Composite Spectra
Paper 1: HR 6902

R. & R. Griffin The Observatories, Madingley Road, Cambridge, England CB3 0HA; Visiting Associates, Mount Wilson & Las Campanas Observatories, Carnegie Institution of Washington

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Abstract. Composite-spectrum binary stars generally consist of a late-type giant and a main-sequence star of type A or B. Their spectra are therefore rather confusing; but by a technique of digital subtraction of the spectra of appropriate single late-type giants, composite spectra can be split into their individual components. In favourable cases the radial velocities of both components can be measured and the mass ratio determined. The procedures are illustrated by reference to HR 6902, a fifth-magnitude composite-spectrum binary. Its components are shown to have spectral types of G9 II and B8 V, with a mass ratio of 1.31, and its orbit is determined. There is some evidence that the system shows eclipses. If it does, the masses of the components are 3.9 and 3.0 $\mu$ respectively, and HR 6902 becomes the sixth known member of the important class of $\zeta$ Aur binaries.

Key words: composite spectra—spectroscopic binaries—stellar masses—stars, individual

1. General introduction to this series of papers

1.1 Descriptive Preamble

1.1.1 Composite spectra

Binary stars exhibiting composite spectra form a particular class of double-lined spectroscopic binary systems. In the sense in which we shall use the term, a composite spectrum is one which arises from a binary or multiple stellar system whose components have a very small angular separation and dissimilar spectral types.

A binary star only gives a recognizably double-lined spectrum if its components have comparable luminosities. If both components are main-sequence stars they cannot have very different spectral types, otherwise the magnitude difference is so great that only the earlier spectrum can in practice be observed. Therefore a main-sequence binary cannot give a composite spectrum. However, a red giant or supergiant has an absolute magnitude quite similar to that of a main-sequence star of type A or B respectively. Thus the binary systems with which this series of papers will principally be concerned are those in which one of the members is an evolved object—a high-luminosity star of more or less late spectral type, to which we will often refer as the ‘cool’ or (sometimes less accurately) as the ‘primary’ star—and the other is an upper-main-sequence object—the ‘hot’ or ‘secondary’ star.
Inasmuch as parts of the evolutionary paths followed in the Hertzsprung-Russell diagram by fairly massive stars are nearly horizontal, it is also possible for composite spectra to arise from binaries both of whose components are evolved. Capella, for instance, would just about qualify as composite; but as an example of a composite spectrum it is a poor one, because the spectroscopic distinction between its components depends more upon the rapid rotation of one of them than it does upon a straightforward difference of spectral type.

Normally the angular separation on the sky between the components of a composite-spectrum binary is so small that there is no possibility of observing the two spectra separately. The binary system is seen on the slit-head of the spectrograph as an apparently single star, and the recorded spectrum consists of the hot and cool spectra superposed upon one another.

The appearance of the typical composite spectrum is distinctive and bizarre. Because the cool-star spectrum is full of narrow and deep absorption lines whereas the hot one is not, only the former can readily be seen in the combined spectrum. The hot star makes its presence felt mainly by its blue continuum and strong Balmer lines. The hot continuum, rapidly increasing in intensity towards shorter wavelengths in relation to the flux from the cool component, progressively veils the late-type spectrum towards the violet. Meanwhile the strong Balmer lines contribute hazy depressions which underlie the mass of sharp lines arising from the cool star, heightening the unnatural appearance of the spectrum.

In a composite spectrum, the region of the H and K lines of Ca II has a characteristic appearance. The H line, which is very strong and wide in the cool spectrum, almost coincides with the Balmer line H\(\epsilon\), which is of comparable strength and width in the hot spectrum, so the H-H\(\epsilon\) blend is a particularly intense feature of the combination. The cool star's K line, on the other hand, has no comparable counterpart in the hot star, so in the composite spectrum the K line is very much filled-in. It usually has a more or less sharp core, which represents the stellar K line from the hot component and/or an interstellar feature. The sharp atomic absorptions contributed by the cool star fade out towards the centre of the feature, where owing to the extreme strength of the K-line in a late-type star the contribution of that component to the combined light is down almost to nothing.

All the above-mentioned characteristics of composite spectra are beautifully exemplified by the object we have selected for our initial study, HR 6902, whose spectrum is illustrated in Fig. 3 and is discussed in the latter part of the present paper.

1.1.2 The desirability of objective methods for separating the constituent spectra

Composite-spectrum binaries are by no means rare; several of them were identified in the original Draper Catalogue by Miss Maury (1897), and numerous examples are to be found amongst the stars of naked-eye brightness. Some of them, such as Capella and (more typically) \(\delta\) Sge, are conspicuously variable in radial velocity and their orbits have long since been studied. Others, such as \(\beta\) Cyg, show little or no change in their radial velocities. Still others have been measured either not at all or else with conflicting results, or (like HR 6902) have simply not been well enough observed for their orbits to be
derived even in cases where radial-velocity variability has been established.

The existence of this last category illustrates a remarkable general fact about composite spectra: they have been systematically neglected. The reason for the neglect is, in all likelihood, that they have been perceived as being too difficult or too troublesome to be effectively investigated. It is possible, certainly, to measure a radial velocity from a composite spectrum; but that is all. The velocity that is found generally belongs largely, sometimes exclusively, to the cool component, which supplies virtually all the spectral features that can be seen or measured. Any attempt on the part of the observer to take a more substantial interest in composite objects is apt to be thwarted by their very compositeness. In particular, no useful spectrophotometry is possible for either component owing to the uncertainty as to the contribution of its companion.

Even such a fundamental parameter as spectral type is difficult to establish. It will become apparent in this series of papers that the spectral types of stars involved in composite systems may be very different from the published types, even those assigned by experts. Indeed, the whole situation concerning the classification of composite spectra has been accurately summarized by Bidelman, in a recent letter (which we were doubtless not expected to quote publicly), as “too horrible to contemplate”!

It is clear that little further general advance is likely to be made by direct measurements on composite spectra. What is needed is the objective allocation of the observed spectral flux, at any and every wavelength, to the two stellar components individually, so that each component's contribution can be studied in isolation.

1.1.3 Principles of separation of the spectra

In principle there are two possible means of disentangling a composite spectrum. One might try to synthesize the observed spectrum from two spectra which represent those of the respective components. Alternatively, one could attempt to isolate the spectrum of one component by subtracting from the composite a spectrum representing that of the other star. Whichever alternative is adopted, a trial-and-error approach seems inescapable: there is no way of establishing a priori the spectrum of either component, so the initial step in either procedure must be to make a guess at the spectrum of at least one of the stars. In the synthesis approach it is necessary to guess at both spectra and also at their mutual wavelength displacement at the outset; in the subtractive procedure only one guess is needed initially, and the estimates of the other spectrum and of the wavelength displacement are revealed as the result of the subtraction. That difference between the methods has prompted us to choose the subtractive option. The input spectra—in the honest terminology above, the “guesses”—might consist of the observed spectra of appropriate single stars, or of spectra computed from stellar models. The former choice has the disadvantages of requiring additional observing time and of being restricted to spectra related to those which exist in Nature in isolated stars; on the other hand it is rooted in reality in a manner that computed spectra may not be. For a variety of reasons we have elected to use observed spectra in our search for spectra similar to those involved in composite objects. In what follows we shall use the expression “standard” to characterize stars or spectra that serve as analogues of a single component of a composite system.
By the considerations outlined above, we have effectively narrowed our choice of procedure to one in which we adopt an observed spectrum as a trial representation of one contributor to the composite, and by subtraction we uncover the other contributor. There is, of course, nothing particularly novel about this principle, which was used by Redman (1936) at our own Observatory fifty years ago: working with equivalent widths rather than complete spectra, he subtracted the secondary components (observed while the primaries were totally eclipsed) of the Algol-type binaries U Cep and U Sge to determine, from spectra taken during the partial phase of eclipses, the centre-to-limb variations of the strengths of absorption lines in the primary spectra. At the Dominion Astrophysical Observatory an heroic effort was made, before the days of digital computers, by Wright (1954), who isolated the shadowy early-type spectrum of Capella by subtracting from the observed spectrum of that binary system the spectrum of \( \beta \) Dra. In collaboration with Lee, Wright went on to derive the secondary spectra of the composite-spectrum eclipsing binaries 31 Cyg (Wright & Lee 1959) and \( \zeta \) Aur (Lee & Wright 1960) by subtracting the spectra observed during eclipse from those seen at other phases. Subtraction is routinely used nowadays to remove the contribution of the sky from the spectra of objects of low surface brightness. We have ourselves used the procedure, in a less developed form than that in which we shall present it here, to demonstrate the existence and nature of the hot secondary star in the previously unrecognized composite system HD 187299 (Griffin 1979a); and Batten et al. (1983) employed it in an effort to measure the radial velocity of the hot secondary in the composite object 93 Leo.

Before spectra can usefully be subtracted from one another, they must be accurately registered in wavelength. Inasmuch as the only reference wavelengths available in the composite spectrum are normally the lines of the cool star, it follows that the spectrum to be subtracted must represent that component, and the spectrum which remains after subtraction is that of its hot companion. The subtraction is necessarily performed in the rest frame of the cool star. No laboratory reference source (such as an iron arc) is involved: the composite and the standard spectra are both reduced to a wavelength scale based upon selected lines identified in the stellar spectra themselves. Successful subtraction is easily monitored by eye: when the requisite proportion of a spectrum that accurately matches that of the cool primary is subtracted from the composite, the lines of the cool star completely vanish and one is left with a spectrum of (usually) a totally different character, much smoother and recognizably that of a hot star.

Such a spectrum is indeed that of the hot component of the composite; and the power of the technique lies in the fact that, once the spectrum of the hot star is thus isolated in a pureform, it can be used for any purpose that a spectrum would normally serve, including spectral classification and the measurement of radial velocity and equivalent widths. Of particular significance is the fact that any radial-velocity difference between the two members of the composite system will be manifested as a displacement of the spectrum that is recovered for the hot component. The wavelength scale used in the subtraction process is that in which the cool star is at rest, and naturally it applies equally to the difference-spectrum: the displacement from it of the latter therefore represents the radial-velocity difference between the components of the composite. The availability of the radial velocities of the members of a composite-spectrum binary permits us, in cases where the binary is not too widely separated and the system accordingly has an adequate radial-velocity amplitude, to determine the relative masses of the components. The use of composite spectra to determine mass ratios has already been urged by Scarfe (1970).
1.1.4 Mass determination from composite spectra

Mass is the most fundamental of all stellar parameters. It is also, in many cases, one of the most difficult to determine with any accuracy. The mass of a single star can in principle be obtained from a detailed spectroscopic analysis. Unfortunately the result is critically sensitive to the model employed to describe the stellar atmosphere, and small changes in the values adopted for the effective temperature and microturbulence require disproportionately large changes in log $g$ and hence in the derived mass. The difficulty of obtaining any consensus upon a spectroscopically derived mass estimate was brought home to us some years ago by our experience concerning Arcturus: different experts, all starting from the same or similar observational premises about which there was no dissent, derived masses ranging over a factor of ten.

Mass estimates that are much less open to casuistry are obtainable from the dynamics of binary systems. There exist eclipsing binaries and visual binaries which include main-sequence stars at least from type A onwards and whose orbits and parallaxes are sufficiently accurately known to give masses that are accurate to the order of 10 per cent (Popper 1980). Unfortunately the positions are less satisfactory as regards giant stars. The masses of the components of Capella—one at least of which is a rather atypical object—are tolerably well determined (Shen et al. 1985) as being near $3 M_{\odot}$ (Wilson 1967). There are six other binaries consisting of pairs of reasonably normal late-type giants in double-lined spectroscopic systems with known orbits: they are $\eta$ And (Gordon 1946), $\phi$ Cyg (Rach & Herbig 1961), HR 2081 (Beavers & Griffin 1979), HR 4665 (Bopp et al. 1979), and HD 44780 and 65 Gem (Griffin 1986). Their minimum masses range from 0.31 to 3.2 $M_{\odot}$ per component, those extreme values being given by $\eta$ And and HR 2081 respectively; but only in the case of $\phi$ Cyg are the actual values determinable (in that case 2.50 and 2.39 $M_{\odot}$ (McAlister 1982)) because the inclinations of the other orbits to the line of sight are unknown.

If the amplitudes of variation of the radial velocities of both components of a composite-spectrum binary can be measured, then the relative masses of the components follow immediately (being in inverse proportion to the amplitudes, by the principle of conservation of momentum). Inasmuch as one of the components is normally a main-sequence star, whose mass can be assigned by reference to its spectral type, the mass of the other component—the cool giant—is determined. It may seem less than satisfactory to have to assign one mass by rule of thumb in order to obtain the other; but if by so doing we have reduced the range of uncertainty of the mass of the giant from a factor of ten down to the order of ten per cent we have obviously achieved a very worthwhile improvement.

Two further comments on this situation may be interjected here. The first concerns the complication which arises from the metallic-lined (Am) nature of the hot secondaries in many composite systems. One needs to decide which of the spectral types (hydrogen-line, metal-line or K-line) to use in entering the tables of main-sequence masses, e.g. Allen (1973), p. 209, to assign the mass. We tentatively suppose that it is the hydrogen type, which more or less corresponds to the effective temperature, which is the important quantity. The other comment is that quite a number of composite-spectrum binaries has already been resolved by speckle interferometry (McAlister & Hartkopf 1984): it is only a question of time before 'visual' orbits for many of these systems become available, and then the masses will follow directly from the observed radial-velocity amplitudes without the need for any assumptions whatsoever. An orbit based on speckle interferometry has already been published (McAlister 1982) for $\gamma$ Per, a composite object for which a spectroscopic orbit was derived long ago.
1.2 Procedural Details

1.2.1 General plan of the observing campaign

Two types of observational input are needed in this investigation of composite spectra. The type to which we have alluded above is that of high-resolution spectroscopic observations, which enable us to separate the combined spectra into their two individual constituents and to determine the radial-velocity difference (if any) between those constituents. Because high-resolution spectroscopy can be quite expensive in terms of observing time, to maximize its effectiveness it is desirable first of all to investigate the radial-velocity behaviour of the composite binary. That requires the second type of input. The radial-velocity of the cool star can be measured directly with a photoelectric radial-velocity spectrometer such as ours at Cambridge (Griffin 1967) and Palomar (Griffin & Gunn 1974). Since the hot star normally lacks the spectral lines upon which the instrument operates, the spectrometer is not sensitive to it at all: it does not confuse or invalidate the measurement, but simply dilutes the late-type spectrum and makes the ‘dip’ in the radial-velocity trace shallower and less easily measured than it would otherwise be.

In cases where the radial velocity of the cool primary in a composite-spectrum binary is constant, there is no preferred moment for obtaining high-resolution spectra, and all that one can hope to do is to identify the spectral types of the components. On the other hand, if the primary velocity proves to be variable, the next step is to determine the orbit. The orbit of the hot secondary is necessarily the inverse of that of the primary, the only unknown quantity being its amplitude of variation. Each spectroscopic observation furnishes one measurement of the relative velocity, i.e., the velocity difference, between the primary and secondary. When the orbit of the primary is known, only one measurement of relative velocity is needed in principle to determine the amplitude of the secondary. Obviously, the greater the relative velocity the greater the relative accuracy of its determination: if the inherent error of the determination of the velocity difference is, say, 5 km s\(^{-1}\), then we shall be able to measure a relative velocity of 100 km s\(^{-1}\) with fair accuracy but one of 10 km s\(^{-1}\) will more or less defeat us.

Thus we use our knowledge of the orbit of the primary to select an epoch of nodal passage, when the displacements from the γ-velocity of the radial-velocities of both stars are at their maxima, for the critical spectroscopic observation. Often the velocity excursion at one node of the orbit is greater than that at the other, so one particular node is the preferred time. From spectroscopy at that time we obtain the value of the radial-velocity difference between the components. The observed difference can then be partitioned between the two components, since the departure of the primary from the γ-velocity can be computed from the orbital elements.

It will be seen that this project is quite appropriate to the authors’ interests and experience, involving as it does a symbiosis of their longstanding commitments both to radial-velocity studies and to high-resolution spectroscopy.

1.2.2 Spectroscopic data

Among the characteristics desired in the spectra used in this work are: (a) a substantial wavelength coverage, to enable lines of different elements, and diagnostic lines in
different spectral types, to be observed simultaneously; (b) a high signal/noise ratio, particularly in view of the fact that in subtracting one spectrum from another we are subtracting the signal but adding the noise; and (c) a high density of sampling points or picture elements, to facilitate interpolation between them so that spectra to be subtracted can be placed accurately in register with one another.

Initially it was our hope and intention to use photoelectric methods to record the required spectra. Not the least of the advantages we foresaw in those methods was the possibility of having the spectra obtained for us by a ‘service observing’ procedure, or by a collaborator who lived near the relevant observatory, rather than by us personally: to obtain spectra at specified times (nodal passages), usually different from one another, for a lot of different objects involves a great deal of travelling when, as in our case, no suitable equipment is available at one’s own observatory. It also represents a heavy imposition upon the hospitality of a host observatory when visitors are constantly wanting to come and observe at specific times.

Through the kindness of Dr H. J. Smith and the McDonald Observatory, we were enabled in 1980 to make trials of the reticon and digicon systems there. It turned out that, for our purposes, the photoelectric detectors were not altogether satisfactory. Because of the very limited numbers of their picture elements it was impossible to fulfil requirements (a) and (c) above simultaneously. Moreover, the digicon, which was the detector of choice in the important K-line region, incorporates electron optics which introduce considerable geometrical distortion, which cannot be determined and allowed for to the necessary accuracy because the small field of view simply does not contain enough unblended lines to serve as wavelength standards.

We were accordingly unexpectedly forced back on photography, which enables high-resolution spectra to be recorded over a wide wavelength region at a single exposure and moreover has quasi-continuous recording rather than the discrete elements of photoelectric detectors. (See desideratum (c) above.) We have been very fortunate in being appointed Visiting Associates of Mount Wilson Observatory, where we were given generous access to the 100-inch reflecting telescope and its coudé spectrograph; all of the spectroscopic data we have so far obtained for our work on composite spectra have come from Mount Wilson. It is, of course, a matter of serious concern (Griffin 1984, 1985a,b) that the 100-inch telescope is now closed.

As a compromise between spectroscopic detail and practical exposure times, we decided at the outset to base the composite-spectrum work on spectrograms whose reciprocal dispersion is 10 Å mm⁻¹. Such spectrograms were efficiently taken at the 100-inch telescope with the 32-inch off-axis Schmidt camera (Dunham 1955) and the ‘46B’ grating—an original 600-line mm⁻¹ grating ruled by Babcock at Mount Wilson Observatory itself. The grating is blazed in the second-order near-ultraviolet. The wavelength region we have observed is from λ 3850 to λ 4650 Å. That region can be covered with satisfactory photographic density in a single exposure for stars of spectral type earlier than about K2; the steep gradient in later types makes two exposures desirable. Above λ 4500 Å the second-order spectra are liable to be contaminated with third-order far-ultraviolet. The contamination of λλ 4500–4650 Å with the overlapping third-order λλ 30000–3100 Å is so slight that we have preferred not to use a filter to avoid it; but it needs to be borne in mind where spectra of hot stars are involved.

The photographic emulsion normally used has been Eastman Kodak IIa-O; but some hot standard stars and hot-dominated composite spectra have been photographed on IIIa-J. High signal/noise ratios have been achieved by taking very wide
spectra; in many cases we have used the maximum width permitted by the optics of
the spectrograph, $2\frac{1}{2}$ mm. The entrance slit has always been set at a width of 100 $\mu$m,
corresponding to 0.27 arcsec on the sky at the $f/30$ coude focus of the 100inch
reflector and to approximately $15 \mu$m or 150 mÅ projected in the spectrum.

The camera gives such good definition that the true spectral resolution is very largely
set by the projected slit width. In relation to the ultimate resolution of the camera the
width we have selected is rather conservative, so small differences of focus etc. from one
observing run to another have little effect on the observed resolution. The camera also
has exemplary stability, and there has been no difficulty in obtaining an extremely
homogeneous series of spectra of composite and standard objects over a period of
years. Spectra of the quality shown in our illustration of HR 6902 later in this paper
(Fig. 3) have been obtained as a matter of routine.

Photometric calibrations have been obtained by the methods outlined in the
Introduction to our Procyon spectrophotometric atlas (Griffin 1979b), and only need
brief description here. In order to make the calibration exposures as similar as possible
to the stellar exposures, which are intermittent because the spectra are widened by
repeated slow trials of the star image along the entrance slit, the calibration too is
broken up into a series of suitable discrete instalments synchronized with the trails. A
series of 12 step slits, illuminated by a tungsten source, is used to generate alongside
the stellar spectrum a series of continuous spectra whose relative intensities are known
from the geometry of the slits. At first we found the scale of the step-slit images to be
rather small for the best photometry: the same slits served not only the 32-inch but also
the 73- and 114-inch cameras where of course they were imaged at correspondingly
larger scales, and their sizes were limited by the need to accommodate their images at
the 114-inch camera. We found it possible to replace the step slits with a set of our own
construction, with a scale better adapted to that of the 32-inch camera. As a precaution
inspired by pessimism, a second calibration has normally been exposed simultaneously
in a separate auxiliary spectrograph with a 24-step entrance slit (Griffin 1979b).

1.2.3 Spectrophotometric technique

The first step in reduction of the photographic spectra is to determine the calibration
curves from microdensitometry of the step slits. The curves steepen systematically with
increasing wavelength on Ilā-O emulsion, and have needed to be established at 200-Å
intervals. It has proved convenient, in fact, to handle the spectrophotometry
throughout in four 200-Å regions centred respectively at $\lambda\lambda$ 3950, 4150, 4350 and
4550 Å, and that is how it will usually be presented in this series of papers; so those are
the wavelengths at which the calibrations have been set up.

The stellar spectra are traced on our largely homemade digital microphotometer at
5-$\mu$m intervals and transferred from plate transmission to a linear scale of light
intensities by means of the appropriate calibration curves. In a second step, the tracing
is placed on a linear scale of wavelengths. The scale is established by the identification of
a number of (almost) unblended stellar lines which are assigned their corresponding
solar wavelengths (laboratory wavelengths in the cases of the hottest standard stars),
the wavelength solution being computed by a trigonometrical relationship based on the
geometry of the spectrograph (Griffin 1979b). Rms wavelength residuals of less than
10 mÅ, corresponding to 1 $\mu$m the plate, are the norm. The intensity values (whose
scale is already linear) are normalized so that the highest point in each 200-Å region is near the value 100; but no effort is made to level the continuum at this stage, since for our purposes it is quite useful to leave the natural gradient in the spectrum.

The final outcome of this phase of the reduction procedure is that each spectrum is reduced to four sets of numbers, 4001 numbers in a set, each set representing the digital intensity values of the spectrum at precise 50-mÅ intervals for the 200-Å region starting at \( \lambda (3850 + 200n) \) Å where \( n \) is 0, 1, 2 or 3. The wavelength scale is that of the rest frame of the star (the cool star in the case of a composite spectrum). These digital data sets can then be added to, or subtracted from, the corresponding sets for other stars and represent the raw material upon which our work on composite spectra is based. From the interest shown in them by others, it is clear that the observations of standard stars are also seen as a valuable resource of spectroscopic data in their own right.

We are quite aware that photography is considered in many quarters to be an obsolete method of recording astronomical spectra, so it is perhaps worth making the points that (a) we deliberately selected it in this instance after trials of alternative methods, and (b) once one has traced a plate with a digital microphotometer the resulting data set is as modern and digital as any data can possibly be.

1.2.4 The disentangling of composite spectra by subtraction

The first step in attempting to disentangle a composite spectrum is to select from the library of late-type standards a spectrum to use as a surrogate for the primary star. The choice may be guided by an existing classification; but even a casual inspection of a 10-Å mm\(^{-1}\) spectrogram of the composite object often furnishes a more accurate indication of the spectral type involved. A series of trial subtractions is then made, the proportion of the late-type standard subtracted being changed by (say) 5 or 10 per cent between successive trials. We have sometimes found it useful to produce a paper plot with the trial subtractions ranged one below another so that one can follow the evolution of any given feature of the spectrum as increasing amounts of the standard are subtracted. What normally happens is that an absorption feature seen in the composite spectrum becomes progressively shallower as more of the standard is subtracted, and ultimately reverses into an emission peak when the optimum proportion has been passed.

Ideally, all the late-type absorption lines should just vanish at the same time. In practice, unless one is very lucky, some lines will disappear at one percentage subtraction of the late-type standard and others will require different percentages. A familiarity with the behaviour of the strengths of different lines as a function of spectral type and luminosity—a familiarity rapidly gained by inspection of the tracings forming the library of standard spectra—then guides one to successively better choices of standard.

Notice that it is not essential to have a dense grid of observed standard spectra. In just the same way as modest gaps in standards are readily bridged by interpolation when one is, for example, classifying spectra, so it is possible here to interpolate simply by averaging two standards—not necessarily in equal proportions. There is in fact a positive advantage in being obliged to interpolate between standards inasmuch as the averaging procedure improves the signal/noise ratio, so the interpolated spectrum is better than either of its constituents! On the other hand, one fact that the present
project has really brought home to us is the individuality of stars: if one takes the spectra of half a dozen stars all classified as (for example) K0 III they are found to be subtly different from each other. Consequently, the more standards at one’s disposal the better is the prospect of finding a good match for the star one is trying to imitate in the composite spectrum.

For the subtraction process to work effectively, very accurate registration in wavelength between the spectra concerned is essential. If one spectrum is even slightly displaced from the other, the result of subtraction is not to cancel the lines which correspond in the two spectra but to turn them into P-Cygni profiles. An analogous phenomenon occurs if there are differences in the effective resolution of the spectra, either for instrumental reasons or because of real differences in the projected rotational velocities of the stars concerned. If the lines in the spectrum being subtracted are too narrow—the light intensities rising too rapidly away from their cores—too much will be subtracted in the wings before enough is subtracted in the cores; and the resulting line profiles appear to have central emission peaks flanked on both sides by absorption. In the opposite case there is central absorption flanked by emission peaks. In practice, the differences in apparent resolution are small except in the cases of certain very luminous stars, and are readily removed through slight blurring of the ‘sharper’ spectrum by a digital broadening function; but the need for precise equalization of the effective resolution of the spectra does represent an additional complication in the subtraction process.

The hoped-for result of this stage of our procedure is the identification of a late-type spectrum which exactly matches the one involved in the composite binary, so subtraction of an appropriate percentage of it completely cancels all the late-type lines. The necessary percentage usually changes more or less progressively along the spectrum, as the relative fluxes from the two stars in the composite change quite rapidly with wavelength. We have routinely assessed the percentage to be subtracted separately for each 50-Å wavelength interval, and then interpolated the actual percentages applicable at every wavelength so as to avoid any discontinuity. The spectrum that is left after cancellation of the late-type lines by subtraction is evidently the spectrum of the other component of the binary: it is normally obvious that it is an early-type spectrum. It is convenient to re-scale its ordinates so that they are comparable with those that would be plotted for a directly-observed spectrum, since subtraction has reduced all the numerical intensities.

1.2.5 The need for accurate assessment of the amount of the late-type Standard to be subtracted

We glibly remarked in Section 1.1.3 that the spectrum that is uncovered for the hot star by subtracting away the cool contributor to a composite spectrum can be used for any purpose, including the measurement of radial velocity. That is certainly true in principle; but in actual practice much effort has had to be expended in attempts to devise procedures which lead to reliable velocities.

The difficulties that we have encountered mainly arise from the existence, in the hot stars of many composite-spectrum binaries, of many of the same spectral lines that characterize the cool components. Examples from a small region in the violet are the
Figure 1. To illustrate the radial-velocity errors incurred through small errors in the subtraction of spectra. Two absorption lines, one deep and one broad and shallow, intended to represent corresponding lines in the spectra of the cool and hot components respectively of a composite object, are mutually displaced by a radial-velocity difference. Their blended profile is what is actually observed. In an effort to separate the components of the blend, we try to subtract the contribution of the deep component, whose strength is not known a priori. An appreciable range of strengths, exemplified here by 97 and 106 per cent of the true strength, leaves plausible apparent profiles for the shallow component; but it is clear that if we assign a wrong strength to the deep component we get a wrong radial velocity for the shallow one.

very strong Fe I lines λλ 4045, 4063 and 4071 Å, Sr II λ 4077 Å, and even the Mn I multiplet near λ 4030 Å: all are present in A-type spectra as well as in K types. In A-type spectra they are often substantially broadened by rotation, and the positions that they seem to occupy in a subtracted spectrum are then exceedingly vulnerable to small errors in the percentage of the cool star subtracted. We have tried to make the situation clear in Fig. 1, which portrays a typical absorption line appearing in both an A and a K spectrum, with a relative velocity shift between them. The illustration shows how a slight residual absorption from the K star, caused by a slight underestimate of the amount to be subtracted, draws the apparent line in the A star towards the position of that in the late-type star, i.e. it leads to an underestimate of the A-star's velocity shift. Conversely, slight over subtraction fills up the A-star line near the zero-velocity position and pushes the centroid of the line too far away: the velocity shift is overestimated. An accurate and objective means of assessing the amount of the late-type standard to be subtracted is essential.

1.2.6 Methods of obtaining accuracy in subtraction

An obvious recourse to see whether any late-type spectrum remains, either in absorption or in reversal, after subtraction is to cross-correlate the result of subtraction, i.e. the proposed secondary spectrum, with the late-type standard spectrum. Any
addition to spectral type, and metallicity is not even a single parameter. It is practically remaining late-type absorption should show up as a peak at zero displacement, whereas a reversed spectrum due to over-subtraction should be manifested as a ‘dip’ or negative peak. A complication immediately arises because, even though the absorption lines may be exactly cancelled, the secondary spectrum nevertheless contains the photographic grain noise from the late-type standard spectrum. Since that spectrum has been subtracted from the original one, its noise is present in negative form: cross-correlation with the (positive) late-type spectrum produces an autocorrelation ‘noise dip’ at zero displacement. At our dispersion of 10 Å mm\(^{-1}\), the spatial frequency of the noise overlaps that of the spectrum. Therefore, if the subtraction is judged by cross-correlating the result of the subtraction with the late-type spectrum used in it, a systematic error will result, viz. not enough of the late-type spectrum will be subtracted, because the cross-correlation function is depressed at the all-important zero position by the autocorrelation of the grain noise. The remedy is clear: the standard spectrum used to judge the subtraction must be a different one—a spectrum of a different star, or another spectrum of the same star—from that used in the subtraction itself.

The next problem is that, because there is only a very limited number of spectral lines in each short interval of wavelength over which the proportion to be subtracted must be judged, the cross-correlation function tends to be everywhere far from level, and it is by no means clear what it should actually look like when the cool-star spectrum has been exactly annulled by subtraction. In fact, the cross-correlation of the late-type spectrum with the hot secondary spectrum itself usually has substantial features in it. A considerable improvement is obtained by reflecting the cross-correlation function about the zero-displacement position and averaging it at equal positive and negative displacements. That effectively removes the first-order gradient at the zero position. In cases where the components of a composite spectrum are sufficiently dissimilar, it may be satisfactory to judge the amount of late-type standard to be subtracted by requiring the reflected-and-averaged cross-correlation between the derived secondary spectrum and the separate, ‘judging’ late-type standard to be locally flat at the zero-displacement position.

Further refinement must be sought through actual modelling of the cross-correlation function. Using the procedure set up above, one obtains a fair initial estimate of the spectral type and radial-velocity shift of the secondary, and attempts to match the secondary from the library of standard stars. It is then necessary to form a cross-correlation of this standard secondary with the same ‘judging’ surrogate primary, over the relevant wavelength region and at the proper velocity shift; and the observed cross-correlation function is required to be locally flat at zero displacement after it has been divided by this model function. Then the procedure may need to be iterated to convergence.

1.2.7 The radial velocity of the secondary star

The final technical problem in this project is to determine the radial velocity from the spectrum we uncover for the hot secondary star.

The hydrogen Balmer lines are always too wide for accurate velocity measurement, and velocities derived from them are very sensitive to small errors in the wing profiles. A drawback to the use of metallic lines is that many of them are superimposed upon the
sloping wings of Balmer lines and their centroids are therefore displaced. We have sought to overcome that difficulty by normalizing the spectra of hot stars—both the one recovered from the composite spectrum and the best-matching available standard—to the local continuum. That is to say, for the purpose of finding the radial velocity we prepare versions of the early-type spectra in which the Balmer lines are effectively removed by normalization to their wing profiles. The deeper parts of the Balmer profiles obviously cannot be treated in this way and are omitted from consideration altogether. The radial velocity of the composite secondary is then determined by cross-correlation with the standard spectrum.

Direct cross-correlation of two early-type spectra is very inefficient, because much of each spectrum consists of continuum which contributes a lot of noise but no useful signal. By ‘inverting’ the spectra, so that the numerical intensity values represent depressions from the continuum, that problem is largely overcome because the cross-product terms arising from points with near-zero depressions are then very small. The noise contributed by non-significant features in the spectra is still further reduced by treating every point whose depression below the continuum is less than some specified value as having a depression that is exactly zero.

The cross-correlation function has a peak which is in general at a non-zero value of the mutual displacement of the spectra. The displacement gives the relative Doppler shift in terms of spectral elements or ‘bins’, which in our case are each 50 mÅ wide. Although the cross-correlation is only computed at shifts of integral numbers of bins, the function is in principle continuous and its maximum may be found by interpolation to whatever accuracy seems useful. For each 200-Å spectral region we also determine an effective mean wavelength, defined as the point at which the moments of the cross-product terms summed in the cross-correlation calculation are in balance. The effective wavelength is needed in order to convert wavelength shifts into velocity shifts. We could obviate the need to compute it if we binned the spectra in equal intervals of frequency or of \( \log \lambda \) at the outset, so that the Doppler shift would be everywhere the same in terms of bins; our adherence to a wavelength scale may be seen as a manifestation of prejudice rooted in the history of astronomical spectroscopy.

1.2.8 Intractable difficulties

Our work on composite spectra has brought to light a number of difficulties associated with the natures of the stars involved in such systems, difficulties which we can scarcely hope to overcome.

In the first place, a surprisingly large number of primary stars proves to be Hertzsprung-gap objects with spectral types near G0 III. Good examples of such stars are almost impossible to find in isolation in the general field. Their preferential existence in composite spectra must be trying to tell us something, but we have not yet grasped what it is. In any case, it makes for difficulty in obtaining analogues for subtraction purposes.

Secondly, the hot components too are difficult to match. In their case the problem is not so much that they are of unusual types but simply that there are too many variable parameters of importance amongst A-type stars. Even if we restrict attention to stars that are on the main sequence, metallicity and rotation are continuously variable in
impossible to acquire a library of standard stars which adequately represents all the variables in such a multi-dimensional grid.

Thirdly, the high rotational velocities of certain secondary stars make for great uncertainty in the determination of their radial velocities.

Finally, too great similarity between the component spectra can create difficulties in identifying unblended lines of the primary for use in determining the wavelength scale upon which the subtraction procedure depends.

Another type of difficulty arises where the observed spectra include features which do not arise in the stars being observed. Such features can naturally vitiate the judgement of the proportions of the standard spectra to be subtracted, and the radial velocities determined for secondary stars, unless appropriate evasive action is taken. Fortunately there are not many of them: the only ones that we have come across are interstellar H and K, and emission lines near \( \lambda 4358 \) and \( 4046 \) Å—mercury lines arising from artificial urban illumination scattered in the atmosphere or (worse) by cirrus cloud. One or two other mercury lines are marginally visible on the worst-affected plates. The great width of most of our spectra is achieved by the use of an entrance slit that is very long, admitting scattered light continuously from a considerable area of sky although the star image itself normally occupies only a small fraction of the slit length. Spectra of even quite bright stars thus become vulnerable to sky contamination; a practical answer to the problem would be a ‘moonlight eliminator’ (Bowen 1962).

1.3 Possible Significance or Otherwise of the Results

We have indicated in Section 1.1.3 that in cases where a composite spectrum arises from a spectroscopic binary whose orbit can be determined we shall hope to obtain the mass ratio of the binary. Even without assignment of the mass of the hot star to give an estimate of that of the other, the mass ratio itself is of great interest. According to the received theories of stellar evolution, the more massive a star is the sooner it evolves off the main sequence. Therefore in a composite-spectrum binary system, in which the two stars are presumably coeval, the more evolved component should be more massive than the one that remains near the main sequence, or at any rate it should have been so when it began its giant-branch evolution. If a case is found in which the cool giant is distinctly less massive than its main-sequence companion, it will provide evidence of substantial mass-loss during the part of its evolution that it has so far undergone.

Concern is naturally felt as to whether the masses and evolution of the members of a binary system are identical with those of corresponding single stars. Certainly there are cases in which mass-exchange is taking place, and the existence of the companion is having a profound influence on the career of the evolving star. In AR Mon and RZ Cnc (Popper 1976), which have orbital periods near 20 days, the larger star has lost most of its mass to its companion. However, disturbance of normal single-star evolution is only probable when the evolving star has at some epoch filled its Roche lobe of the Lagrangian surface. Scarfe (1970) has given reasons why, if an observed giant does not now fill its Roche lobe, it is unlikely to have done so in the past, He has also put forward a conservative criterion for non-interactive evolution in a binary system in the form of an inequality

\[
\frac{P^2(1-e)^3}{R_*^3} > 40
\]
where $P$ is the orbital period in years, $e$ is the orbital eccentricity and $R_e$ is the radius of the giant star in astronomical units. Most of the binary systems that we hope to discuss seem likely to meet this criterion.

2. HR 6902

2.1 Introduction: Existing Knowledge Concerning H R 6902

HR 6902 is a fifth-magnitude object in the Milky Way in the eastern part of Ophiuchus, about 20° preceding Altair. It has been selected for the first paper in this series because the secondary has sharp lines and is of very different spectral type from the primary; so the disentangling of the components is relatively easy, and our procedures can perhaps be seen to work at least as convincingly as if we started with a more challenging example.

Although Miss Cannon was not the first to classify the spectrum of HR 6902, she appears to have been the first to recognize it as composite. Adams (1914) took four spectrograms of it in 1914, but simply called its type F7. Miss Cannon noted its composite nature in the Henry Draper Catalogue (Cannon & Pickering 1919), where she assigned it two numbers, HD 169689 and HD 169690, with spectral types G0 and A3 respectively.

During the 1920s the then new techniques of estimating stellar luminosities spectroscopically were energetically applied to a large number of stars. Adams et al. (1921) considered the absolute magnitude of HR 6902 to be –0.8 and the spectral type G0. Another Mount Wilson syndicate (Adams et al. 1935) offered a revision to $M_v = +0.9$ and type F9. Meanwhile Young & Harper (1924) had each independently estimated the absolute magnitude and found it to be –0.6 and +0.4 respectively; they jointly gave the spectral type of G5. In none of those investigations is there any mention that the spectrum is composite.

Much later, several further classifications of HR 6902 were made. The object appears in the Radial Velocity Catalogue (Wilson 1953) with type gG2. Kuhi (1963) sought to overcome the problem caused by the composite nature of the system by observing the spectrum in the red, where the contribution of the hot star is greatly reduced; he gave a type of G8 IV. Markowitz, in his doctoral dissertation (1969), found G8 III + A0 V and also supplied $\nu \sin i$ estimates of 60 km s$^{-1}$ for the secondary and < 50 km s$^{-1}$ for the primary. Most recently Hendry (1978) offered a classification of F8 V + A, but at the same time remarked that the F star is the more luminous component.

Cousins (1963) gave the photometric data $V=5^m.64$, $(B-V) = 0^m.92$, $(U-B) =0^m.45$; Häggkvist & Oja (1966) obtained the identical value of $V$, and $(B-V) =0^m.898$. In addition, $V$ magnitudes of 6.62 [sic] and 5.638 have been published by Eggen (1955) and Heck & Manfroid (1980) respectively.

The first radial velocities of HR 6902 were measured at Mount Wilson. Four plates, all taken in a three-month interval in the summer of 1914, showed a range of more than 40 km s$^{-1}$ and prompted Adams (1914) to announce the variability of the velocity. Details of the four plates only became available in Abt’s (1973) compilation. Harper (1934) reported 11 velocities obtained at the Dominion Astrophysical Observatory in the summers of 1919 and 1920; but in contrast to the experience at Mount Wilson they only showed a range of 11 km s$^{-1}$. This anomaly is readily understood in
R. & R. Griffin

retrospect, now that the orbit is known to be quite eccentric and to have a period 20
days longer than a year: there is a relatively short interval of rapid change, and it
migrates around the calendar in a cycle of about 18 years. In 1914 it coincided with the
season when HR 6902 was in opposition, but by 1919/1920 it did not.

Three velocities were obtained on almost consecutive nights in 1938 by Tremblot
(1938). Fourteen more, measured at Perkins and Lindheimer and ranging in date from
1940 to 1976, were listed by Hendry (1978), who also published an orbit with period of
208 days and later withdrew it (Hendry 1984). Woolley et al. (1981) provided eight
measures taken in 1966 with the 36-inch Yapp reflector at Herstmonceux.

2.2 New Radial Velocities and Orbit

HR 6902 was placed on the observing programme of the Cambridge photoelectric
radial-velocity spectrometer (Griffin 1967) in 1981, and 42 observations have been
made of it. It gives quite satisfactory radial-velocity traces, as the dilution of the light of
the primary star by the secondary is fairly small in the region of the spectrum (roughly
corresponding to the $\beta$ band in $UBV$ photometry) used by the radial-velocity
instruments. These last few years have been a difficult time to establish the orbit of
HR 6902, because the interval of rapid change in velocity, mentioned above, has
occurred near the time of conjunction with the Sun. Nevertheless we have managed to
cover the cycle reasonably well, and our own observations taken in isolation yield an
orbit with a period of $385.7 \pm 0.5$ days. Their rms residual from that orbit is less than
0.7 km s$^{-1}$.

Armed with the preliminary orbit, we were enabled to take a critical look at the five
series of radial-velocity observations found in the literature. It is likely, but not certain
in all cases, that the published velocities are measured from the lines of the primary star
alone; if the hydrogen lines were also measured they would be a fruitful source of
confusion. The velocities published by Hendry (1978) and by Woolley et al. (1981) were
found not to be of utility for our purposes, having residuals reaching over 20 km s$^{-1}$.
The three measures by Tremblot (1938) are equally without value, since they cover less
than one-hundredth of the period. The early velocities published by Adams (1914) and
Harper (1934) seem fairly reliable, the largest residual in each series being 6 km s$^{-1}$.
Unfortunately Harper’s 11 observations span less than a fifth of the period, at a phase
which is well covered by the much more accurate photoelectric measurements and at
which the velocity is almost constant. The observed range of 11 km s$^{-1}$ among Harper’s
velocities is to be compared with the computed variation of only 2 km s$^{-1}$ during the
interval of phase that they cover. Those velocities, therefore, cannot help us in refining
the orbit.

Adams’ four velocities have tolerable residuals and fall on the steep descent in the
velocity curve. Taken 70 years ago, when the uncertainty in the phase derived by
extrapolating the photoelectric orbit backwards in time has grown to about a month, they
clearly can assist in determining the period. An orbit solution which includes them
with the weight (0.05) to which their residuals entitle them has a period of $384.95 \pm 0.05$
days. They have an algebraic mean residual of $+1.9$ km s$^{-1}$ according to that solution.
However, since they all fall on the descending branch of the velocity curve, any
systematic discrepancy between their zero-point and ours will cause a shift in their
phases (Griffin 1981). If we apply a ‘correction’ of $-1.9$ km s$^{-1}$ to them, the mean
residual does not become zero because the re-computed orbital period takes the opportunity to increase again towards the photoelectric value and largely negates our correction. It requires a ‘correction’ of $-5.6\ \text{km\ s}^{-1}$ to Adams’ velocities to reduce their mean residual to zero. The residuals themselves are then very small too, their values being $+1.2$, $0.0$, $-0.3$ and $-0.8\ \text{km\ s}^{-1}$ respectively; the period is $385.10\ \text{days}$, with a formal standard error of $0.05\ \text{days}$. There is no way of deciding what, if any, correction is really warranted; and to make a change of $-5.6\ \text{km\ s}^{-1}$ just in order to overcome a systematic discrepancy of $+1.9\ \text{km\ s}^{-1}$ seems a rather extreme reaction. We consider that if we set the period at $385\ \text{days}$ exactly we are likely to be within $0.1\ \text{day}$ of the truth; and by holding the period at that value and using only our own velocities in the orbit solution we shall avoid any further effects of uncertain zero-point discrepancies. It is on that basis that we have performed the final orbit calculation.

Table 1 lists all the radial velocities, old and new, for the primary of HR 6902, together with the phases and residuals according to the adopted orbit, whose elements are listed in Section 2.6 below. The radial-velocity curve is illustrated in Fig. 2; the figure also includes the curve for the secondary, whose data are discussed in Section 2.5.

![Figure 2. Radial-velocity orbit of HR 6902. The orbit is based upon our photoelectric measurements (filled circles) of the radial velocity of the primary star. Its period has been refined by appeal to the early photographic measurements of Adams (1914), represented here by open diamonds. Open triangles represent other velocities found in the literature but disregarded in the derivation of the orbit. Those with vertices up are by Harper (1934), vertices right by Tremblot (1938), vertices left by Hendry (1978) and vertices down by Woolley et al. (1981). The four observations of the secondary are derived from our photographic work, as described in Section 2.5 below.](image)
Table 1. Radial-velocity measurements of the late-type primary star of HR 6902.

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<td>16.88</td>
<td>832.88</td>
<td>-2.6</td>
<td>.578</td>
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<tr>
<td>26.87</td>
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<td>-1.1</td>
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<td>Sept 3.85</td>
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<td>C</td>
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<tr>
<td>12.86</td>
<td>859.86</td>
<td>-1.3</td>
<td>.648</td>
<td>-0.7</td>
<td>C</td>
</tr>
<tr>
<td>18.88</td>
<td>865.86</td>
<td>-0.5</td>
<td>.663</td>
<td>+0.1</td>
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<tr>
<td>Oct 2.81</td>
<td>879.81</td>
<td>-1.6</td>
<td>.700</td>
<td>-0.7</td>
<td>C</td>
</tr>
<tr>
<td>12.81</td>
<td>889.81</td>
<td>-1.1</td>
<td>.725</td>
<td>+0.4</td>
<td>C</td>
</tr>
<tr>
<td>25.19</td>
<td>902.19</td>
<td>-3.2</td>
<td>.758</td>
<td>-0.4</td>
<td>C</td>
</tr>
<tr>
<td>1982 Mar  5.24</td>
<td>45033.24</td>
<td>-47.9</td>
<td>1.098</td>
<td>-0.2</td>
<td>C</td>
</tr>
<tr>
<td>May 4.07</td>
<td>093.07</td>
<td>-25.1</td>
<td>.253</td>
<td>+0.5</td>
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</tr>
<tr>
<td>24.02</td>
<td>113.02</td>
<td>-19.0</td>
<td>.305</td>
<td>+0.6</td>
<td>C</td>
</tr>
<tr>
<td>June 26.00</td>
<td>146.00</td>
<td>-12.0</td>
<td>.391</td>
<td>-0.2</td>
<td>C</td>
</tr>
<tr>
<td>July 9.97</td>
<td>159.97</td>
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<tr>
<td>26.91</td>
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<tr>
<td>Aug 11.92</td>
<td>192.92</td>
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<tr>
<td>31.85</td>
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<tr>
<td>Sept 13.86</td>
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<td>Oct 27.75</td>
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<td>.712</td>
<td>+0.9</td>
<td>C</td>
</tr>
<tr>
<td>1983 Feb  3.61</td>
<td>45368.61</td>
<td>-40.2</td>
<td>1.969</td>
<td>-0.3</td>
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</tr>
<tr>
<td>23.24</td>
<td>388.24</td>
<td>-50.8</td>
<td>2.020</td>
<td>-1.2</td>
<td>C</td>
</tr>
<tr>
<td>Mar 11.17</td>
<td>404.17</td>
<td>-50.1</td>
<td>.061</td>
<td>+0.7</td>
<td>C</td>
</tr>
<tr>
<td>15.18</td>
<td>408.18</td>
<td>-49.6</td>
<td>.072</td>
<td>+1.6</td>
<td>C</td>
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<tr>
<td>Apr 16.08</td>
<td>440.08</td>
<td>-41.3</td>
<td>.155</td>
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<td>Dec 4.70</td>
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<td>24.12</td>
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<tr>
<td>May 13.06</td>
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<td>-0.5</td>
<td>C</td>
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<tr>
<td>June 9.07</td>
<td>860.07</td>
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<td>C</td>
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<tr>
<td>1985 Jan   24.28</td>
<td>46089.28</td>
<td>-10.7</td>
<td>3.841</td>
<td>0.0</td>
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</tr>
<tr>
<td>Feb 17.56</td>
<td>113.56</td>
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<td>.904</td>
<td>-0.2</td>
<td>V</td>
</tr>
<tr>
<td>24.20</td>
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<td>.921</td>
<td>0.0</td>
<td>C</td>
</tr>
<tr>
<td>Mar 11.20</td>
<td>135.20</td>
<td>-37.2</td>
<td>.960</td>
<td>+0.4</td>
<td>C</td>
</tr>
</tbody>
</table>

* Source code:
MtW  Mount Wilson (Adams 1914; Abt 1973)
DAO  Dominion Astrophysical Observatory (Harper 1934)
Tr   Tremblot (1938)
Hn   Hendry (1978)
RGO  Royal Greenwich Observatory (Woolley et al., 1981)
C    Cambridge spectrometer (Griffin 1967)
V    Victoria spectrometer (Fletcher et al. 1982)
P    Palomar spectrometer (Griffin & Gunn 1974)
2.3 Spectroscopic Observations

We have taken six 10Å mm\(^{-1}\) plates of HR 6902 at Mount Wilson, all on IIa–O emulsion. The first two were exploratory ones, taken in 1982 before we knew the orbit, with the comparatively small trailed width of 1 mm. In August of the following year, when we visited Mount Wilson principally for other purposes but knew HR 6902 to be quite near a node, we obtained a 2\(\frac{1}{2}\)-mm-wide spectrogram. Finally, in 1984 March we took three plates on consecutive nights during a visit especially timed to enable HR 6902 to be observed at the more favourable node when the velocity difference between the components would be the maximum possible. The total width of the three spectra was 5\(\frac{1}{2}\) mm; on two of the nights observing conditions did not allow a spectrogram of the full 2\(\frac{1}{2}\) mm width permitted by the spectrograph to be obtained during the limited time that HR 6902, which is far over to the east in the morning sky in March, was accessible. Fig. 3 shows prints made from short regions of spectrograms taken at both nodes, and also of spectra whose types most nearly match those of the components of HR 6902.

In Section 1.2.1 we pointed out that in principle only one spectrogram is needed to determine the amplitude of radial-velocity variation of the secondary component of a composite-spectrum binary and thereby to find the mass ratio. That is perfectly true; but if there were a systematic—or any other—error in the measurement of velocities from our subtracted spectra, it would simply vitiate the mass ratio and would not be apparent if we relied on just a single spectrogram. In this initial paper, therefore, we have thought it wise to treat an object for which we have several spectra, including ones taken near each node. The shape of the radial-velocity curve is almost entirely defined by the spectrometer observations of the primary star; all that the velocities of the secondary are called upon to do is to fix the amplitude of variation of that component, so only one degree of freedom is thereby lost. The residuals of the secondary velocities can therefore furnish a true idea of the errors inherent in our procedures. Those velocities will be seen not to suffer from any significant nonrandom error, and their Standard deviation will be shown to be better than 1 km s\(^{-1}\), corresponding to little more than 1 \(\mu\)m on our 10-Å mm\(^{-1}\) spectrograms.

2.4 Recovery of the Secondary Spectrum

Reduction of the spectrograms proceeded in the manner outlined in the earlier part of this paper. The digital data sets resulting from the three plates taken on consecutive nights were averaged, with weighting according to their relative widths, and were thereafter treated as a single observation. The best match to the primary spectral type from among the available standards was found to be an equally-weighted average of \(\varepsilon\) Sct (G8 II) and \(\varepsilon\) Boo A (K0 II–III), so the derived type could be said to be G9 II. The result of subtracting that hybrid cool standard from HR 6902 was a spectrum that is matched very well by that of \(\pi\) Cet (B7 V) and even more accurately by an equal hybrid of \(\pi\) Cet and \(\nu\) Cap (B9.5 V). Those stars are said in the Bright Star Catalogue (Hoffleit 1982) to have projected rotational velocities of only 18 and 17 km s\(^{-1}\) respectively; the lines of the B star in HR 6902 are if anything sharper still. The radial velocity of \(\pi\) Cet has been considered variable (Moore & Albrecht 1910) but there is no hint of any secondary star in the spectrum, so for our purposes \(\pi\) Cet is satisfactory as a standard.
Figure 3. Small parts of some of the spectra upon which this paper is based. The middle two spectra are of HR 6902, taken near opposite nodes of the orbit on 1983 August 31 (upper) and 1984 March 24 respectively. The top spectrum is of ε Set (G8 II) and the bottom one is of π Cet (B7 V). These spectra were taken at 10 A min⁻¹ with the coude spectrograph of the 100-inch Hooker reflector on Mount Wilson.

R. & R. GRIFFIN
Figure 4. Subtraction of spectra. The top spectrum is adopted as the analogue of the late-type component of HR 6902. Subtraction of an optimal proportion of it from the spectrum of HR 6902 (second tracing down) isolates the early-type component (third tracing), which may be compared with the standard spectrum at the bottom. The horizontal line below each tracing is the zero-intensity level.

R. & R. Griffin
Figure 5. The spectrum of HR 6902 B (upper tracing in each panel) compared with the spectrum of the B8 V standard. The horizontal line below each tracing represents zero intensity for HR 6902 B; the B8 V spectrum has been displaced by \(-20\) per cent of the continuum height. R. & R. GRIFFIN
The process of subtraction is well illustrated by Fig. 4, which shows just one of the four 200-Å regions into which our spectra are divided. It will be seen that the B-type spectrum, visible in the tracing of HR 6902 itself as little more than a loss of contrast of the spectrum of the cool star, emerges as a result of the subtraction with astonishing clarity and detail; and its correspondence with the B8 hybrid standard is remarkably exact. The radial-velocity displacement, of about 1Å to the red, in the spectrum uncovered for the hot component of HR 6902 is noticeable even at the small scale of Fig. 4.

An interesting feature marginally visible in Figs 3 and 4 is the duplicity of the sharp K line in HR 6902. The line contains an interstellar component as well as one arising in the B star. HR 6902 is probably some 300 pc away and has a Galactic latitude of about 9°, so the appearance of an interstellar K line is not surprising. Any effort to measure the radial velocity of the secondary star by means of the K line, such as was proposed by Hendry (1978), would be likely to give misleading results unless it were carried out on spectra of sufficiently high quality and resolution to enable a clear distinction to be drawn between the two contributors to the line. However, now that the metal lines in the secondary are known to be so sharp, a sufficiently shrewd investigator might find it possible to measure some of them directly in the composite spectrum, especially at wavelengths that are shorter than those we have illustrated and where the contribution of the primary star is smaller.

The signal/noise ratio in the spectrum of the secondary star can be maximized by averaging the results of all the available plates. The individual spectra need to be shifted back to zero displacement before they are averaged. An interesting point concerns the signal/noise requirements of the cool standard spectra: there is only one spectrogram of each of the two stars that make up the hybrid G9 II standard. Because the secondary spectra have to be shifted before they are ready to be co-added, the noise due to the G9 standard (which was used in every reduction) is out of register when the spectra are summed, so to all intents and purposes the same standard spectrum becomes different ones when it is used at sufficiently differing radial-velocity shifts. The critical shift needed to ensure independence of the noise is equal to the autocorrelation length of the noise. The noise is manifested as photographic grain, which is probably under-sampled by our 5-µm microdensitometer intervals. The limit is therefore set by our own procedures. Actual trials on featureless spectra show that the autocorrelation of the noise is down to 10 per cent at a displacement of ± 2 spectral elements (10 mÅ) and is zero at ± 3 elements (15 mÅ) and beyond. Two elements correspond to about 7 km s\(^{-1}\), so spectra whose shifts differ by more than that are effectively independent as far as the noise from the standard spectra is concerned.

Fig. 5 shows our final results on the secondary spectrum, over the complete wavelength range for which we have recovered it. The signal/noise ratio naturally tends to fall off towards longer wavelengths, where the relative brightness of the secondary star is rapidly diminishing in comparison with the primary. We repeat here the caveat given in Section 1.2.2, that the part of the spectrum above λ 4500 Å is liable to be slightly contaminated with the overlapping spectrum from the next (third) order of diffraction of the grating.

We think that the initial impression given by Fig. 5 is a generally favourable one: the spectrum that our procedure has revealed for HR 6902 B is indeed extremely similar to the genuine B-star spectrum that we have provided for the purposes of visual comparison. However, a sufficiently critical appraisal does disclose some deficiencies;
among the ones that we find most objectionable are the residual traces of the primary star's G band near $\lambda 4310$ Å, and the apparent emission lines near $\lambda 4215$ Å. These imperfections, and other lesser ones which may be detected by the discerning eye, no doubt arise because we have been unable to find an absolutely exact match for the primary star among our library of late-type standard spectra. Near the spectral type of the primary, G9 II, the G band (due to the CH radical) shows little systematic change of strength with either spectral type or luminosity, so there is no hope of finding a better match by looking for a more appropriate spectral type. What we need is another G9 II standard spectrum with a slightly stronger G band, no doubt reflecting a slightly greater carbon abundance or minor differences in atmospheric structure. It must be realized that the difference between the present G9 II standard and HR 6902 A is nothing like as great as the apparent defects in the spectrum of HR 6902 B; especially at the longer wavelengths, the composite spectrum is so dominated by the primary that imperfections in the match with its surrogate are magnified several-fold in the difference-spectrum that is left to represent the secondary. Just as the G band fails to match perfectly, so the CN bands, whose principal heads are at $\lambda \lambda 3850$ and 3883 Å, are not of identical strength in HR 6902 A and in the G9 II standard. CN dominates the late-type spectrum between $\lambda \lambda 3850$ and 3883 Å; the subtraction process (shown in detail in Fig. 4) therefore ensures that the right amount of CN is subtracted, but that is done at the slight expense of the atomic lines. Just above the head of the CN band is a ‘window’ where the spectrum rises nearly to the continuum; it forms an arresting feature, looking almost like an emission line, in Fig. 3. The CN band is subtracted so accurately that the spectrum attributed to HR 6902 B passes over the region of this profound disturbance with little sign of discontinuity; but then the profile of the Hζ line at $\lambda 3888$ Å is a trifle disturbed by imperfect matching of certain metal lines near its centre. Similarly, the $\lambda 4215$-Å CN band has disappeared without trace in the spectrum of HR 6902 B, but only at the cost of slight mismatching of some atomic lines just beyond its head.

Except by taking spectra of more and more late-type stars of about the right spectral type in the hope that one of them might prove to be a better match to the relevant component of the composite object, there is nothing we can do to improve on Fig. 5, unless indeed HR 6902 proves to exhibit total eclipses (see Section 2.7.1) and the primary spectrum can then be observed directly free of contamination. We can only say that we are aware of the defects; that they stem in large measure from the fact that late-type spectra are not uniquely characterized by just the two parameters spectral type and luminosity alone; and that we interpret them as showing that the fundamental limitation to the accuracy with which composite spectra can be disentangled by the method we are using is set not so much by our own shortcomings as by ‘cosmic scatter’ in the properties of stars.

2.5 The Radial Velocity of the Secondary

The derivation of the radial velocity from the secondary spectrum has been partly described in Section 1.2.7. The spectra of HR 6902 B and the B-type standard are renormalized to a ‘continuum’ which follows the wings of the Balmer lines, and are then inverted and cross-correlated with one another. The Balmer cores, and also the K line which we have seen to be partly interstellar, are omitted in the cross-correlation, care being taken to site the ends of the sections being correlated in featureless parts of the
Table 2. Redial velocities derived from spectra, recovered by the substraction produced, of the hot component of HR 6902.

<table>
<thead>
<tr>
<th>Date</th>
<th>MJD</th>
<th>Width of spectr. mm</th>
<th>Wt.</th>
<th>Radial velocities*</th>
<th>Phase (O – C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1982 Apr 22.49</td>
<td>45081.49</td>
<td>1.0</td>
<td>0.4</td>
<td>-29.5 +25.8 -3.7</td>
<td>1.223 + 2.2</td>
</tr>
<tr>
<td>1982 Aug 24.18</td>
<td>45205.18</td>
<td>1.0</td>
<td>0.4</td>
<td>-3.0 -37.8 -40.8</td>
<td>1.545 - 0.1</td>
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<tr>
<td>1983 Aug 31.24</td>
<td>45577.24</td>
<td>2.5</td>
<td>1.0</td>
<td>-4.4 -34.1 -38.5</td>
<td>2.511 + 0.4</td>
</tr>
<tr>
<td>1984 Mar 23.49†</td>
<td>45782.49</td>
<td>5.5</td>
<td>2.2</td>
<td>-51.0 +73.2 +22.2</td>
<td>3.044 - 0.1</td>
</tr>
</tbody>
</table>

* \(P\) is the radial velocity computed from the orbit solution for the primary at the relevant epoch; \((S - P)\) is the velocity, determined from the recovered spectrum, of the secondary in the rest frame of the primary; and \(S\) is the resulting velocity of the secondary.

† Sum of three spectrograms taken on consecutive nights.

The velocity data are used, finally, in a double-lined orbit solution for HR 6902. In the expectation that the principal source of uncertainty in the relative velocities is the noise on the spectra of HR 6902, we have thought it appropriate to weight the four velocities of the secondary in direct proportion to the widths of the spectra upon which they are based. It is convenient to regard the unit of weight for the secondary velocities as being that corresponding to a spectrogram with a trailed width of 2½ mm, because then the four measurements have a total weight of exactly 4. In the orbit solution the secondary velocities have been globally weighted by the further multiplicative factor 0.7, in order to cause their weighted mean-square errors to be equal to those of the primary velocities. The actual value of those errors is 0.53 \((\text{km s}^{-1})^2\), so a secondary observation of unit weight has a means-square error of 0.53/0.7, or 0.76, \((\text{km s}^{-1})^2\). In other words, a consideration of the external errors of the secondary velocities, as judged by the orbital residuals, leads to the conclusion that an observation of unit weight (based on a 2½-mm spectrogram) has a standard deviation of \((0.76)^{1/2}, \text{i.e.} \ 0.87 \ \text{km s}^{-1}\) According to our assumption that the weight of an observation is directly proportional to the width of the original spectrogram, the standard deviation of a velocity based on a spectrogram of
width \( W \) mm is \((1.9/W)^{1/2}\) km s\(^{-1}\). Of course these numbers cannot be expected to be very accurate, in view of the fact that we are purporting to do statistics on only four observations; but they certainly should be of the right order.

The final orbit computed from the radial velocities given in Tables 1 and 2 has the elements given in Table 3. The corresponding radial-velocity curves have already been shown in Fig. 2.

### Table 3. Orbital elements for HR 6902.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>385.00 days (fixed, see text)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>(-19.31 \pm 0.11) km s(^{-1})</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>25.26 \pm 0.19 km s(^{-1})</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>33.1 \pm 0.6 km s(^{-1})</td>
</tr>
<tr>
<td>( q )</td>
<td>(1.310 \pm 0.021 (m_1/m_2))</td>
</tr>
<tr>
<td>( e )</td>
<td>0.311 \pm 0.007</td>
</tr>
<tr>
<td>( \omega_1 )</td>
<td>145.9 \pm 1.2 degrees</td>
</tr>
</tbody>
</table>

\[ \text{rms error (unit weight)} = 0.73 \text{ km s}^{-1} \]

The mass ratio \( q \), which is probably the most important single quantitative datum to emerge from this investigation, is seen from Table 3 to be determined to an accuracy of better than 2 per cent. The more evolved star in the system is the more massive one, as is to be expected according to stellar evolution theory; the difference of mass has a Statistical significance of 14 standard deviations.

Masses of main-sequence stars have been conveniently tabulated by Allen (1973) and in Landolt-Börnstein (Schaifers & Voigt 1982). Entering the tables at spectral type B8 V (by interpolation in the case of Allen), we find the mass to be 4.3 or 3.8 \( M_\odot \) respectively. The impression is given, therefore, that the mass of the B8 component of HR 6902 might be taken as close to 4 \( M_\odot \). However, the review by Popper (1980), although cited in Landolt-Börnstein as providing the most reliable data on stellar masses, presents a rather different picture. Popper’s Table 2 lists the properties of main-sequence eclipsing binaries whose components are mutually detached, and includes five systems whose primaries are attributed spectral types of B8 or B9. The most massive of the five is \( \chi^2 \) Hya, whose mass is given as 3.61 \pm 0.08 \( M_\odot \) in a \( (\log T, \log L) \) diagram that star is right at the upper edge of the main-sequence band, and indeed it is said by Andersen (1975) to be on the point of commencing a phase of rapid evolution. AS Cam, V451 Oph and RX Her are more or less in the middle of the main-sequence band and have masses of 3.31 \pm 0.07, 2.78 \pm 0.06 and 2.75 \pm 0.06 \( M_\odot \) respectively. AR Aur is on the zero-age main-sequence line and has a mass of only 2.48 \pm 0.10 \( M_\odot \) respectively.

From those data, therefore, we could conclude that the mass of a B8/B9 star is about 3.0 \( M_\odot \) with an uncertainty of about 20 per cent. However, the lower part of that range is excluded, since our Table 3 shows from the spectroscopic orbit that the minimum mass
of HR 6902 B is 2.95 ± 0.09 \(M_\odot\). If the actual mass is close to that lower limit there must be eclipses in the HR 6902 system. Primary eclipse would take place at phase .252, or 97 days after periastron passage, and could last up to about 13 days if central. Opportunities to watch for eclipses will occur around 1987 August 10 and then 20 days later in each succeeding year. On the first few occasions HR 6902 will be conveniently placed in the evening sky.

Table 1 shows that on 1982 May 4 a radial-velocity trace of HR 6902 was obtained at phase .253, almost exactly the time of a possible eclipse. There is no doubt that that trace shows dips that are significantly deeper than normal. The photon detection rate also suggests that the star was unusually faint on that occasion in comparison with neighbouring stars, although such evidence—acquired through a spectrometer whose entrance slit does not pass all of the star’s light, and moreover through a sky which was probably far from photometric—cannot be conclusive. The trace of 1984 June 9, whose phase of .246 places it little more than two days before the computed mid-time of a possible eclipse, was unfortunately compromised by thick and variable cloud, but it does not support the idea that an eclipse was in progress then. Nevertheless we adopt as a working hypothesis, pending explicit photometric and/or spectroscopic observations at the time of a predicted eclipse, that eclipses do occur and that therefore the factor sin\(^3\)i in the expression for the stellar masses must be almost unity. Thus the mass of the B8 V star is very close to 3.0 \(M_\odot\) and that of the G9 II component is 3.9 \(M_\odot\).

If eclipses do indeed take place, then HR 6902 is to be seen as an important new addition to the small class of bright \(\zeta\) Aur stars, eclipsing systems consisting of a late-type supergiant and a B star. There are five presently recognized members of the class: \(\zeta\) Aur itself, 31 and 32 Cyg, VV Cep and 22 Vul. Their particular significance lies in the fact that, around the times of eclipses, the line of sight to the B star passes through the outer layers of the distended atmosphere of the supergiant. Spectra taken at those times are apt to show circumstellar absorption features, and a time sequence of such spectra (e.g. Wright 1970, Plate I) can give information on the properties of the chromosphere as a function of radius from the star. Because the supergiant in HR 6902 is probably less luminous than that in any of the other \(\zeta\) Aur systems, it is perhaps to be expected that the duration of any episode of circumstellar absorption will be relatively short (only a few days at most on either side of the actual eclipse). The first Mount Wilson spectrogram was exposed 11 days before a prospective eclipse date, and does not show any unusual features.

### 2.7.2 Luminosities

Appeal to the same authorities as before, this time for information on stellar luminosities, produces for types B8 V and G9 II the absolute magnitudes 0.0 and (by rather risky interpolation) – 2 respectively from Allen (1973) and – 0.25 and – 2.3 from Landolt-Bornstein (Schaifers & Voigt 1982). Popper’s (1980) late-B stars are in tolerable agreement, having absolute magnitudes near zero though with a range amounting to ± 1 magnitude.

One can see immediately that a difference in luminosity of the order of two magnitudes in the \(V\) band is entirely consonant with the apparent near equality of the stars in the \(\lambda\) 4000 Å region. However, we ought in principle to be able to establish the luminosity difference quite accurately, using information already available in the
spectrum-subtraction process in order to obtain the true flux ratio as a function of wavelength. When we subtract the late-type spectrum from the composite one, we are in effect dividing the light in the composite spectrum between the two components in carefully defined proportions. By summing the intensities assigned to each component in every successive wavelength element within a given interval, we should be able to determine the relative luminosities of the two stars in any desired wavelength interval within the range covered by our observations.

Notice that the division of light between the components is largely independent of the photometric and spectroscopic characteristics of the standard stars employed and of such external factors as interstellar reddening and our treatment of the continuum in the various stars concerned. If, for example, we reduced the height of the continuum of the late-type standard star by a factor of two, we should simply find that we needed to subtract twice as large a proportion of it, and the result would be exactly the same. Since the proportion to be subtracted is optimized in every 50-Å band independently, effects which vary slowly with wavelength, such as reddening and the apparent slopes of the stellar continua, have negligible baselines over which to operate.

A suitable source of absolute photometry for comparison with the relative intensities derived from our spectrophotometry appears to be the tabulation by Willstrop (1965), which offers intensities in sharply-bounded 50-Å bands for stars of a number of representative spectral types. Included in the tabulation are two B8 V stars, $\phi$ Eri and $\eta$ Aqr. For our purposes their fluxes are hardly distinguishable from one another, and we have arbitrarily selected $\phi$ Eri. The only late-G star of luminosity class II observed by Willstrop is $\epsilon$ Sct (G8 II), which is one of the contributors to our hybrid analogue of the late-type primary of HR 6902, the other being $\epsilon$ Boo A (K0 II–III).

In Table 4 we give the flux ratios found from our splitting of the spectrum of HR 6902 into its two components. To obtain an exact correspondence with Willstrop’s table, the intensities whose ratios are given have been summed in successive 50-Å bands. The ratios have been averaged from all the available plates; we ourselves were somewhat disappointed at the level of interagreement between different determinations, and the mean values shown in the Table have accuracies no better than a few per cent. The Table also lists the ratio of fluxes between $\epsilon$ Sct and $\phi$ Eri, as derived from Willstrop’s photometry whose errors are said to be 1 or 2 per cent. The $\epsilon$ Sct/$\phi$ Eri ratios have been multiplied throughout by the empirical factor 5.6, which represents the relative brightness we need to attribute to $\epsilon$ Sct relative to $\phi$ Eri (in the $V$ band, to which Willstrop’s data are normalized) in order to mimic the ratio observed for HR 6902. With such a factor, representing 1.9 mag in magnitude terms, there is seen to be a close correspondence between the observed flux ratios and those obtained from the absolute photometry of $\epsilon$ Sct and $\phi$ Eri. This result reassures us that the spectral types that we have assigned on purely spectroscopic grounds also provide a satisfactory photometric representation of the HR 6902 system, and it provides a rather accurate estimate of the visual luminosity difference between the components, again in close correspondence with the difference expected on the basis of the spectral types.

2.7.3 Rotation periods

The spectral lines of the primary star in HR 6902 are slightly but distinctly broadened by (presumably) rotation: the single Palomar radial-velocity trace furnishes an estimate
Table 4. Relative fluxes of $\epsilon$ Sct and $\phi$ Eri in 50-Å bands, compared with the ratios between the components of HR 6902. The $\epsilon$ Sct/$\phi$ Eri ratios have all been multiplied by the empirical factor 5.6, which represents the relative visual luminosities of the components of HR 6902.

<table>
<thead>
<tr>
<th>$\lambda$ (Å)</th>
<th>From Willstrop (1965)</th>
<th>HR 6902 (observed)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\epsilon$ Sct</td>
<td>$\phi$ Eri</td>
</tr>
<tr>
<td>3850</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>3900</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>3950</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>4000</td>
<td>170</td>
<td>944</td>
</tr>
<tr>
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<td>170</td>
<td>831</td>
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<td>632</td>
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<tr>
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<tr>
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<td>651</td>
</tr>
<tr>
<td>4600</td>
<td>309</td>
<td>630</td>
</tr>
</tbody>
</table>

of $v \sin i \approx 12$ km s$^{-1}$, so $v = 12$ km s$^{-1}$. The broadening is too small to be apparent in Fig. 3. If the radius of the star is estimated at 40 $R_\odot$ (28 Gm), the corresponding rotation period is about 15 Ms or 170 days. The rotation is therefore probably not synchronized with the 385 day orbital period: that would require $v \sin i \approx 5$ km s$^{-1}$ or $R_* \approx 100$ $R_\odot$ (both of which are well outside the likely range of uncertainty) or a combination of revisions (still almost unacceptable) to both $v \sin i$ and $R_*$. The lines of the secondary are so sharp that we can only set an upper limit of about 15 km s$^{-1}$ to $v \sin i$: the rotation period is longer than about 10 days.

Acknowledgements

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Eggleton who saw a preprint of the paper, led us to look for (and probably discover) eclipses in HR 6902; we had previously discounted the possibility of eclipses because we were misled by the exaggerated masses tabulated for late-B stars by certain reference handbooks.

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**Note added in proof**

The Dominion Astrophysical Observatory, Victoria, very kindly provided us with a generous allocation of observing time on the 48-inch reflector to monitor HR 6902 spectroscopically around the time of conjunction in 1986 July. An eclipse was seen; preliminary indications are that it was total for about 5 days. Chromospheric absorption, persisting for about 3 days after the end of totality, was also observed. Photometric observers in Arizona were alerted at short notice to the likelihood of the eclipse through the good offices of Mr R. M. Genet, but we do not yet know the outcome.