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Accretion Effects on Compact Members of Binary Stars

N. Spyrou *Astronomy Department, University of Thessaloniki, 54006 Thessaloniki, Greece*

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Abstract. We explore the change in the period of axial rotation and in the radius of a magnetized compact star in a binary system, induced by the accretion on it of mass with angular momentum from the surface of its non-compact companion. No specific assumption is made concerning the accretion model, and the primary's interior is described by the Fermi-Dirac statistics for degenerate matter. The rate of change with time of the period and radius is expressed in terms of the compact primary's physical parameters and total absolute luminosity. The conditions are fully derived under which the above changes can be positive, negative or even vanish. In the case of the millisecond pulsars in binary X-ray sources the predicted values of the period time derivative, depending on the values of the accretion rate and. the absolute luminosity, can be positive or negative—if not vanishing—and they fall absolutely in the range 10^{-21} –10^{-17} ss^{-1}, in good agreement with current observational data. The corresponding rate of change of radius, either positive or negative, fall in the range of 10^{-3} –10^{-1} cm y^{-1}. Finally, it is proved that the well-known bursters can be explained by thermonuclear flash due to gravitational instability in the accreted matter, but their explanation as a result of direct contraction could be possible only for quite high accretion rates (>10^{-7} $M_\odot$ y^{-1}). This last result indicates that, in contrast to the accretion-induced change in period, which can be of either sign irrespective of the primary's age, the accretion-induced non-catastrophic contraction is impossible, while according to repent results the contraction in general is possible for young compact objects.

Key words: binary stars—collapsed stars—period and radius change—accretion

1. Introduction and Motivation

One of the most interesting areas of current research in astrophysics, relativistic or not, from either theoretical or observational point of view, is related to the compact stars in binary systems. Much effort has been devoted to the study of the astrophysical and dynamical behaviour of members of such a system and of the binary as a whole. Thus in semi-detached binaries, white dwarfs give rise to cataclysmic variables, while neutron stars give rise to binary X-ray sources, bursters and quasi-periodic X-ray sources (Lewin & van den Heuvel 1983; Shapiro & Teukolsky 1983; Lamb & Paterson 1985). The main idea underlying the above is the accretion of mass with angular momentum on the surface of the compact primary from the photosphere of a normal, giant or supergiant companion star (Ghosh & Lamb 1977; van den Heuvel 1984; for recent results see *e.g.* Hayakawa 1985; Taylor & Stinebring 1986; van den Heuvel, van Paradijs, Taam 1986).

According to standard results, the members, whether compact or not, of a clean binary act on each other gravitationally as point masses. This means that the linear dimensions of the members are small compared to their mutual separation. So the tidal interactions between the members can be ignored as compared with their point-mass gravitational attraction, either Newtonian or relativistic. Of particular interest in this case are the clean binary pulsars, and especially the clean relativistic binary, containing the pulsar PSR 1913+16, which provides strong indication for the detection of gravitational radiation (Weisberg & Taylor 1984; Ehlers & Walker 1984; Spyrou & Papadopoulos 1985). Furthermore, we shall assume that the same dynamical description is applicable even in the case of mass and angular-momentum exchange between the members, due to either gravitational fields or stellar winds from the non-compact member. Such an assumption is justified, especially if the non-compact member is in a stage of stellar evolution, in which it does not fill its Roche lobe. In this case, we shall assume that the mass exchange between the members is through the inner Lagrangian point only, and through the subsequent formation of an accretion disc around the compact member. Under these conditions we are allowed to apply the results of Spyrou (1981a, b) for a dynamical description of the orbital motion of a realistic binary valid to the first post-Newtonian approximation of general relativity.

Due to interaction, with or without mass-exchange, the physical characteristics and parameters of the members change in time due to evolutionary reasons (in case of no mass exchange) or non-evolutionary reasons (in case of mass exchange). For this reason, recently we examined (Spyrou 1985; quoted here as paper 1) the possibility of the change of the radius of a compact star in a binary. More precisely we explored the change of the orbital period of a clean relativistic binary induced by the *evolutionary change* of the radii and axial periods of its members. By applying to the binary pulsar PSR 1913 + 16 we proved that a slow contraction of the younger companion could explain part of the possibly non-vanishing difference between the observed and the predicted values of shortening of the orbital period (see also Spyrou 1987). Finally we proved that the slowing down of axial rotation and the possible contraction, in principle, can occur in young compact stars and contraction is more likely for white dwarfs than for neutron stars, the inverse being true for the slowing down of the axial period.

In this paper we explore the changes of radius and of period of axial rotation of a compact member of a binary system, induced by the accretion on it of mass with angular momentum from the surface of its non-compact companion. This problem is of obvious interest, in view of the multiplicity of interesting phenomena occurring in such systems, especially in those involving millisecond pulsars (Alpar *et al.* 1982), which are being discovered continuously (Dewey *et al.* 1986; Segelstein *et al.* 1986), and at least seven such systems are known at present. From the problems to be examined here, although the accretion-induced change of the radius has been studied (for a neutron-star primary: Ghosh, Lamb & Pethick 1977; Ghosh & Lamb 1979; Shapiro & Teukolsky 1983), the solution has not been explicitly obtained and applied in terms of the binary's physical characteristics. On the other hand, the problem of accretion-induced change of period cannot be considered as generally solved, because the sign of the time derivative of the period is not certain over the whole range of permitted total mass and equations of state for the interior of a neutron star or a white dwarf. The obvious difference with the results of Paper 1 is that there the change of the radius of the *young* compact star, considered as the relic of the violent event, during which the compact star was born, is a decrease, while the change of period is an increase, as due to the loss of electromagnetic radiation (or/and to starquakes, in the form of the *glitches*). Finally, these results themselves could be of some interest to the very problem of the evolutionary history of non-clean binaries with a compact star.

In Section 2 we describe the physics of the binary star and evaluate its total absolute luminosity as a result of the accretion of mass with angular momentum on the magnetized compact member. Hence the rates of change of the period and of the radius are expressed in terms of the accretion rate and the (observationally derived) total absolute luminosity. In Section 3 we apply the results to the millisecond pulsars, binary X-ray sources and bursters, and finally in Section 4 we discuss the results.

2. The physics of the binary system

We consider a binary system composed of a compact primary star (neutron star or white dwarf) and a non-compact companion (normal star, giant or supergiant). As in Paper 1, we assume for simplicity that the compact primary remains spatially homogeneous, spherically symmetric, and uniformly and rigidly rotating, magnetized collapsed star. Also its interior will be described by the Fermi-Dirac statistics of non-relativistic, degenerate matter composed of noninteracting particles (electrons or neutrons).

The binary system is at the stage of accretion of mass with angular momentum on the compact star from the surface of its companion, due mainly to the strong gravitational action of the primary on the photosphere of the companion, or to a lesser degree due to the stellar wind from the companion. Moreover, we shall assume that practically there are no mass losses (*e.g.* through the inner Lagrangian point) or angular-momentum losses, from the system. So the total angular momentum is conserved and the mass removed from the surface of the companion, most probably through an accretion disc, eventually is transfered fully to the surface of the primary. Due to this transfer of mass with angular momentum, the internal characteristics of the two members change in time, and the same is true for the total *inertial mass* (or total mass energy) of the binary. As explained in Spyrou (1981a, b), in order to examine the influence on the orbital motions of the internal characteristics of the binary members, we have to know the orbital motion to post-Newtonian accuracy. In this case the binary's total inertial mass, namely the system's effective mass,

$$M = m_{\mathrm{p}} + m_{\mathrm{c}} \tag{1}$$

in terms of its total *rest mass*

$$\bar{M} = \bar{m}_{\mathrm{p}} + \bar{m}_{\mathrm{c}} \tag{2}$$

and its total *Newtonian self-energy*

$$E^{(\mathrm{s})} = \varepsilon_{\mathrm{p}}^{(\mathrm{s})} + \varepsilon_{\mathrm{c}}^{(\mathrm{s})} \tag{3}$$

is expressed as

$$M = \bar{M} + c^{-2} E^{(\mathrm{s})} \tag{4}$$

where c is the velocity of light in vacuum, the subscripts p and c denote the primary and the companion, respectively, and the symbols $\bar{m}$ and $\varepsilon^{(s)}$ with the proper subscript stand for the rest mass and the total self-energy of the corresponding member.

As a consequence of the above definitions, the binary's absolute luminosity L is defined as

$$L \equiv -\dot{M}c^2 = -\dot{\bar{M}}c^2 - \dot{E}^{(s)} \tag{5}$$

with the dot denoting total time derivative. The luminosity L results from the changes of the rest masses and self-energies of generally both members. Furthermore, according to our basic assumption that no mass (and angular momentum) is lost from the system during the accretion, we shall have

$$\dot{\bar{M}} = 0 \tag{6a}$$

or equivalently,

$$\dot{\bar{m}} \equiv \dot{\bar{m}}_p = -\dot{\bar{m}}_c > 0. \tag{6b}$$

The condition (6a) does not imply that there is no transfer of mass between the two members, as was the case in Paper 1. In its form (6b) it simply implies that the rest mass removed from the companion's surface, is eventually transfered fully to the compact primary's surface. Thus the definition (5) reduces to

$$L = -\dot{E}^{(s)} \tag{7}$$

according to which the absolute luminosity of the pair is due to the change of only the self-energy of the members, resulting from the accretion of mass with angular momentum from the companion to the primary.

Moreover, in a binary like the one considered here, the change of the internal characteristics, due to the mass and angular momentum changes, is not so important for the non-relativistic companion. In fact, it is rather obvious that, unless the accretion rate is extremely heavy, the changes of the companion's radius and rest mass are negligible, and hence the same is true for the change of its various forms of energy (potential, kinetic etc.). Under such conditions, if additionally the linear dimensions of the members are considerably smaller than their mutual distance, then the mutual dynamical interaction of the members will not differ from that of two point-masses. On the other hand, the accretion-induced change of the internal characteristics of the relativistic compact primary can be important, so that structural changes of the latter become important. In view of the above we shall reasonably assume that the pair's luminosity L is practically due to the change of the self-energy of only the primary, namely

$$L \sim -\dot{\varepsilon}_p^{(s)} \equiv -\dot{\varepsilon}^{(s)}. \tag{8}$$

Furthermore, since the accreting matter has also angular momentum, quite generally the conservation of the total angular momentum requires (Shapiro & Teukolsky 1983)

$$\dot{\mathbf{S}} = \dot{\mathbf{s}} + \mathbf{N} \tag{9}$$

where **S** and **s** are the angular momenta of the primary's axial rotation and of the accreting matter, respectively, while **N** is the torque acting on the accreting matter due to *e.g.* viscous effects and the primary's magnetic field *etc.* Here we shall assume that **N** vanishes, postponing for a later study the examination of the case $\mathbf{N} \neq O$ (see, however, Ghosh & Lamb 1979 and Section 3 below). Then, Eq (9) reduces to

$$\dot{\mathbf{S}}=\dot{\mathbf{s}} \quad (\mathbf{N}=0). \tag{10}$$

The two basic Eqs (8) and (10) relate the observationally derived (for known distance of the binary) absolute luminosity (L) with the characteristics of the accreting matter ($\bar{m}$, $\mathbf{s}$) and the subsequently changing internal characteristics ($\varepsilon^{(s)}$, $\mathbf{S}$).

In order to apply the above basic equations, we need a physical model for the primary's interior and of the accretion on it of mass with angular momentum. To this purpose we shall assume, as in Paper 1, that the primary is composed of a perfect fluid with a Fermi–Dirac equation of state of degenerate matter in the form of non-interacting particles (neutrons for neutron stars, and electrons for white dwarfs). The dynamical behaviour of a realistic binary composed of two such bodies has been studied in Spyrou (1981a, b and 1983). These results were then applied in Paper 1 in studying the influence on the binary's orbital period of the changing internal characteristics of the members, namely slowing down and contraction of the axial rotation. Here we shall assume moreover that the primary's perfect-fluid matter is magnetized and has infinite electrical conductivity. The far-field of such an isolated gravitating source has been studied by Spyrou (1984), where it was proved that the inertial (and gravitational) mass of this source is a generalization of the corresponding quantity of Spyrou (1981a, b), such as to include also the energy of the electromagnetic field.

All these results will finally be specialized to the case of a primary, spherical star of radius R, axially rotating in a uniform and rigid-body manner with period. P. In this case in the notation of Paper 1 (see Equation A7 in Paper 1)

$$\frac{\varepsilon^{(s)}}{\bar{m}c^2}=\frac{4\pi^2}{5c^2}\frac{R^2}{P^2}-\frac{3}{5}\frac{G\bar{m}}{c^2R}+A\bar{g}(x)x^{-3}+\frac{B^2R^3}{6\bar{m}c^2} \tag{11}$$

and

$$S=\frac{2\pi}{P}I, \quad I=\frac{2}{5}\bar{m}R^2, \quad xR=\beta\bar{m}^{\frac{1}{3}} \quad (\beta=\text{const.}). \tag{12}$$

Moreover the rate of change, $l\dot{\bar{m}}$, of the angular momentum of the primary and its neighbourhood up to the radial distance r, satisfies

$$\dot{s}=l\dot{\bar{m}} \tag{13}$$

where, according to Eq (14) of Spyrou (1981a), for relativistic orbits,

$$l=(Gm_pr)^{\frac{1}{2}} \tag{14}$$

is the specific angular momentum of the accreting matter (to post-Newtonian accuracy). In the case of a pulsar with a dipole magnetic field $\mathbf{B}$, the above neighbourhood is simply its magnetosphere located at the '*Alfven radius*

$$r_A=(\mu^4/2Gm_p\dot{\bar{m}}^2)^{\frac{1}{7}}, \quad B=2\mu/r^3 \tag{15}$$

where μ is the dipole's magnetic moment. If, moreover, the rate of change of the angular momentum of the pulsar's magnetosphere is negligible, then $l\dot{\bar{m}}$ can practically be identified with the rate of change of simply the pulsar's angular momentum, as is exactly expressed by Eqs (9) and (13).

From the above it becomes transparent that, as in Paper 1, we distinguish m from $\bar{m}$ as far as their inertial properties are concerned. Also we distinguish $\dot{m}$ from $\dot{\bar{m}}$, because here the rate of change of the accreting matter's angular momentum is composed of a part l, depending on the primary's inertial mass, and a part $\dot{\bar{m}}$, depending on the properties of the accreting matter.

Due to the accretion of matter, the primary's physical parameters $\bar{m}$, R, x, B and, hence, $\varepsilon^{(s)}$, s and $\bar{m}$ will change, and this will result in the change of L. The change of $\varepsilon^{(s)}$, for constant $\bar{m}$, has been evaluated in Paper 1, not taking into account the magnetic field. Here this result is generalized, with the aid of the conservation of magnetic flux in the form

$$BR^2 = \text{const.}$$

from which we find

$$\frac{\dot{B}}{B} = -\frac{2\dot{R}}{R}. \tag{16}$$

We stress that the above change of B is different from the decay of magnetic field due to ohmic dissipations, and results simply from the change of R due to accretion ($\dot{R} \gtrless 0$).

In view of that mentioned above the two basic Eqs (8) and (10) can be put in the form

$$\begin{aligned}&\left\{-\frac{4\pi^2}{5}\,\bar{m}\,\frac{R^2}{P^2}+\frac{6}{5}\frac{G\bar{m}^2}{R}-\bar{m}c^2A\left[\frac{\bar{g}}{x^3}+\frac{x}{3}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\bar{g}}{x^3}\right)\right]\right\}\left(\frac{\dot{\bar{m}}}{\bar{m}}\right)\\&+\left[-\frac{8\pi^2}{5}\,\bar{m}\,\frac{R^2}{P^2}-\frac{3}{5}\frac{G\bar{m}^2}{R}+\bar{m}c^2A\,\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\bar{g}}{x^3}\right)+\frac{B^2R^3}{6}\right]\left(\frac{\dot{R}}{R}\right)\\&+\frac{8\pi^2}{5}\,\bar{m}\,\frac{R^2}{P^2}\left(\frac{\dot{P}}{P}\right)=L,\end{aligned} \tag{17}$$

and

$$\left(\frac{l}{l_\mathrm{p}}-1\right)\left(\frac{\dot{\bar{m}}}{\bar{m}}\right)-\frac{2\dot{R}}{R}+\frac{\dot{P}}{P}=0, \tag{18}$$

where

$$l_\mathrm{p}=\frac{4\pi}{5}\frac{R^2}{P} \tag{19}$$

is the specific angular momentum of axial rotation of the primary. Finally, we notice that direct consequence of Eqs (14), (15) and (19) is the useful relation

$$\frac{l}{l_\mathrm{p}}=\frac{[2^{-4}G^3\lambda^{-3}(\mu\bar{m})^2]^{\frac{1}{7}}}{\frac{4\pi}{5}\frac{R^2}{P}}\left(\frac{\dot{\bar{m}}}{\bar{m}}\right)^{-1/7} \tag{20}$$

where

$$\lambda=\frac{\bar{m}}{m}. \tag{21}$$

Eqs (17) and (18) constitute a system of equations for $\dot{P}$ and $\dot{R}$ in terms of L, $\dot{\bar{m}}$ and the companion's physical parameters m, R, P, x and B. Its solution can be written as

$$\frac{\dot{P}}{P}=\frac{1}{S_\mathrm{P}}(L-L_\mathrm{P}) \tag{22}$$

and

$$\frac{\dot{R}}{R}=\frac{1}{2S_{\mathrm{P}}}(L-L_{\mathrm{R}}) \tag{23}$$

where by definition

$$S_{\mathrm{P}}=\frac{4\pi^2}{5}\,\bar{m}\,\frac{R^2}{P^2}-\frac{3}{10}\frac{G\bar{m}^2}{R}+\frac{3}{10}\,\bar{m}c^2x^2+\frac{B^2R^3}{12}, \tag{24}$$

$$L_{\mathrm{P}}=\left[-\frac{4\pi^2}{5}\,\bar{m}\,\frac{R^2}{P^2}\frac{l}{l_{\mathrm{p}}}-\frac{3}{10}\frac{G\bar{m}^2}{R}\left(\frac{l}{l_{\mathrm{p}}}-1\right)+\frac{3}{10}\,\bar{m}c^2x^2\left(\frac{l}{l_{\mathrm{p}}}-\frac{8}{3}\right)+\frac{B^2R^3}{12}\left(\frac{l}{l_{\mathrm{p}}}-1\right)\right]\left(\frac{\dot{\bar{m}}}{\bar{m}}\right), \tag{25}$$

and

$$L_{\mathrm{R}}=\left[-\frac{4\pi^2}{5}\,\bar{m}\,\frac{R^2}{P^2}\left(2\frac{l}{l_{\mathrm{p}}}-1\right)+\frac{6}{5}\frac{G\bar{m}^2}{R}-\frac{1}{2}\,\bar{m}c^2\chi^2\right]\left(\frac{\dot{\bar{m}}}{\bar{m}}\right). \tag{26}$$

We notice that in writing down Eqs (24)–(26) we assumed that the primary's degenerate matter is nonrelativistic, namely

$$x \ll 1 \tag{27}$$

and applied Eqs (A4)–(A6) of the Appendix of Paper 1. Finally we notice that the symbols L_{P} and L_R have entirely different meaning compared to Paper 1.

A rather interesting, although not transparent, feature of Eqs (22)–(26) is that *e.g.* in Eq (22), which is explicitly independent of $\dot{R}$, the right-hand side depends also on the coefficient of $\dot{R}$ in Eq (18) (which happens to be simply a constant). Consequently, it is obvious that if $\dot{R}$ was set equal to zero, the resulting value fo $\dot{P}$ would be different. Analogous arguments are valid for Eq (23).

Finally, from Eqs (22) and (23) it becomes transparent that P and R in general can either increase or decrease or even remain constant during the accretion phase. This problem for a rapidly rotating neutron-star primary is examined in the next section, while the case of a white-dwarf primary will be examined elsewhere.

3. The change of the period and radius of a neutron star

The accretion-induced values of $\dot{P}$ and $\dot{R}$ are evaluated, in terms of the physical characteristics of the primary and the accreted matter, directly and generally via Eqs (22)–(26) under the assumption of a vanishing total external torque. However, before applying these equations we recall the following general form of Eq (10) due to Ghosh, Lamb & Pethick (1977; Eq 58)

$$\frac{\dot{P}}{P}+\frac{\dot{\bar{m}}}{\bar{m}}\left(\frac{l}{l_{\mathrm{p}}}-\frac{\mathrm{d}\ln I}{\mathrm{d}\ln\bar{m}}\right)=0 \tag{28}$$

from which one deduces that the logarithmic derivative d ln I /d ln $\bar{m}$ is related to the accretion-induced change of the primary's radius. Actually, for a spherical star, Eqs

(10) and (28) [or the second of Eqs (12) upon differentiation] imply

$$\frac{2\dot{R}}{R}+\frac{\dot{\bar{m}}}{\bar{m}}\left(1-\frac{\mathrm{d}\ln I}{\mathrm{d}\ln\bar{m}}\right)=0$$

and so, for a given $\dot{\bar{m}}$, both $\dot{P}$ and $\dot{R}$ can be evaluated provided that the value of the logarithmic derivative d ln I/d ln $\bar{m}$ is known. As evaluated by Ghosh, Lamb & Pethick (1977; Fig. 5) using the equation of state of Baym, Pethick & Sutherland (1972) and Pandharipande, Pines & Smith (1976), for neutron stars of intermediate mass ($0.5 \lesssim \bar{m} \lesssim 1.2\, m_\odot$), this logarithmic derivative is positive and slightly above unity and so $\dot{R}$, of either sign, generally is expected to be small, but not necessarily vanishing. Under the same conditions (d ln I/d ln $\bar{m}$ ~ 1), for fast-rotating ($4\pi^2/P^2 \lesssim Gm_p/R^3$) neutron stars, it can be verified that in Eq (28) l/l_p exceeds unity ($r_A \gg R$) for strongly magnetized stars ($B \sim 10^{12}$ G), while it is close to unity ($r_A \sim R$) for weakly magnetized ones ($B \lesssim 10^9$ G).

In view of all the above we shall restrict the applications to the case of fast-rotating, weakly-magnetized neutron stars of intermediate mass, for which the accretion-induced $\dot{P}$ and $\dot{R}$ are expected to be small. It is obvious that for such neutron stars the effect of accretion on P and R is underestimated, but on the other hand it is obvious that such neutron stars are supposed to participate in the interesting phenomena of the X-ray binaries, bursters and quasi-periodic X-ray sources. Hence in the units

$$\hat{m}=1m_\odot,\quad \hat{R}=10^6\text{ cm},\quad \hat{P}=10^{-3}\text{s},\quad \hat{L}=10^{37}\text{ erg s}^{-1},$$
$$\hat{\mu}=10^{30}\text{G cm}^3,\quad \hat{\dot{m}}=10^{17}\text{ g s}^{-1}$$

the physical parameters involved will be denoted by $\bar{m}_{(\odot)}$, $R_{(6)}$, $P_{(-3)}$, $\mu_{(30)}$ and $\dot{\bar{m}}_{(17)}$, respectively, and should not differ greatly from unity. For the above typical values, the last term on the right of Eq (24) is negligibly small compared to the difference of the second and third terms. Also, by comparison with a static neutron star (with λ = 1.1; see Table 1 of Paper 1) for which

$$0.519 \leqslant [\bar{m}_{(\odot)}^{1/3} R_{(6)}]_{\text{static}} \leqslant 0.522, \tag{29}$$

we verify that the condition

$$\frac{G\bar{m}}{R} < c^2 x^2 \tag{30a}$$

reduces to the generally correct condition

$$R < \gamma R_{\text{static}},\quad 2.368 \leqslant \gamma \leqslant 2.382 \tag{30b}$$

so that

$$S_P > 0. \tag{31}$$

It is interesting that the last result does not affect the sign of $\varepsilon^{(s)}$ or, equivalently, the possibility for the star to be considered or not under conditions of stable hydro-dynamical equilibrium ($\varepsilon^{(s)} < 0$).

Similarly Eq (20) takes the form

$$\frac{l}{l_p} = 79.238\, \bar{m}_{(\odot)}^{-3/7} \mu_{(30)}^{2/7} R_{(6)}^{-2} P_{(-3)} \dot{\bar{m}}_{(17)}^{-1/7}. \tag{32}$$

In view of Eqs (20) and (25) the condition

$$L_{\mathrm{P}} \gtreqless 0$$

bounds the value of m and is equivalent to

$$\dot{\bar{m}} \gtreqless \dot{\bar{m}}_{\mathrm{P}} \tag{33}$$

where, by definition,

$$\dot{\bar{m}}_{\mathrm{P}} = \frac{\left(\frac{G\bar{m}}{\lambda}\right)^3 \mu^2}{2^{\frac{1}{2}}\left(\frac{4\pi}{5}\frac{R^2}{P}\right)^7} \left[\frac{\frac{4\pi^2}{5}\frac{R^2}{P^2} + \frac{3}{10}\frac{G\bar{m}}{R} - \frac{3}{10}c^2x^2 - \frac{B^2R^3}{12\bar{m}}}{\frac{3}{10}\frac{G\bar{m}}{R} - \frac{4}{5}c^2x^2 - \frac{B^2R^3}{12\bar{m}}}\right]^7 \tag{34}$$

As a consequence of the condition (30a), the denominator in brackets in Eq (34) is negative. Furthermore, for the numerator we may have

$$A_{\mathrm{P}} \equiv \frac{4\pi^2}{5}\frac{R^2}{P^2} + \frac{3}{10}\frac{G\bar{m}}{R} - \frac{3}{10}c^2x^2 - \frac{B^2R^3}{12\bar{m}} \gtreqless 0, \tag{35}$$

or equivalently

$$P \lesseqgtr P_{\mathrm{P}}, \tag{36}$$

where

$$P_{\mathrm{P}} = \left\{\frac{4\pi^2}{5}R^2\left[-\frac{3}{10}\left(\frac{G\bar{m}}{R} - c^2x^2\right) + \frac{B^2R^3}{12\bar{m}}\right]^{-1}\right\}^{\frac{1}{2}}, \tag{37a}$$

or, equivalently,

$$P_{\mathrm{P}(-3)} = 0.400 R_{(6)}^2 \bar{m}_{(\odot)}^{-1/3}[1 - 0.808\,\bar{m}_{(\odot)}^{1/3} R_{(6)}]^{-\frac{1}{2}}. \tag{37b}$$

We observe that in view of the condition (29) the positivity of P^2 is always guaranteed provided that the condition (30b) is satisfied. Hence P_{P} is a real number, and all the three conditions (35) are meaningful.

In view of the above and since $m > 0$, we can distinguish the following three cases: (i) If

$$A_{\mathrm{P}} > 0, \tag{38a}$$

then

$$\dot{\bar{m}}_{\mathrm{P}} < 0, \quad \dot{\bar{m}} > \dot{\bar{m}}_{\mathrm{P}}, \quad L_{\mathrm{P}} > 0, \tag{38b}$$

and so

$$\dot{P} \gtreqless 0, \quad \text{if} \quad L \gtreqless L_{\mathrm{P}}. \tag{38c}$$

(ii) If

$$A_{\mathrm{P}} = 0, \tag{39a}$$

then

$$\dot{\bar{m}}_{\mathrm{P}} = 0, \quad \dot{\bar{m}} > \dot{\bar{m}}_{\mathrm{P}}, \quad L_{\mathrm{P}} > 0, \tag{39b}$$

and so

$$\dot{P} \gtreqless 0, \quad \text{if} \quad L \gtreqless L_{\mathrm{P}}. \tag{39c}$$

(iii) If

$$A_{\mathrm{P}} < 0 \tag{40a}$$

then

$$\dot{\bar{m}}_{\mathrm{P}} > 0, \quad \dot{\bar{m}} \gtreqless \dot{\bar{m}}_{\mathrm{P}}, \quad L_{\mathrm{P}} \gtreqless 0, \tag{40b}$$

and so we distinguish three subcases

$$\begin{aligned}&\text{(a) for } L_{\mathrm{P}}>0, \quad \dot{P} \gtrless 0, \quad \text{if} \quad L \gtrless L_{\mathrm{P}},\\ &\text{(b) for } L_{\mathrm{P}}=0, \quad \dot{P}>0, \quad \text{because} \quad L>0,\\ &\text{(c) for } L_{\mathrm{P}}<0, \quad \dot{P}>0, \quad \text{because} \quad L-L_{\mathrm{P}>0}.\end{aligned} \tag{40c}$$

The above cases fully determine the conditions under which the accretion causes the compact primary to accelerate (P<0), slow down (P>0) or even retain constant (P=0) its period during the accretion.

During the accretion, radius and density also generally change. In view of Eqs (20) and (26) the condition

$$L_{\mathrm{R}} \gtrless 0 \tag{41}$$

again bounds the value of m and is equivalent to

$$\dot{m} \gtrless \dot{m}_{\mathrm{R}} \tag{42}$$

where by definition

$$\dot{\bar{m}}_{\mathrm{R}}=\frac{\left(\frac{G\bar{m}}{\lambda}\right)^{3}\mu^{2}}{2^{1/2}\left(\frac{4\pi}{5}\frac{R^{2}}{P}\right)^{7}}\left[\frac{\frac{8\pi^{2}}{5}\frac{R^{2}}{P^{2}}}{\frac{4\pi^{2}}{5}\frac{R^{2}}{P^{2}}+\frac{6}{5}\frac{G\bar{m}}{R}-\frac{1}{2}c^{2}x^{2}}\right]^{7}. \tag{43}$$

Noting that m_{R} must be finite, putting

$$A_{\mathrm{R}} \equiv \frac{4\pi^{2}}{5}\frac{R^{2}}{P^{2}}+\frac{6}{5}\frac{G\bar{m}}{R}-\frac{1}{2}c^{2}x^{2} \gtrless 0, \tag{44}$$

and noting that m > 0, we can distinguish the following two cases:

(i) If

$$A_{\mathrm{R}}<0, \tag{45a}$$

then

$$\dot{\bar{m}}_{\mathrm{R}}<0, \quad \dot{\bar{m}}>\dot{\bar{m}}_{\mathrm{R}}, \quad L_{\mathrm{R}}>0, \tag{45b}$$

and so

$$\dot{R} \gtrless 0, \quad \text{if} \quad L \gtrless L_{\mathrm{R}}. \tag{45c}$$

(ii) If

$$A_{\mathrm{R}}>0, \tag{46a}$$

then

$$\dot{\bar{m}}_{\mathrm{R}}>0, \quad \dot{\bar{m}} \gtrless \dot{\bar{m}}_{\mathrm{R}}, \quad L_{\mathrm{R}} \gtrless 0, \tag{46b}$$

and so we distinguish three subcases:

$$\begin{aligned}&\text{(a) for } L_{\mathrm{R}}>0, \quad \dot{R} \gtrless 0, \quad \text{if} \quad L \gtrless L_{\mathrm{R}},\\ &\text{(b) for } L_{\mathrm{R}}=0, \quad \dot{R}>0, \quad \text{because} \quad L>0,\\ &\text{(c) for } L_{\mathrm{R}}<0, \quad \dot{R}>0, \quad \text{because} \quad L-L_{\mathrm{R}}>0.\end{aligned} \tag{46c}$$

The above cases fully determine the conditions, under which the accretion causes the compact primary to expand (R > 0), contract (R < 0) or even retain constant (R = 0) its radius during the accretion.

Although today there is no general consensus on the intensity of the surface magnetic field of a pulsar in binary X-ray sources, bursters and quasi-periodic X-ray sources, we shall conform with the case of an old, weakly-magnetized, millisecond pulsar with typical parameters

$$m_{(\odot)}=1, \quad R_{(6)}=1, \quad P_{(-3)}=1.5, \quad \mu_{(30)}=5\times 10^{-3}.$$

Then from Eqs (24), (25), (32), (34) and (37b) we find

$$S_{\mathrm{P}}=1.40\times 10^{53}\,\mathrm{erg}, \quad \dot{m}_{\mathrm{P}}=4.026\times 10^{18}\,\mathrm{g\,s^{-1}}=6.389\times 10^{-8}\,M_{\odot}\,\mathrm{y}^{-1},$$
$$P_{\mathrm{P}}=0.914\times 10^{-3}\,\mathrm{s}\,(A_{\mathrm{P}}<0), \tag{47}$$

and

$$L_{\mathrm{P}}\gtreqless 0, \qquad \text{if} \qquad \dot{m}_{(17)}\lesseqgtr 40.262, \tag{48}$$

whence Eq (22) takes the form

$$\dot{P}=1.064\times 10^{-19}[L_{(37)}+\dot{m}_{(17)}(0.915-1.552\,\dot{m}_{(17)}^{-1/7})]\,\mathrm{s\,s^{-1}}, \tag{49}$$

and yields

$$\dot{P}\gtreqless 0, \qquad \text{if} \qquad L_{(37)}+\dot{m}_{(17)}(0.915-1.552\,\dot{m}_{(17)}^{-1/7})\gtreqless 0. \tag{50}$$

So the value(47) of the accretion rate *m*(~ 6.39 × 10 $^{-8}$ *M* y $^{-1}$), which is critical of the sign of *P*, is very close to its generally accepts values (~ 10 $^{-9}$ *M* y $^{-1}$) in the binary X-ray sources, bursters and quasi-periodic X-ray sources. Some typical values of *P*, shown in Table 1, either positive or negative, fall in the range 10 $^{-21}$–10 $^{-17}$SS $^{-1}$, and are in satisfactory aggreement with the measured ones (Alpar *et al.* 1982). From the conditions (48) we conclude that small accretion rates result always in a period increase, irrespective of the luminosity, while, for heavier accretion rates, both the period increase and decrease are possible depending on the luminosity.

As far as the change of the radius is concerned, from Eqs (26), (43) and (44) we find

$$A_{\mathrm{R}}<0, \quad \dot{m}_{\mathrm{R}}<0, \quad \dot{m}>\dot{m}_{\mathrm{R}}, \tag{51}$$

and

$$L_{\mathrm{R}}\gtreqless 0, \qquad \text{if} \qquad \dot{m}_{(17)}\gtreqless 2.456, \tag{52}$$

whence Eq (23), due to Eqs (47) takes the form

$$\dot{R}=1.119\times 10^{-3}[L_{(37)}+\dot{m}_{(17)}(1.836\dot{m}_{(17)}^{-1/7}-1.615)]\,\mathrm{cm\,y^{-1}}, \tag{53}$$

and yields

$$\dot{R}\gtreqless 0, \quad \text{if} \quad L_{(37)}+\dot{m}_{(17)}(1.836\dot{m}_{(17)}^{-1/7}-1.615)\gtreqless 0. \tag{54}$$

So, again, the value of the accretion rate *m*(~ 3.903 × 10 $^{-9}$ M y $^{-1}$), which is critical of the sign of *R*, is very close to its generally accepted values (~ 10 $^{-9}$ M y $^{-12}$) in the same sources as in the case of *P*. Some typical values of *R*, shown in Table 1, either positive or negative, fall in the range 10 $^{-3}$–10 $^{-1}$ cm y $^{-1}$ for typical absolute lumen-osities 10 ^{37}–10 ^{39} ergs $^{-1}$. From the conditions (52) we conclude that small accretion rates result always in the star's expansion, irrespectively of the luminosity, while for heavier accretion rates both changes are possible depending on the luminosity.

The special case of a burster (L ~ 10^{39} ergs $^{-1}$) is attributed to instabilities in the accretion of matter or to thermonuclear flashes in the smoothly accreted matter on the

Table 1. Typical values of P and R for binary X-ray sources(**BXS**) and bursters (**BST**).

$(\bar{m}_{(\odot)} \equiv 1,\ R_{(6)} = 1,\ P_{(-3)} = 1.5,\ \mu_{(30)} = 5 \times 10^{-3},\ \dot{\bar{m}}_{P(17)} = 40.262,\ \dot{\bar{m}}_R < 0)$

$\dot{\bar{m}}_{(17)}$	$L_{(37)}$	$\dot{P}(\mathrm{s\,s^{-1}})$	$\dot{R}(\mathrm{cm\,y^{-1}})$	Object
1	1	3.870×10^{-20}	1.367×10^{-3}	BXS
1	10^2	1.060×10^{-17}	1.122×10^{-1}	BST
2.456	1	-1.122×10^{-20}	1.119×10^{-3}	BXS
2.456	10^2	1.052×10^{-17}	1.119×10^{-1}	BST
5	1	-6.288×10^{-20}	9.447×10^{-4}	BXS
5	10^2	1.047×10^{-17}	1.117×10^{-1}	BST
40	1	-2.570×10^{-21}	-2.264×10^{-2}	BXS
40	10^2	1.060×10^{-17}	8.815×10^{-2}	BST
41	1	1.160×10^{-19}	-2.341×10^{-2}	BXS
41	10^2	1.060×10^{-17}	8.739×10^{-2}	BST
100	10^2	1.182×10^{-17}	3.763×10^{-2}	BXS
150	10^2	1.313×10^{-17}	-3.098×10^{-1}	BST

neutron-star's surface. In the latter case the potential energy of the (practically static) accreted matter, in the pulsar's gravitational field, changes and the flash takes place, when the extra potential energy will exceed the kinetic energy of thermal motion of accreted matter, namely (k is the Boltzamann's .constant)

$$\left(1 - \frac{kT}{5G\bar{m}\mu_m}\right) 2\frac{\dot{\bar{m}}}{\bar{m}} - \frac{\dot{R}}{R} = 0, \tag{55}$$

where T and μ_m are absolute temperature and molecular weight of the accreted matter. For hydrogen burning ($T \sim 10^7$ **K**, $\mu_m = 1.673 \times 10^{-24}$g). Eq (55) reduces to

$$\frac{\dot{R}}{R} = 2\frac{\dot{\bar{m}}}{\bar{m}} > 0, \tag{56}$$

implying a radius increase (due to the mass, accretion) followed by the thermonuclear flash (due to the compression) Eqs (53) and (56) hold simultaneously under the condition

$$\dot{\bar{m}}_{(17)} > 2.033 \times 10^{-3} \tag{57}$$

giving the smallest permissible accretion rate for thermonuclear flashes to take place, independently of the luminosity. Moreover, for $m_{(17)}$ in the range 10-100, the luminosity is typical for bursters, $L_{(37)} \sim 10^2$. However, the upper limit of this range should not be considered as a normal feature of the binary and could possibly be met during only the final catastrophic coalescence of a pair's members. On the other hand, *e.g.*, for $m_{(17)} \sim 41$ (see also Eq 48) the increase of mass and radius are 10^{-14} M and 10^{-8} cm for type-**I** bursters (preparation time ~ 10s: Kourganoff 1980), and 10^{-11} M and 10^{-5} cm for type-**II** bursters (preparation time ~1 h). Finally from Eq (53) we verify that for $L_{(37)} \sim 10^2$, direct contraction ($R < 0$) is possible provided that $m_{(17)} > 141$, whence (see last row of Table 1) the corresponding radius change is very large: ~ —10^{-1} cm y^{-1}. In view of the above, the bursters most probably cannot be explained as a result of the pulsar's direct contraction, because both the accretion rate and the radius decrease required are very large.

4. Concluding remarks and outlook

The content of the present article constitutes an as complete and consistent as possible theoretical framework for examining the changes, structural or not, induced on a spherical compact star from the accretion of mass with angular momentum from its non-compact companion in a binary star. The present contributions can be summarized as follows:

(i) To the extent of our knowledge, the article's theoretical framework is the first one in the literature, in which the changes of the compact star's radius (apart from the period) is explicitly evaluated, presented and used.

(ii) The theoretical framework is valid irrespectively of the nature of the compact star (white dwarf or neutron star), and of the specific way of accretion.

(iii) The theoretical framework takes into account all the internal characteristics of the members, assuming the primary's interior to be a uniform degenerate Fermi gas (of electrons or protons) either relativistic or not. It is important that the assumption of relativistic or non-relativistic Fermi gas is only slightly changing the dependence of the results on x (see Eqs 29 and 30 on p. 362 of Chandrasekhar 1939).

(iv) The theoretical framework takes into account and is based on the binary's relative motion correct to post-Newtonian accuracy of general relativity, and not simply the Newtonian, Keplerian orbit. In connection with this, according to Spyrou (1981a, b), such a relativistic dynamical description of a realistic binary requires the use of the binary's total inertial mass (see Eqs 1–4 in the text). This mass generalizes the concept of the Einstein-Infeld-Hoffmann (1938) point mass in the sense that the relativistic correction is described by the mass which is equivalent to the total Newtonian self-energy $\varepsilon^{(s)}_{p}$.

(v) Furthermore, the self-energy is not the same as the one derived in Paper 1, because it contains additionally the contribution of the compact star's magnetic field to its total self energy.

(vi) The compact star's absolute luminosity is defined (see Eq 8 in the text) in terms of its total Newtonian self-energy and not simply its potential energy (namely $L \sim G\bar{m}\dot{\bar{m}} /R$) as elsewhere (see *e.g.* Shapiro & Teukolsky (1983; Eqs 15.1.5, 15.2.20); Ghosh *et al* (1977; Eq 73); Ghosh & Lamb (1979; Section IIIc).

(vii) The theoretical framework provides a full determination of the conditions under which, due to the accretion, the axial period and the radius of the compact star increase, decrease or remain constant.

(viii) In the examined cases of the millisecond pulsars participating the binary X-ray sources, the predicted values for the change of the axial period, either positive or negative, conform with current observational data. Also the predicted change of the radius, for small accretion rates is always an increase irrespectively of the luminosity, while for heavier accretion rates it is either positive or negative depending on the luminosity. In terms of such changes of the radius, the usual phenomenon of the bursters is explained as a result of an accretion-induced thermonuclear flash.

(ix) The theoretical framework applied to the case of old pulsars, naturally completes and extends the results of Paper 1 valid for young compact stars.

In spite of all the advantages above, we have to stress that our results, although conforming with observational data, are approximate. Thus, the compact star has been treated as spherical and homogeneous with its interior described by the relatively simple Fermi–Dirac statistics. Although we did not check it, we believe that these simplifications will affect our numerical results in a non-significant way. Also, in the currently unknown 2nd Post-Newtonian-Approximation (PNA) extension of the author's dynamical description of the binary's relative orbital motion, a 1st-PNA correction to $\varepsilon_p^{(s)}$ would be necessary. But this has not been examined up to now. Moreover, we neglected the external torque acting on the accreting matter. This assumption, however, at least in the case of the X-ray binaries, cannot be considered as always correct, so that it can change the sign of $\dot{P}$ (and perhaps $\dot{R}$) or even invalidate our results. In any case our results are true for at least a weakly-magnetized neutron-star companion. Furthermore, it could present some interest to examine more closely the case, in which the mass and angular-momentum losses are non-negligible. Finally, our numerical results refer only to pulsars and it certainly will be of particular interest to examine also the case of a white-dwarf primary. Such problems are currently under investigation in the context of the more general research programme initiated by Spyrou (1985, 1987) and continued with the present article.

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