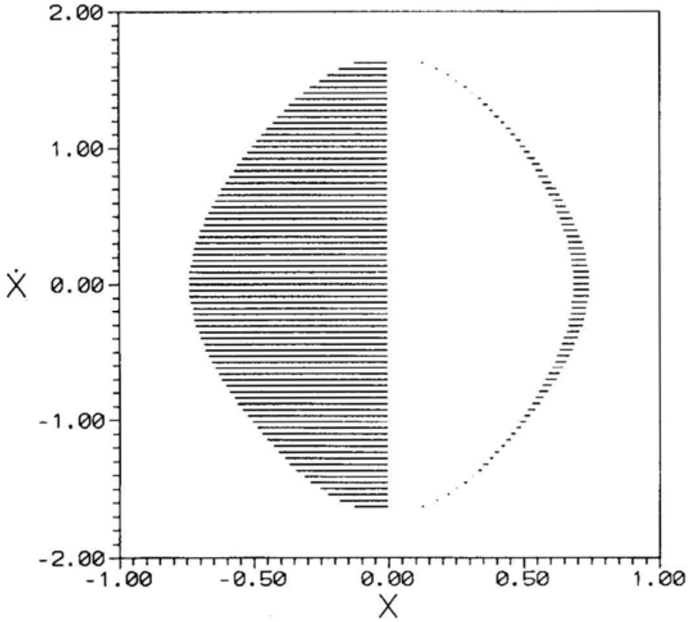

Figure 5. Area of direct orbits when $M_c = 0.1\ M_g$, $R = 1.5$, $h = -2.3$.

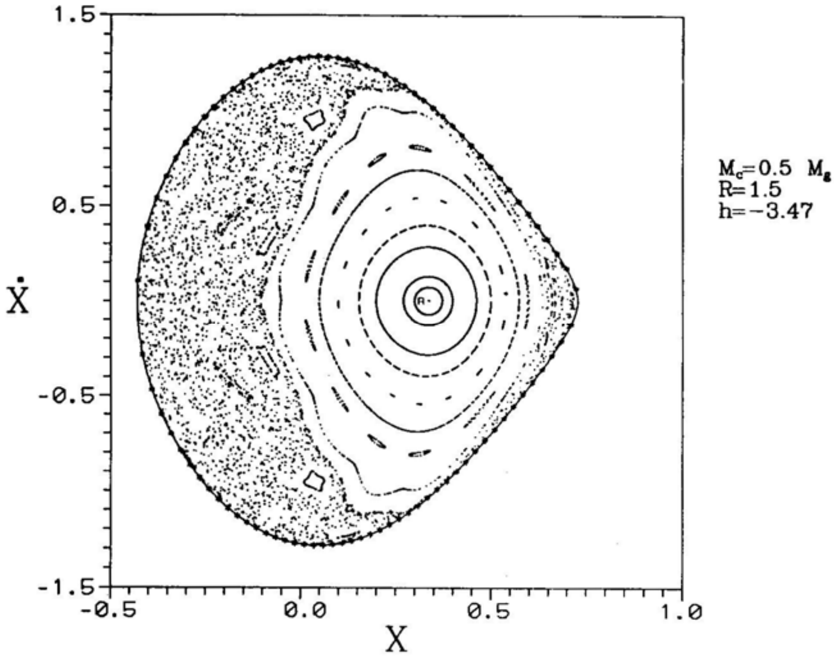

Figure 6. The x -x surface of section when $M_c = 0.5\ M_g$, $R=1.5, h= -3.47$

be closed and computing the corresponding surface of section. Our numerical experiments always showed that the corresponding retrograde motion is regular. Fig. 6 shows a surface of section when M_c =0.5M_g, R = 1.5, h = – 3.47. One can see that almost all the direct orbits are stochastic while the retrograde orbits are regular. We did not feel it was necessary to go to larger masses for the companion, because then, due to the shape of the zero velocity curves, the majority of the orbits would escape. Thus, our numerical calculations strongly suggests that, in the case of stochastic motion induced by a companion galaxy, this motion is present only in the direct orbits.

4. Discussion

The present work was devoted to the study of regular and stochastic orbits in the primary galaxy of a binary system. It is evident that, for a companion of a given mass, the motion in the primary galaxy is regular when the value h of the Jacobi's integral is small. As the value of the Jacobi's integral increases part of the orbits become stochastic. Our numerical calculations show that only direct orbits become stochastic.

In order to check this we computed a large number of surfaces of section using the mass of the companion as a parameter which was increased up to 0.5M_g. For all these numerical experiments the value of h was chosen properly so that the test particle would be able to cover nearly all the galaxy while the corresponding curve of zero velocity was closed. The results of this experiment showed that, even when nearly all direct orbits were stochastic, the retrograde orbits were still regular!

It is natural then to ask: why do only the direct orbits become stochastic? To answer this question we could say that, as the star in the direct orbits rotate at the same direction as that of the companion, it suffers a greater perturbation, because it is closer to the companion during its motion. Therefore, it seems reasonable to expect these orbits to display stochastic behaviour, rather than the retrograde ones, where the star spends only a small amount of time near the companion.

Furthermore, it is well known that, the interaction of a companion with the direct orbits produces spiral structure (see for instance Sundelius *et al.* 1987; Noguchi 1988). In order to relate the spiral structure with the stochasticity found in the present work, one needs a self-consistent model. Since our model is not self-consistent, it is not possible to make such a comparison.

All numerical calculations were made by means of a Bulirch-Stoer method in double precision while the accuracy of the calculations was checked by the constancy of the Jacobi's integral which was conserved up to the fifteenth significant figure. The time scale for the calculation of each orbit on the surface of section was between $0.6 - 1\times10^{10}$yr.

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