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Energy Loss of Electrons in Solar Atmosphere taking Many-Body Interactions into Account

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Abstract. Role of many-body interactions on the energy loss of electrons accelerated at neutral point during solar flares has been studied. Energy loss with and without many-body interactions has been computed for different electron-density models as function of height. The energy loss increases by a factor of two by inclusion of many-body interactions for incident electron energies greater than 10 keV. Role of this on the generation of hard X-rays is discussed.

Key words: Sun, flares—Sun, hard X-rays

Hard X-ray bursts ($\geqslant 20$ keV) from solar flares are believed to be generated by collisional bremsstrahlung mechanism by the interaction of nonthermal electrons with the ambient dense plasma of the chromosphere (Korchak 1967; Cline, Holt & Hones 1968). These electrons are accelerated to nonthermal energies in the current sheet near an X-type neutral point. During the downward motion of these electrons towards the photosphere they interact with regions of constantly increasing density and undergo strong deceleration resulting in the emission of hard X-ray bursts (Takakura 1969; Holt & Ramaty 1969). Most of the processes involved in the energy loss by these nonthermal electrons in flare conditions have been discussed in some detail by Brown (1971). In most of the calculations, the author considers only the short-range binary collisions and does not account for the long-range Coulomb interaction responsible for the collective behaviour of the ambient plasma. We propose, therefore, to improve upon this assumption and want to incorporate the effect of collective behaviour on the energy loss of nonthermal electrons in thick target model.

The accelerated stream of electrons moving down towards the photosphere interacts with the ambient plasma particles through electron-electron and electron-ion collisions. The electron-ion collisions result mostly in the scattering of the incident electrons and thus determine the trajectory of the precipitating electron beam. The electron-electron interactions, however, dominate the energy loss of the incident electrons. The bremsstrahlung X-ray spectrum depends on electron energy and angular distributions. The energy distribution of electrons depends on the energy loss. Therefore, the energy loss of electrons plays a role in X-ray spectrum calculations. Accounting for Coulomb interaction, the mean rate of energy loss suffered by an electron moving with a velocity v through an electron gas at a temperature T is given by

$$-\frac{1}{N_e}\frac{\mathrm{d}E}{\mathrm{d}x} = \left(\frac{8\sqrt{\pi}e^4}{m_e v_e v}\right) F\left(\frac{v}{v_e}\right) \ln \Lambda \tag{1}$$

where N_e is the number density of ambient electrons, v_e is the thermal velocity of electrons and the function $F(x)$ is defined as

$$F(x) = \frac{1}{x}\int_0^x \exp(-x^2)\,dx - \exp(-x^2)$$

$$\simeq \begin{cases} -1 + \frac{5}{3}x^2 + \ldots & (x < 1) \\ \frac{\sqrt{\pi}}{2x} + \ldots & (x > 2). \end{cases} \quad (2)$$

Equation (1) is derived on the basis of binary encounter theory in which the interaction of the incident electron with the electron gas *is* looked upon as a superposition of many independent electron-electron collisions in the Coulomb field. However, the Coulomb interaction being of a long-range character, the incoming electron will interact simultaneously with the large number of ambient electrons within the Debye sphere.

The effect of many-body interactions on the rate of energy loss by an electron has quantitatively been studied in the recent past by a number of authors (Perkins 1965; Itikawa & Aono 1966; Ray 1969). In the unified theory of Kihara & Aono (1962) the rate of energy loss of an electron moving through an electron gas is given by

$$-\frac{1}{N_e}\frac{dE}{dx} = \frac{8\sqrt{\pi}\,e^4}{m_e v_e v}\left[F\left(\frac{v}{v_e}\right)\ln\left(\frac{2kT_e}{\gamma e^2}\lambda_D\right) + G\left(\frac{v}{v_e}\right)\right]$$

$$= \frac{8\sqrt{\pi}\,e^4}{m_e v_e v}\left[F\left(\frac{v}{v_e}\right)\ln\Lambda + \left\{F\left(\frac{v}{v_e}\right)\ln\frac{2}{3\gamma} + G\left(\frac{v}{v_e}\right)\right\}\right] \quad (3)$$

where $\gamma = 0.577$, is Euler's constant. The function $G(u)$ gives the dependence of the Coulomb logarithm on the velocity of the incident electron (Itikawa & Aono 1966). Analytical expression for this function has been given by Takayanagi & Itikawa (1970) in a different context. We reproduce it below for convenience.

$$\frac{2}{\sqrt{\pi}}G(u) = 4\left[\frac{1}{u}\phi(u) - \frac{d\phi(u)}{du}\right] + \left[\frac{1}{u}\tilde{\phi}(u) - 2\frac{d\tilde{\phi}(u)}{du}\right] - 2ug(u) + \frac{2}{\sqrt{\pi}}\exp(-u^2)C(u),$$

where ϕ is the error function

$$\phi(u) = \frac{2}{\sqrt{\pi}}\int_0^u \exp(-x^2)\,dx,$$

and

$$\tilde{\phi}(u) = \frac{u}{\sqrt{\pi}}\int_0^\infty dx\,\sinh(2ux)\ln x\exp[-(u^2 + x^2)],$$

$$g(u) = \frac{2}{\sqrt{\pi}}\frac{1}{u^2}\int_0^u dx\,x^2\exp(-x^2)C(x),$$

$$C(x) = \frac{1}{4}\ln\left[\{\xi(x)\}^2 + \{\eta(x)\}^2 + \frac{\xi(x)}{2|\eta(x)|}\left[\frac{\pi}{2} - \tan^{-1}\frac{\xi(x)}{|\eta(x)|}\right]\right].$$

The functions ξ and η in the above equation are given by

$$\xi(x) = 1 + x \operatorname{Re} Z(x)$$
$$\eta(x) = x \operatorname{Im} Z(x)$$

where Re Z and Im Z are, respectively, the real and imaginary parts of the plasma dispersion function (Fried & Conte 1961)

$$Z(x) = -2\exp(-x^2)\int_0^x \exp(t^2)\,dt + i\sqrt{\pi}\exp(-x^2)$$

which characterizes the collective interaction between the incident electron and the plasma. For $u \gg 1$, the form of the function $G(u)$ reduces to

$$\frac{2}{\sqrt{\pi}}G(u) \simeq \frac{1}{u}\ln(\sqrt{2}\,u^3).$$

Equation (3) which gives us the rate of energy loss by an electron differs from the corresponding equation obtained on the basis of binary encounter theory by the terms in the curly brackets. Corresponding to the incident electron energy of the order of a few electron volts we find that in the solar flare region $v/v_e \ll 1$. For such small values of v/v_e the values of F and G functions are approximately equal so that in this range of incident electron energies the use of the unified theory predicts the rate of electron energy loss which does not differ appreciably from that predicted on the basis of binary encounter theory. However, the electrons that are responsible for the emission of hard X-rays from flare regions have their energies of the order of 10 keV and even more. The value of v for nonthermal electrons is in the range of 4.2×10^7–1.88×10^8 m s^{-1} and the value of thermal electron velocity v_e varies from 1.23×10^6 m s^{-1} for $T = 10^5$ K to 1.23×10^7 m s^{-1} for $T = 10^7$ K. The ratio of v/v_e for $T = 10^7$ K lies between 3.41 and 15.29 for electron energies 5 keV and 100 keV respectively, and increases at lower temperatures. For electrons of such large energies ($v/v_e \gg 1$) the terms in the curly brackets assume a significant role in the rate of electron energy dissipation. For incident electron energies of the order of 10 keV or more, Equation (3) above can be rewritten as

$$\frac{1}{N_e}\frac{dE}{dx} \simeq \frac{4\pi e^4}{m_e v^2}\left[\ln\Lambda + \left\{\ln\frac{2\sqrt{2}}{3\gamma} + \ln\left(\frac{v}{v_e}\right)^3\right\}\right]. \tag{4}$$

Table 1. Electron velocities at different energies.

Energy E keV	Velocity v $5.95 \times 10^5\,[E(\text{eV})]^{1/2}$ m s^{-1}	v/v_e $T(\text{K}) = 10^5$ $v_e(\text{m s}^{-1}) = 1.23 \times 10^6$	10^6 3.89×10^6	10^7 1.23×10^7
5	4.2×10^7	34.15	10.79	3.41
10	5.95×10^7	48.37	15.29	4.84
15	7.28×10^7	59.19	18.71	5.92
20	8.41×10^7	68.37	21.62	6.84
30	1.03×10^8	83.73	26.48	8.37
40	1.19×10^8	96.75	30.59	9.68
50	1.33×10^8	108.13	34.20	10.81
100	1.88×10^8	152.85	48.33	15.29

Equation (4) gives a much larger rate of electron energy dissipation and stands out in contrast to Equation (1) used previously in computing the rate of energy loss of electrons moving through the solar atmosphere.

Using Equations (1) and (4) we have computed the energy-loss rate of the incident electron as a function of its energy for different values of electron densities, the ambient temperature and Coulomb logarithm. At 10 keV, the energy loss calculated by Equations (1) and (4) differ by less than five per cent. We show the loss rate computed with and without collective effects in Fig. 1 for various values of temperature T_e, electron density N_e and corresponding Coulomb logarithm In Λ. The values of T_e and

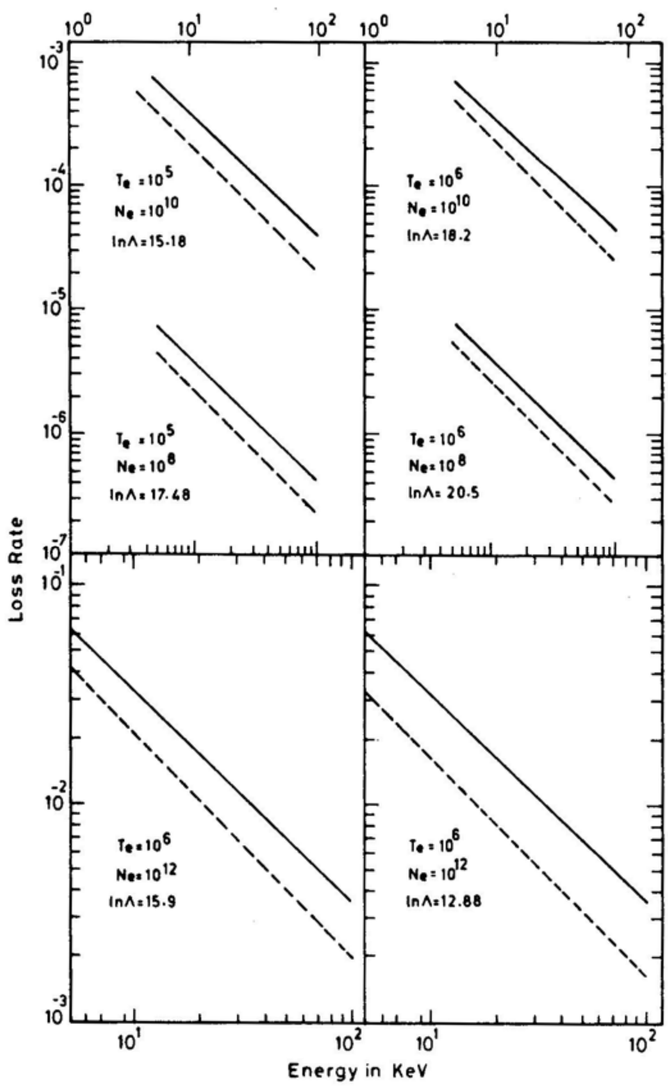

Figure 1. Variation of electron energy loss rate as a function of incident electron energies for different values of electron temperature T_e(K), electron densities N_e(cm^{-3}), and Coulomb logarithm In Λ, Solid line: with collective effect; dashed line: without collective effect.

N_e are representative of temperatures and electron densities found at different heights in solar atmosphere, at different times. A look at the figures shows that at a fixed temperature, the energy loss increases with an increase in electron number density. For a fixed electron density the energy loss becomes more at higher temperatures because the Coulomb logarithm becomes higher.

From these results we find that the rate of energy loss after the inclusion of the many-body effect is invariably larger than the corresponding one in the absence of collective effects. The enhanced energy loss must, therefore, be reflected in the polarization and energy of the bremsstrahlung X-rays generated by such electrons during their deceleration process. This role of the many-body effect would become more significant in the case of nonthermal incident electrons which travel a longer distance than the corresponding electrons of smaller energy.

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