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Planetary Distance Law and Resonance

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Abstract. The relation between the planetary distance law and the resonant structures in the solar system and in the satellite systems is shown, in that, the resonance relation has been expressed in terms of Roche's (Bode's) constant defined in the text. This brings forth a coherent, elegant and unified picture of the formation and structure of the solar system and the satellite systems. The Roche's (Bode's) constant is seen to play a central role in this unified picture, in that, it also appears to govern the resonance phenomenon in the systems.

Key words: distance law—resonance—unified picture

1. Introduction

Elaborating the modern Laplacian theory of the formation of the solar system (Prentice 1978a, b) on the basis of Roche limit, Rawal (1984,1986,1989) has arrived at the planetary distance relation in the solar system and in the satellite systems of planets in the following form:

$$R_p = R_* a^p \qquad (p = 1, 2, \ldots, k) \tag{1.1}$$

where R_p is the distance of the pth secondary from the primary (the Sun or a planet), R_* is the present radius of the primary and a the Roche's (Bode's) constant. In the case of the solar system the value of a is $\simeq$ 1.442 and in the cases of satellite systems a has the value $\simeq$ 1.26 (see, Rawal 1984,1986,1989). It is emphasized here that the values of a are not just arbitrary irrational numbers but, indeed, they bear simple relations to whole numbers.

In this, the Kepler's third law assumes the following form:

$$T_p = T_*(a^{3/2})^p \qquad (p = 1,2, \ldots, k) \tag{1.2}$$

where T_p is the revolution period of the pth secondary, T_* is the rotational period of the primary at the time when it attained the present radius (Rawal 1986).

Resonance theory in the planetary/satellite systems states that if n_p, n_{p+1}, n_{p+2} ($n_p = 2\pi/T_p$, T_p being the corresponding orbital period, $n_p > n_{p+1} > n_{p+2}$) denote the mean motions of three successive secondaries going around a primary (orbits assumed circular and coplanar), then a necessary condition for the frequent occurrence of mirror configuration (Dermott 1968a, b, 1973; Rawal 1981, 1986, 1989) is

$$\alpha n_p - (\alpha + \beta) n_{p+1} + \beta n_{p+2} = 0 \tag{1.3}$$

where α, β are small positive integers which are mutually prime. It follows from Equation (1.3) that a reference frame revolving with the mean motion of any one of the three secondaries, the relative mean motions n'_i ($i = p + 1, p + 2$) of the other two are commensurate and that in a frame I (that of the innermost secondary), we have $n'_{p+1} = n_p - n_{p+1}$ and $n'_{p+2} = n_p - n_{p+2}$ and the ratio of these relative mean motions is given as follows:

$$n'_{p+1}/n'_{p+2} = (n_p - n_{p+1})/(n_p - n_{p+2}) = \beta/(\beta + \alpha). \tag{1.4}$$

In terms of revolution periods, Equation (1.4) becomes

$$n'_{p+1}/n'_{p+2} = (T_P - T_{P+1})\,T_{p+2}/(T_p - T_{p+2})\,T_{p+1} = \beta/(\beta + \alpha). \tag{1.5}$$

For a stable threebody resonance, the relative mean motion ratio $\beta/(\beta + \alpha) = 2/3$ and the Equation (1.5) assumes the form:

$$n'_{p+1}/n'_{p+2} = (n_p - n_{p+1})/(n_p - n_{p+2}) = 2/3. \tag{1.6}$$

This is Laplace's resonance relation and the three successive orbits following this relation represent stable motion (Laplace 1805).

There has been a planetary distance law Equation (1.1) and there is a resonance relation Equation (1.3), a special case of which is the Laplace's resonance relation Equation (1.6). Obviously, the link between them is Kepler's third law Equation (1.2). One may now ask the question: Do the planetary distance law and the resonance relation reconcile each other? Naturally, they should. It is shown here that they do.

2. Relation between the planetary distance law and the resonance

Substituting the values of T_i ($i = p, p+1, p + 2$) from Equation (1.2) into the Equation (1.5), we get

$$n'_{p+1}/n'_{p+2} = a^{3/2}/(a^{3/2} + 1). \tag{2.1}$$

In the case of the solar system a has the value ~ 1.442 and hence $n'_{p+1}/n'_{p+2} = 0.6331$ which is very close to 2/3. This shows that a triad of successive planets approximately follow the Laplace's resonance relation, a conclusion already arrived at by Rawal (1981, 1986). In the cases of satellite systems, a has the value $\simeq 1.26$ (see Rawal 1989) and hence $n'_{p+1}/n'_{p+2} = 0.586$ which is close to 4/7 or 3/5, again a conclusion already arrived at by Rawal (1989).

From Equation (2.1), we could see that if we take $\beta = a^{3/2} = f$ and $\alpha = 1$, we could write the original resonance relation Equation (1.3) in the form:

$$n_p - (1 + f)\,n_{p+1} + f n_{p+2} = 0 \tag{2.2}$$

Equation (2.2) could be termed the resonance relation in terms of Roche's (Bode's) constant.

The pairwise resonance is given by the ratio n_2/n_1. In terms of orbital period, it is T_1/T_2. In the case of the solar system, the resonance in a pair of successive orbits $n_2/n_1 = T_1/T_2 = a^{-3/2} = 0.5794$ which is very close to 4/7 or 3/5 as the Roche's (Bode's) constant a has the value $\simeq 1.442$. This has been observed. In the cases of the satellite systems the resonance in a pair of successive orbits turn out to be 0.71 which is very close to 2/3 as a has the value $\simeq 1.26$. This has also been observed.

It has been known that there is a marked preference for near commensurability among pairs of mean motion in the planetary/satellite systems. The reason for this preference, however, was unknown (Roy & Ovenden 1955; Ovenden *et al.* 1974; Goldreich 1965a, b; Dermott 1968a, b, 1973; Greenberg 1973). From the discussion here, it is clear that the preference for near commensurability among pairs of mean motion in the planetary/satellite systems is a consequence of the 'Roche limit-dependent' contraction of the solar nebula and subsolar nebulae (see, Rawal 1984, 1986, 1989).

3. Conclusion

This brings forth a coherent, elegant and unified picture of the formation and structure of the solar system and the satellite systems. The Bode's constant is seen to play a central role, in that, it also appears to govern the resonance phenomenon in the systems.

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