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Thermal-Convective Instability of a Composite Rotating Plasma in a Stellar Atmosphere with Finite Larmor Radius

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Abstract. Thermal-convective instability of a hydromagnetic, composite, rotating, inviscid and infinitely conducting plasma in a stellar atmosphere has been studied in the presence of finite Larmor radius. It is found that the criterion for monotonic instability holds good in the presence of the effects due to rotation and finite Larmor radius.

Key words: Stellar atmosphere—convective instability, composite plasma

1. Introduction

The instability in which motions of a thermally unstable atmosphere are driven by buoyancy forces, has been termed as 'thermal convective instability'. Defouw (1970) has generalized the Schwarzschild criterion for convection to include departures from adiabatic motion and has shown that a thermally unstable atmosphere is also convectively unstable, irrespective of the atmospheric temperature gradient. He has found that an inviscid stellar atmosphere is unstable if

$$D = \frac{1}{C_p}(L_T - \rho\alpha L_\rho) + \kappa k^2 < 0, \tag{1}$$

where L is the heat-loss function and α, κ, k, L_T, L_ρ denote, respectively, the coefficient of thermal expansion, the coefficient of thermometric conductivity, the wave number of perturbation, the partial derivative of L with respect to temperature T and the partial derivative of L with respect to density ρ, both evaluated in the equilibrium state. C_p is the specific heat at constant pressure.

The effects of uniform rotation and a uniform magnetic field on thermal convective instability of a stellar atmosphere have been studied by Defouw (1970) and independently by Bhatia (1971), Sharma & Prakash (1977) have studied the finite Larmor radius effect on thermal convective instability of a stellar atmosphere. It has been found in the above studies that inequality (1) is a sufficient condition for monotonic instability, for situations of astrophysical interest. In the above studies, a fully ionized plasma has been considered. Quite frequently the plasma is not fully ionized and may, instead, be permeated with neutral atoms. As a reasonably simple approximation the plasma may be idealized as a mixture of a hydromagnetic (ionized) component and a neutral component, the two interacting through mutual collisional effects. Recently, Sharma (1986) has studied the thermal convective instabiity of a composite plasma in a stellar atmosphere with Hall currents.

It may, therefore, be of importance and is the objective of the present paper to study the result of simultaneous inclusion of finite Larmor radius and collisional effects on the thermal convective instability of a composite, inviscid and infinitely conducting plasma in a stellar atmosphere. The effect of a uniform rotation has also been included.

2. Perturbation equations

Here we consider an infinite horizontal composite layer consisting of an inviscid and infinitely conducting, hydromagnetic, incompressible fluid layer of denisty ρ and a neutral gas of density ρ_d, which is in a state of uniform rotation $\mathbf{\Omega}$ = (0,0, Ω) acted on by a vertical magnetic field $\mathbf{H}$ = (0, 0, H) and gravity force $\mathbf{g}$ = (0,0, $-g$). This layer is heated from below such that a steady temperature gradient $\beta(= |\mathrm{d}T\ /\ \mathrm{dz}|\ )$ is maintained. Regarding the model under consideration we assume that both the ionized fluid and the netural gas behave like continuum fluids and that the effects on the neutral component resulting from the presence of gravity and pressure are neglected which is justified in the present context (Hans 1968). The magnetic field interacts with the ionized components only. It is to be noted that in a rotating star which contains a magnetic field, both the axis of rotation and the direction of the magnetic field will differ from that of gravity at an arbitrary point on the star's surface and thus the present investigation may be regarded as a special case of the more general, real, physical situation.

The linearized equations governing the motion of the mixture of the hydromagnetic fluid and a neutral gas are:

$$\rho\frac{\partial \mathbf{u}}{\partial t} = -\nabla\delta p + \frac{1}{4\pi}(\nabla \times \mathbf{h}) \times \mathbf{H} + \mathbf{g}\delta\rho + 2\rho(\mathbf{u} \times \mathbf{\Omega}) + \rho_d \nu_c(\mathbf{u}_d - \mathbf{u}), \tag{2}$$

$$\frac{\partial \mathbf{u}_d}{\partial t} = -\nu_c(\mathbf{u}_d - \mathbf{u}), \tag{3}$$

$$\frac{\partial \mathbf{h}}{\partial t} = (H \cdot \nabla) \cdot \mathbf{u}, \tag{4}$$

$$\nabla \cdot \mathbf{u} = 0, \qquad \nabla \cdot \mathbf{h} = 0, \tag{5}$$

Where $\mathbf{u}(u,\ \upsilon,\ w)$,$\mathbf{h}(h_x,\ h_y,\ h_z)$, $\delta\rho$ and $\delta\rho$ denote the perturbations in velocity, magnetic field $\mathbf{H}$, density ρ, and pressure p, respectively; $\mathbf{g}$, u_d, u denote respectively the gravitational acceleration, the velocity of the neutral gas and the collisional frequency between the two components of the composite medium.

For the vertical magnetic field $\mathbf{H}$(0, 0, H) the stress tensor components $\overleftrightarrow{\mathbf{P}}$ taking into account the finite-ion gyration, have the components

$$P_{xx} = -\rho\nu_0\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \quad P_{xy} = P_{yx} = \rho\nu_0\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right),$$

$$P_{xz} = P_{zx} = -2\rho\nu_0\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right), \quad P_{yy} = \rho\nu_0\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),$$

$$P_{yz} = P_{zy} = 2\rho\nu_0\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right), \quad P_{zz} = 0. \tag{6}$$

In Equation (6), $\rho\nu_0 = NT / 4\omega_H$, where ω_H, is the ion-gyration frequency, while N and T denote, respectively, the number density and the ion temperature.

The first law of thermodynamics may be written as

$$C_v \frac{\mathrm{d}T}{\mathrm{d}t} = -L + \frac{K}{\rho}\nabla^2 T + \frac{p}{\rho^2}\frac{\mathrm{d}\rho}{\mathrm{d}t}, \tag{7}$$

where C_v, K, T, and t denote respectively, the specific heat at constant volume, the thermal conductivity, the temperature, and time.

Following Defouw (1970), the linearized perturbation form of Equation (7) is:

$$\frac{\partial\theta}{\partial t} + \frac{1}{C_p}(L_T - \rho\alpha L_\rho)\theta - \kappa\nabla^2\theta = -\left(\beta + \frac{g}{C_p}\right)w, \tag{8}$$

where θ is the perturbation in temperature. In obtaining (8), use has been made of the Boussinesq equation of state

$$\delta\rho = -\alpha\rho\theta. \tag{9}$$

We consider the case in which both the boundaries are free and the medium adjoining the fluid is non-conducting. The case of two free boundaries is the most appropriate for stellar atmosphere (Spiegel 1965). The boundary conditions appropriate for the problem (*cf.* Chandrasekhar 1961) are

$$w = 0, \quad \theta = 0, \quad \frac{\partial^2 w}{\partial z^2} = 0, \quad \frac{\partial\zeta}{\partial z} = 0, \tag{10}$$

$\xi=0$ and h_x, h_y, h_z are continuous with an external vacuum field. If the fluid is bounded by an infinitely conducting boundary, no disturbance within it can change the electromagnetic quantities outside. Hence, we have

$$h_z = 0 \tag{11}$$

at a surface bounded by an ideal conductor and, if we further assume that there are no surface charges or surface currents, we may as well take

$$D\xi = 0. \tag{12}$$

The symbols ζ and ξ denote the z-component of vorticity and current density, respectively.

3. Dispersion relation and discussion

Analyzing in terms of normal modes, we seek solutions whose dependence on space and time coordinates is of the form

$$\exp(ik_x x + ik_y y + nt)\sin k_z z, \tag{13}$$

where k_z is an integral multiple of π divided by the thickness of the fluid layer, $k = (k_x^2 + k_y^2 + k_z^2)^{1/2}$ is the wave number of the perturbation and n is the growth rate.

Eliminating u_d between Equations (2) and (3), Equations (8) and (2)–(5) give

$$n'\nabla^2 w = g\alpha\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+\frac{H}{4\pi\rho}\frac{\partial}{\partial z}\nabla^2 h_z - 2\Omega\frac{\partial\zeta}{\partial z}+\nu_0\left(\nabla^2-3\frac{\partial^2}{\partial z^2}\right)\frac{\partial\zeta}{\partial z}, \tag{14}$$

$$n'\zeta = \frac{H}{4\pi\rho}\frac{\partial\xi}{\partial z}+2\Omega\frac{\partial w}{\partial z}-\nu_0\left(\nabla^2-3\frac{\partial^2}{\partial z^2}\right)\frac{\partial w}{\partial z}, \tag{15}$$

$$nh_z = H\frac{\partial w}{\partial z}, \tag{16}$$

$$n\xi = H\frac{\partial\zeta}{\partial z}, \tag{17}$$

$$(n+D)\theta = -\left(\beta+\frac{g}{C_p}\right)w, \tag{18}$$

where

$$n' = n\left(1+\frac{\alpha_0\nu_c}{n+\nu_c}\right) \quad \text{and} \quad \alpha_0=\frac{\rho_d}{\rho}. \tag{19}$$

Eliminating θ, ζ, ξ and h_z from Equations (14)–(18) and using (13), we obtain the dispersion relation in the form

$$n^7+A_6n^6+A_5n^5+A_4n^4+A_3n^3+A_2n^2+A_1n+A_0=0, \tag{20}$$

where

$$A_6 = D+2\nu_c(1+\alpha_0),$$

$$A_5 = 2V_A^2k_z^2+2D\nu_c(1+\alpha_0)+\nu_c^2(1+\alpha_0)^2+\Gamma\left(\beta+\frac{g}{C_p}\right)+E,$$

$$A_4 = 2V_A^2k_z^2\{\nu_c(1+\alpha_0)+D+\nu_c\}+D\nu_c^2(1+\alpha_0)+E(D+2\nu_c)$$
$$+\Gamma\left(\beta+\frac{g}{C_p}\right)\nu_c(2+\alpha_0),$$

$$A_3 = 2V_A^2k_z^2D\nu_c+2\nu_c(1+\alpha_0)V_A^2k_z^2D+\nu_c^2(1+\alpha_0)V_A^2k_z^2+(V_A^2k_z^2)+E(2D+\nu_c)+$$
$$+\Gamma\left(\beta+\frac{g}{C_p}\right)\{V_A^2k_z^2+\nu_c^2(1+\alpha_0)\},$$

$$A_2 = \nu_c^2(1+\alpha_0)V_A^2k_z^2D+(V_A^2k_z^2)^2(D+2\nu_c)+ED\nu_c^2+2\Gamma\left(\beta+\frac{g}{C_p}\right)+V_A^2k_z^2\nu_c,$$

$$A_1 = (V_A^2k_z^2)^2\nu_c(2D+\nu_c)+\Gamma\left(\beta+\frac{g}{C_p}\right)V_A^2k_z^2\nu_c,$$

$$A_0 = (V_A^2k_z^2)^2\nu_c^2D,$$

and

$$\Gamma=\frac{g\alpha(k_x^2+k_y^2)}{k^2}, \quad V_A^2=\frac{H^2}{4\pi\rho}, \quad E=\frac{k_z^2}{k^2}\{2\Omega+\nu_c(k^2-3k_z^2)\}^2.$$

When $D < 0$, i.e., the inequality (1) is satisfied, the constant term in Equation (20) is negative. This means that Equation (20) has a positive real root leading to monotonic instability. The criterion for instability (1) is, thus, the same in the presence of finite Larmor radius on the thermal convective instabiliity of a composite rotating plasma in a steller atmosphere.

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