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Equilibrium Configuration of the Magnetosphere of a Star Loaded with Accreted Magnetized Mass

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Abstract. Equilibrium configuration of the magnetosphere of a star loaded by the gravitationally accreted plasma having its own magnetic field is investigated. Axisymmetry around the star's magnetic axis is assumed for simplicity. It is seen that two distinct configurations appear for the cases of parallel and antiparallel magnetic field of the accreted plasma with respect to the star's magnetic moment. If the external field is antiparallel to the star's magnetic moment, the stellar magnetosphere is confined within a spherical region surrounded by the external field with a separatric surface between them. This is an extension of the case of the spherical accretion of non-magnetic plasma dealt with thus far in connection with the mass accretion by the degenerate stars in X-ray binaries. It is noticed that the mass slides down along the field lines to the point closest to the star and is stratified hydrostatically in equilibrium to form a disk in the equatorial plane. The mass loading compresses the sphere as a whole in this case. If, on the other hand, the external field is parallel to the star's magnetic moment, there appears a ring of magnetic neutral point in the equatorial plane. Polar field is open and extends to infinity while the low-latitude field is closed and faces the external field of opposite polarity across the neutral point. The increase of the loaded mass in this case causes a shrink of the closed field region, and the open polar flux is increased. Therefore, the transition between equilibria with small and large amount of the loaded mass requires the reconnection of magnetic lines of force, and the reconnection of the flux through the magnetic neutral ring is proposed as the mechanism of the steady or the intermittent mass leakage like the ones postulated for some X-ray bursters.

Key words: stellar magnetospheres—accretion of magnetized mass—X-ray binaries

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1. Introduction

The magnetosphere of a celestial object treated as a physical system is a useful concept encountered in diverse astrophysical circumstances. Examples of it include the magnetospheres of the earth and planets, the magnetospheres of the stars ranging from that of the sun to that of degenerate objects such as white dwarfs and neutron stars, and the ‘ magnetospheres ’ of the galaxies, *i.e.* galactic magnetic halos. The magnetosphere embodies the physical interaction between the central object and its environment through the magnetic field of the celestial body. Depending on the nature of the central object and its environment, quite different physical situations exist. For example the underlying effect in the case of earth’s magnetosphere is the inter action between the slowly rotating earth and the directed flow of plasma in the solar wind. Stellar magnetosphere may involve the trapping of the heated plasma (corona) and the control of the outflow of the plasma by magnetic field (stellar wind). In contrast, the magnetosphere of a neutron star in X-ray binaries embodies the interaction of the gravitationally accreted plasma with rapidly rotating star having a strong magnetic field.

Although there have been discussions of the phenomena occurring in these magnetospheres and a considerable progress has been achieved, only few attempts have been made to obtain the self-consistent configurations of the global magnetic field structure in them. It is needless to say, however, that the global model field is important because it provides the stage for those various physical processes and may affect even the physical interpretation of the observed phenomena.

In the present paper we confine ourselves to the problem of the axisymmetric non-rotating magnetosphere in equilibrium with the magnetized mass accreted by the central gravitating star, as a first step to more complex problems. This situation may correspond to a magnetic star embedded in a dense cloud of interstellar matter which has its own large-scale weaker magnetic field, as an example. In Section 2, we derive the basic equation describing the axisymmetric magnetosphere with loaded mass. A class of exact solutions are obtained and analysed in Section 3. In Section 4, we give discussion and mention the implication and the application of the results obtained.

2. Axisymmetric magnetostatic equilibrium

Consider an axisymmetric plasma surrounding a spherically symmetric star which has the gravitational potential,

$$\Phi = -\frac{GM}{r} \tag{1}$$

where G is the gravitational constant, M is the mass of the star and r is the distance from the centre. The magnetostatic equilibrium of the plasma is governed by the equation,

$$\frac{1}{c}(\mathbf{j} \times \mathbf{B}) - \nabla p = \rho \nabla \Phi \tag{2}$$

where **B** is the magnetic field, p and ρ are the pressure and the density of the plasma, respectively. **B** fulfils $\nabla \cdot \mathbf{B} = 0$, and $p = \mathcal{R}\ \rho\ T$, where T is the temperature and $\mathcal{R}$ is the gas constant. The task at hand is to derive the solution of equation (2) with these auxiliary relations*.

Let us use the spherical coordinate $(r,\ \theta,\ \phi)$ with the pole coinciding with the axis of symmetry. Then, the condition $\nabla \cdot \mathbf{B} = 0$ and the assumption of axisymmetry imply that B is expressible as

$$\mathbf{B} = \left\{\frac{1}{r \sin\theta} \frac{\partial (\sin\theta\, A_\phi)}{\partial \theta},\ -\frac{1}{r} \frac{\partial (r\, A_\phi)}{\partial r},\ B_\phi\right\}, \tag{3}$$

where A_ϕ is the ϕ–component of the magnetic vector potential. Note that the magnetic field **B** is defined in terms of two scalar functions, namely, A_ϕ and B_ϕ the current density **j** is given by

$$\frac{4\pi}{c}\mathbf{j} = \nabla \times \mathbf{B} = \left[\frac{1}{r \sin\theta} \frac{\partial (\sin\theta\, B_\phi)}{\partial\theta},\ -\frac{1}{r} \frac{\partial (r\, B_\phi)}{\partial r},\right.$$
$$\left. -\frac{1}{r}\left\{\frac{\partial^2 (r\, A_\phi)}{\partial r^2} + \frac{\partial}{\partial\theta}\left(\frac{1}{r \sin\theta} \frac{\partial (\sin\theta\, A_\phi)}{\partial\theta}\right)\right\}\right]. \tag{4}$$

Substituting equations (3) and (4) into equation (2) and considering the force balance in the azimuthal direction, we have

$$\frac{\partial (A_\phi\, r \sin\theta,\ B_\phi\, r \sin\theta)}{\partial (r,\ \theta)} = 0, \tag{5}$$

and this demands that

$$\tilde{B} = \text{func}\,(\tilde{A}) \tag{6}$$

where $\tilde{A} = A_\phi\ r \sin\theta$ and $\tilde{B} = B_\phi\ r \sin\theta$ (Lust and Schlüter 1954; Chandrasekhar 1956).

By using this, the r and θ-components of eqution (2) then take the form,

$$\frac{\partial\tilde{A}}{\partial r}\left\{\frac{\partial^2\tilde{A}}{\partial r^2} + \sin\theta \frac{\partial}{r\,\partial\theta}\left(\frac{1}{\sin\theta} \frac{\partial\tilde{A}}{r\,\partial\theta}\right) + \frac{1}{2}\frac{d\tilde{B}^2}{d\tilde{A}}\right\} = -4\pi\, r^2 \sin^2\theta \left(\frac{\partial p}{\partial r} + \rho \frac{\partial\Phi}{\partial r}\right), \tag{7}$$

$$\frac{\partial\tilde{A}}{r\partial\theta}\left\{\frac{\partial^2\tilde{A}}{\partial r^2} + \sin\theta \frac{\partial}{r\,\partial\theta}\left(\frac{1}{\sin\theta} \frac{\partial\tilde{A}}{r\,\partial\theta}\right) + \frac{1}{2}\frac{d\tilde{B}^2}{d\tilde{A}}\right\} = -4\pi\, r^2 \sin^2\theta \frac{\partial p}{r\,\partial\theta}, \tag{8}$$

*Comfort, Tandberg-Hanssen and Wu (1979) and Hundhansen and Zweibel (1981) have recently dealt with this problem in a different approach. Approaches in these are ultimately equivalent to ours, as they should be, but we believe that our formulation is more convenient in having insight into the type of the problem to be dealt with in this paper.

respectively. Multiplying equation (7) by $\partial\tilde{A}/r\partial\theta$ and equation (8) by $\partial\tilde{A}/\partial r$ and subtracting one from the other, we obtain

$$\frac{\partial(p,\tilde{A})}{\partial(r,\theta)}+\rho\frac{\partial\Phi}{\partial r}\frac{\partial\tilde{A}}{\partial\theta}=0. \tag{9}$$

The magnetic field is now expressed in terms of $\tilde{A}$ and $\tilde{B}$ in the form,

$$\mathbf{B}=\frac{1}{r\sin\theta}\left(\frac{1}{r}\frac{\partial\tilde{A}}{\partial\theta},-\frac{\partial\tilde{A}}{\partial r},\tilde{B}\right) \tag{10}$$

and the magnetic lines of force are given by integrating the line-of-force equation,

$$\frac{d\mathbf{x}}{ds}=\frac{\mathbf{B}}{|\mathbf{B}|} \tag{11}$$

where $d\mathbf{x}/ds \equiv (dr/ds,\ rd\theta,\ r\sin\theta\ d\phi/ds)$ and s is the length measured along the field line. If we look at this equation in a meridional plane, ϕ = const, it is easy to see by using equations (10) and (11) that $d\tilde{A}=0$ along the projection on this plane of each field line. In other words, the curves $\tilde{A}(r,\theta)$ = const represent the lines of force projected on the $r\theta$–plane.

Given $\tilde{A}(r,\theta)$, we may transform an arbitrary function of r and θ into a function of r and $\tilde{A}$. We denote by $\partial/\partial_{\tilde{A}} r$ the partial derivative with respect to r in this transformed system. This technique of transformation was used by Low (1975) for a magnetostatic problem in the planar geometry.

Transforming $p(r,\theta)$ into $p(r,\tilde{A})$, we have from equation (9),

$$\frac{\partial p}{\partial_{\tilde{A}} r}+\rho\frac{\partial\Phi}{\partial r}=0, \tag{12}$$

which may be integrated to obtain

$$p(r,\tilde{A})=p(r_0,\tilde{A})\exp\left\{-\int_{r_0}^{r}\frac{GMdr}{\mathscr{R}T(r,\tilde{A})\,r^2}\right\}. \tag{13}$$

In deriving equation (13) we made use of the ideal gas law $p=\mathscr{R}\rho T$ and transformed $T(r,\theta)$ into $T(r,\tilde{A})$. The integration is to be carried out with $\tilde{A}$ held constant, r_0 is a constant and $p(r_0,\tilde{A})$ is a free function of $\tilde{A}$ arising from the integration with respect to r. Substituting equation (12) into equations (7) and (8), we obtain

$$\mathscr{L}\tilde{A}+\frac{1}{2}\frac{d\tilde{B}^2}{d\tilde{A}}+4\pi r^2(1-\mu^2)\frac{\partial p(r,\tilde{A})}{\partial\tilde{A}}=0, \tag{14}$$

where

$$\mathscr{L} \equiv \frac{\partial^2}{\partial r^2} + \sin\theta \frac{\partial}{r\,\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{r\,\partial\theta}\right) \equiv \frac{\partial^2}{\partial r^2} + \frac{1-\mu^2}{r^2}\frac{\partial^2}{\partial\mu^2}, \tag{15}$$

$\mu \equiv \cos\theta$, and $\partial/\partial\tilde{A}$ is the partial differential operator with respect to $\tilde{A}$ when r and $\tilde{A}$ are taken to be independent variables.

In our approach to the magnetostatic problem, we must specify a priori the functional forms of $\tilde{B}(\tilde{A})$, $p(r_0, \tilde{A})$ and $T(r, \tilde{A})$. Then equation (13) fixes the functional form of $p(r, \tilde{A})$ and equation (14) poses a problem for $\tilde{A}$ as an unknown. In the final step, the quantities **B**, p, ρ and T are expressed in terms of the spatial coordinates through the explicit solution $\tilde{A}(r, \theta)$. In principle, the functional forms of $\tilde{B}(\tilde{A})$, $p(r_0\ \tilde{A})$ and $T(r, \tilde{A})$ should be determined in an unambiguous way from some initial dynamical state if we perform an integration of the equations of motion, mass conservation, energy conservation, and magnetic induction, which govern the dynamical evolution of the system. For the present, however, we shall confine ourselves to a simple approach in which we deal with the equation of the mechanical equilibrium by specifying the functional forms of $\tilde{B}(\tilde{A})$, $p(r, \tilde{A})$ and $T(r, \tilde{A})$ in a reasonable manner and investigate what magnetostatic states are generated by them.

3. A class of exact solutions

For a given set of field lines $\tilde{A}$ = const, the functions $\tilde{B}(\tilde{A})$ and $p(r, \tilde{A})$ describe the amount of the ϕ-component of the magnetic field and the pressure along the individual field lines. Let us consider a simple case in which $\tilde{B} \equiv 0$ and

$$p(r, \tilde{A}) \equiv Q(r)\tilde{A} + S(r) \tag{16}$$

where $Q(r)$ and $S(r)$ are free functions. This example leads to analytic solutions and is best suited for demonstrating the basic behaviour of magnetostatic solutions. Substituting for $p(r, \tilde{A})$ in equation (14), we obtain the linear equation,

$$\frac{\partial^2\tilde{A}}{\partial r^2} + \frac{1-\mu^2}{r^2}\frac{\partial^2\tilde{A}}{\partial\mu^2} + 4\pi r^2(1-\mu^2)\,Q(r) = 0. \tag{17}$$

The density and temperature are given from equations (12) and (16) as

$$\rho = -\frac{r^2}{GM}\left(\tilde{A}\frac{dQ}{dr} + \frac{dS}{dr}\right) \tag{18}$$

And

$$T = -\frac{GM}{\mathfrak{R}\, r^2}\,\frac{\tilde{A}\,Q + S}{\tilde{A}\dfrac{dQ}{dr} + \dfrac{dS}{dr}} \tag{19}$$

where $\widetilde{A}$ is to be a solution of equation (17).

To solve equation (17), we obtain a particular solution of equation (17) and add it to the general solution of the homogeneous version of equation (17),

$$\frac{\partial^2 \widetilde{A}}{\partial r^2} + \frac{1-\mu^2}{r^2}\,\frac{\partial^2 \widetilde{A}}{\partial \mu^2} = 0. \tag{20}$$

The particular solution A_1 of equation (17) can be obtained by introducing the transform $\widetilde{A}_1$ to u,

$$\widetilde{A}_1 = (1-\mu^2)\, r^2\, u(r). \tag{21}$$

Equation (17) then reduces to

$$r\frac{d^2u}{dr^2} + 4\frac{du}{dr} + 4\pi r\, Q(r) = 0, \tag{21}$$

which can be shown to have a particular solution of the form

$$u(r) = -4\pi \int_{r_0}^{r} \frac{dr'}{r'^4} \int_{r_0}^{r'} r''^4\, Q(r'')\, dr''. \tag{23}$$

Equation (20) is just the equation for a potential magnetic field. The complete set of solutions to equation (20) corresponds to the set of multipole potential expansion in an axisymmetric system (Marion 1965). For the purpose of this paper, we consider the case where the solution of equation (20) is made of only the dipole and the uniform field. This solution represents the situation of a stellar dipole field superposed on a uniform interstellar magnetic field in vacuum. The solution of equation (20) which we want is,

$$\widetilde{A}_0 = (1-\mu^2)\left(\frac{C_1}{r} + C_2\, r^2\right) \tag{24}$$

where C_1 and C_2 are constants. Direct substitution into equation (10) shows that the terms C_1/r and $C_2\, r^2$ represent a dipole and a uniform field, respectively. The solution to equation (17) that we seek is then

$$\begin{aligned} \widetilde{A} &= \widetilde{A}_0 + \widetilde{A}_1 \\ &= (1-\mu^2)\left(\frac{C_1}{r} + C_2\, r^2 - 4\pi\, r^2\, \Psi_1(r)\right) \end{aligned} \tag{25}$$

where we define

$$\Psi_1(r) \equiv \int_{r_0}^{r} \frac{dr'}{r'^4} \int_{r_0}^{r'} r''^4\, Q(r'')\, dr''. \tag{26}$$

By substituting $\tilde{A}$ into equation (10), we have,

$$B_r(r,\theta) = \frac{1}{r^2 \sin\theta} \frac{\partial \tilde{A}}{\partial \theta}$$

$$= 2\left(\frac{C_1}{r^3} + C_2 - 4\pi\, \Psi_1(r)\right)\cos\theta \tag{27}$$

$$B_\theta(r,\theta) = -\frac{1}{r\sin\theta}\frac{\partial \tilde{A}}{\partial r}$$

$$= 2\left(\frac{C_1}{2r^3} - C_2 + 4\pi\, \Psi_1(r) + 2\pi\, \Psi_2(r)\right)\sin\theta \tag{28}$$

where

$$\Psi_2(r) = \frac{1}{r^3}\int_{r_0}^{r} r'^4\, Q(r')\, dr'. \tag{29}$$

The current density is given from equation (4) as

$$j_\phi(r,\theta) = c\left(r\, Q(r) - \frac{2}{r}\Psi_2(r)\right)\sin\theta. \tag{30}$$

Other components of **B** and **j** all vanish.

Boundary conditions to be set are: (a) The distribution of the magnetic flux on the stellar surface is not altered from the original one since the field lines are anchored by the heavy stellar material at its surface,

$$B_r(r_0', \theta) = B_p \cos\theta. \tag{31}$$

where B_p is the field strength at the star's magnetic pole, and (b) the field at infinity is not affected by the mass-loading around the star,

$$\mathbf{B}(\infty,\theta) = (B_\infty \cos\theta, -B_\infty \sin\theta, 0). \tag{32}$$

These boundary conditions demand that

$\Psi_1(r)$ converges to a finite value as $r \to \infty$, (35)

$\Psi_2(r)$ converges to zero as $r \to \infty$ (36)

and

$$\begin{cases} C_1 = r_0^3\left(\dfrac{B_p - B_\infty}{2} - 4\pi\, \Psi_1(\infty)\right), & (35) \\[2ex] C_2 = \dfrac{B_\infty}{2} + 4\pi\, \Psi_1(\infty). & (36) \end{cases}$$

We finally have the magnetic field components fulfilling the boundary conditions as

$$B_r(r,\theta)=\left\{[B_p-B_\infty-8\pi\Psi_1(\infty)]\left(\frac{r_0}{r}\right)^3+B_\infty+8\pi[\Psi_1(\infty)-\Psi_1(r)]\right\}\cos\theta, \quad (37)$$

$$B_\theta(r,\theta)=\left\{\frac{[B_p-B_\infty-8\pi\Psi_1(\infty)]}{2}\left(\frac{r_0}{r}\right)^3 - B_\infty-8\pi[\Psi_1(\infty)-\Psi_1(r)]+4\pi\Psi_2(r)\right\}\sin\theta. \quad (38)$$

We discuss simple illustrative examples in the rest of this section in order to demonstrate the properties of our solution. We first note that $S(r)$ does not come into equation (17) for the magnetic vector potential, A The effect of the mass loading comes only through $Q(r)$. A simple interpretation of this may be that $S(r)$ represents the background distribution of the mass in the absence of the magnetic field. $S(r)$ and — dS/dr are required to be positive so that the background pressure and density are positive. The field-related portion of the pressure and the density, A $Q(r)$ and —(r^2/GM) A dQ/dr, can be either positive or negative, corresponding to the enhancement over, or depletion below the background. If we assume that the field-related component of the density is positive outside the boundaries between the stellar and the accreted field, dQ/dr should be negative outside these boundaries and vanish inside, if we want to consider the accreted mass piling up outside these boundaries, deforming the magnetic field due to the effect of the gravity on it.

A simple example of $Q(r)$ fulfilling these conditions may be given by connecting a constant to an inverse-a power function smoothly across the boundary region as

$$Q(r)=Q_0\left[\frac{\left\{1-\tanh\left(\frac{r-r_b}{d}\right)\right\}}{2}+\frac{\left\{1+\tanh\left(\frac{r-r_b}{d}\right)\right\}}{2}\left(\frac{r_0}{r}\right)^\alpha\right] \quad (39)$$

and we may assume $S(r)$ to be an inverse-a power function as

$$S(r)=S_0\left(\frac{r_0}{r}\right)^\beta \quad (40)$$

as an example, where r_b is the distance to the boundary from the centre of the star, d is the thickness of the transition between internal and external regions across the boundary and Q_0 and ; are constants. The sign of Q_0 is chosen to make the additional component of the density positive in the external region so that it represents the mass pile-up by accretion.

Two cases arise corresponding to the signs of B_∞ relative to B_p which is assumed to be positive and $|B_p| \gg |B_\infty|$, $|8\pi\ \psi_1(\infty)|$. For the convenience of computation, we assume a ratio of $|B_p|/|B_\infty|$ of the order of $10^3 \sim 10^4$ in the following calculation. Much larger ratio of $|B_p|/|B_\infty|$ of degenerate star case is equally permissible except that the calculation requires more time and accuracy.

Case A: $B_{\infty} > 0$

A neutral point in $r\theta$-plane (actually a ring in three dimensions) appears at $r = r_n$ and $\theta = \pi/2$. In particular, for the case without mass loading ($Q_0=0$, $\psi_1 = \psi_2 = 0$), r_n turns out to be

$$r_{n_0} = \left(\frac{B_p - B_\infty}{2B_\infty}\right)^{1/3} r_0. \tag{41}$$

For $Q_0 \neq 0$ (> 0 in this case to make the additional density positive), we solve an equation,

$$\left\{8\pi\left[\Psi_1(\infty) - \Psi_1(r)\right] - 4\pi\,\Psi_2(r) + B_\infty\right\}\left(\frac{r}{r_0}\right)^3 - \tfrac{1}{2}\left[B_p - B_\infty - 8\pi\,\Psi_1(\infty)\right] = 0 \tag{42}$$

to obtain r_n. This equation comes from the condition that the coefficient of sin θ in B_θ vanishes. An iterative method is adopted in finding r_b and r_n by assuming $r_b = (1 + \epsilon)\, r_n$, where ϵ *is* a small positive constant.

The field lines in the meridional plane for a typical set of parameter values are given by integrating equation (11) with B_r and B_θ given in equations (37) and (38), and are shown in Figs 1(b), (c). This is to be compared with the case of no mass-loading ($Q_0 = 0$) as given in Fig. 1(a). It is seen that r_n decreases as Q_0 increases, as expected from the gravitational effect of the additional mass introduced by nonzero Q_0. The density and the temperature distributions in rθ-plane are calculated from equations (18) and (19) by using the solution for $\tilde{A}$ and expressions (39) and (40) for Q and S. The distributions of $\tilde{A}$, p, and T *in* r, for given θ for the case of Fig. 1(c) are given in Fig. 2 as examples. It is seen that the present solution has a concentration near the equatorial plane of the mass component due to Q, which interacts with the magnetic field.

Case B: $B_\infty < 0$

A different type configuration appears in this case; singular points occur above the poles. Now the coefficient of cos θ in B_r has a zero point which in the mass-free case occurs at $r = r_{s0}$ and $\theta = 0$ and π, where

$$r_{s0} = \left(\frac{B_p - B_\infty}{-B_\infty}\right)^{1/3} r_0. \tag{43}$$

In this case, it is seen from equation (25) that $\tilde{A}$ changes sign from positive to negative as r increases, and we set $Q_0 < 0$ in. order to make the additional density in the region $r \gtrsim r_s$, positive. In order to obtain r_s in the mass-loaded case, $Q_0 \neq 0$ (< 0), we solve an. Equation

$$\left\{8\pi\left[\Psi_1(\infty) - \Psi_1(r)\right] + B_\infty\right\}\left(\frac{r}{r_0}\right)^3 - \left[B_p - B_\infty - 8\pi\,\Psi_1(\infty)\right] = 0, \tag{44}$$

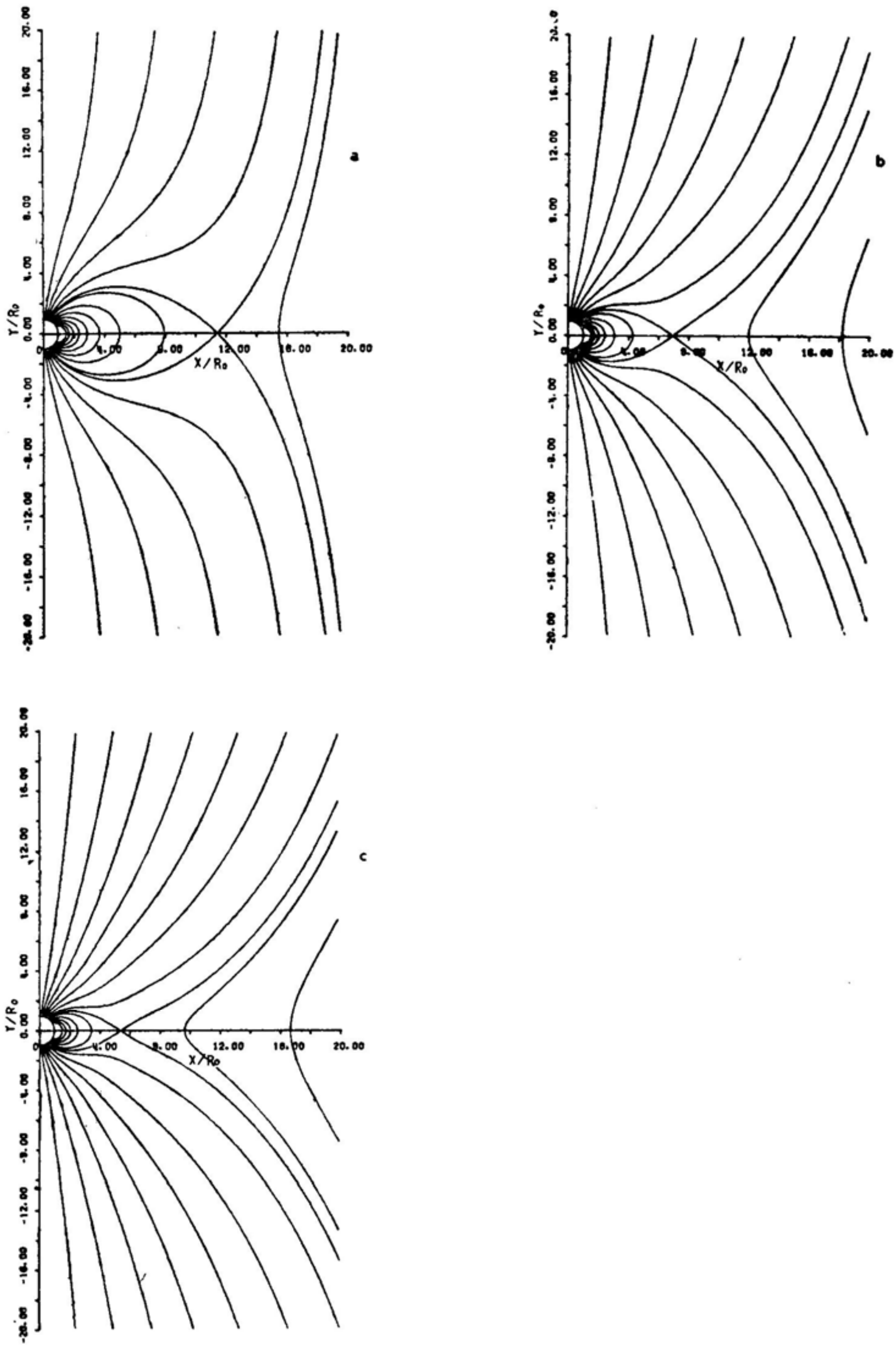

Figure 1. Magnetic field configuration in the meridional plane for Case **A** with $\boldsymbol{B_p} = 3 \times 10^3$ and B_∞, = 1. (a) $Q_0 = 0$, (b) $Q_0 = 1 \times 10^{-2}$, (c) $Q_0 = 4 \times 10^{-2}$. In this figure and in the following, *a* and *b* in expressions (39) and (40) are both taken to be 4 and $\epsilon \equiv (r_b / r_n - 1)$ is 0·2, to show an example.

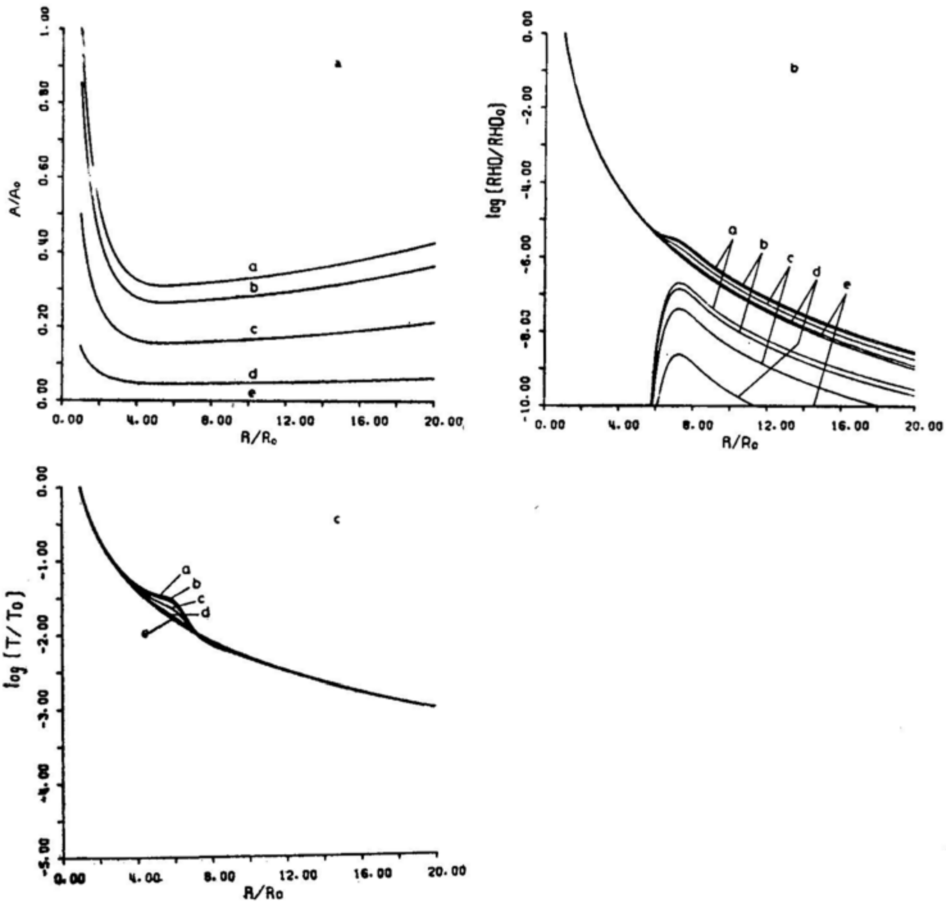

Figure 2 Distributions in r of (a) $\tilde{A}$, (b) the total density, ρ, and the density related to the magnetic field, — $(r^2/GM)\ A\ dQ/dr$, and (c) the temperature, T, for the case of Fig. I. (c), all normalized to the values at the equatorial surface. Curves (a) through (e) in the figures are for θ= 90°, 67·5°, 45°, 22·5° and 0°, respectively.

which comes from the condition that the coefficient of cos θ in B_r vanishes. r_s is r_s is obtained by the same iterative scheme as in Case A. Fig. 3(a) ($Q_0 = 0$) and 3(b), (c) ($Q_0 \neq 0$) show that the field configuration in Case B is very different from that in Case A. The distance to the singular point is seen to decrease with increasing Q_0 also in this case due to the shrink of the domain of the stellar field by mass-loading. Fig. 4 shows the distributions of $\tilde{A}$ ρ and T for the case of Fig. 3(c) as examples. The mass slides down along the field lines to the closest point from the star and distributes itself hydrostatically, and thus its distribution becomes disk-like in the equatorial plane in this case also.

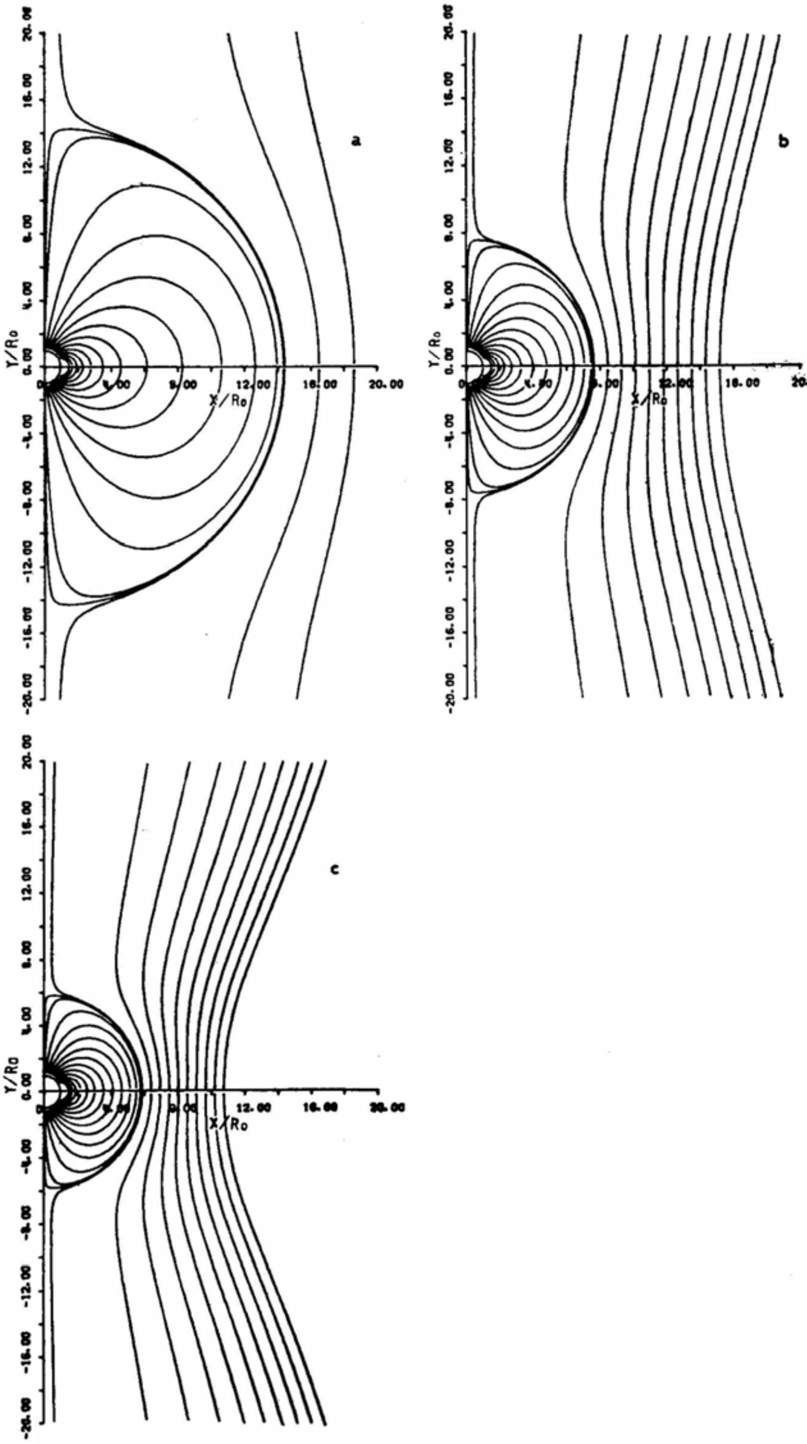

Figure 3. The same as Fig. 1 for Case B, with $B_p = 3 \times 10^3$ and $B_\infty = 1$; (a) $Q_0 = 0$ (b) $Q_0 = 1 \times 10^{-2}$ and (c) $Q_0 = -4 \times 10^{-2}$.

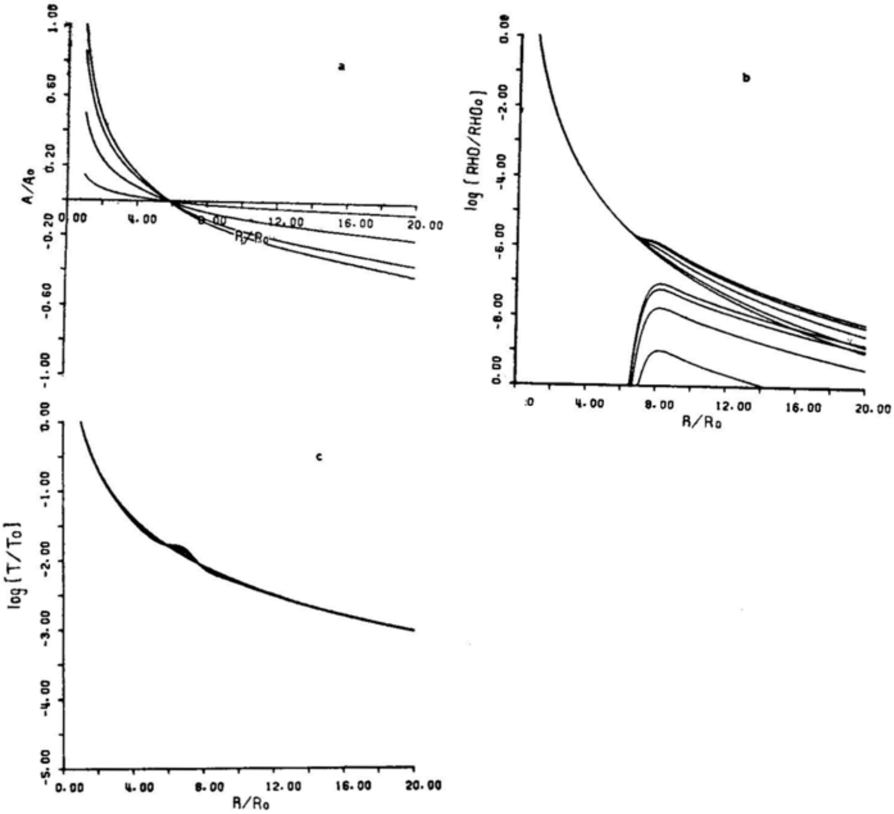

Figure 4. The same as fig. 2 for Case B.

4. Discussion

We have demonstrated simple solutions for the axisymmetric magnetogravitational equilibrium governed by equation (14) in order to obtain the global self-consistent magnetic field configuration of a star loaded by the mass which is, for example, accreted from dense (magnetic) cloud surrounding the star*. It is noteworthy that the inclusion of the axisymmetric magnetic field alone results in the formation of a disk-like density structure. Physically, this is due to the fact that the mass slides down along the field lines to the closest possible points which are—in both Cases A and B—in the equatorial plane. The density is stratified hydrostatically along each tube of force with a scale height corresponding to the temperature given by equation (19), and it has a concentration in the equatorial plane as seen from Figs 2 and 4.

Many models of 'spherical accretion' have been discussed (see review by Lamb 1979) in the context of mass accretion by neutron stars in X-ray binaries, but the global solution of the self-consistent field was not dealt with in full. The configuration

*The situation may also be applicable to a degenerate magnetic star in the companion's magnetic atmosphere if the effect of both rotation of the degenerate star and revolution of the accreted gas around the degenerate star be small. It is, however, likely that these effects are important in this case and we intend to take into account these effects in later papers in certain approximate way.

derived in Case B is related to the shielded-dipole type model argued by Midgley and Davis (1962). They considered no external field, and sought the shielding current layer which cuts off the effect of the internal field to zero beyond certain radius, representing the effect of the gas pressure of the external plasma. Models of the closed magnetosphere extended to include the effect of the accretion disk have been discussed after the introduction of the notion of the accretion disk by Pringle and Rees (1972) and Shakura and Sunyaev (1973). Inoue (1976) and Ghosh and Lamb (1978) have assumed the formation of the indentation of the field configuration at the equator due to the gravity effect on the disk material. It may be noted in this context, however, that the disk-like structure, at least in our solution, squeezes the stellar field as a whole rather than pushing into it like a sharp knife-edge as assumed by them.

In relation to the problem of the mass take-in into the stellar field region, it is often suggested that the mass flows into the polar region. It may be pointed out, however, that the singular openings above the poles are covered up by the field lines in the presence of the external field, and this may affect the mass flow into the polar region assumed in some models. Magnetic field lines should be reconnected in order to allow the mass infall into the closed field region. An alternative mechanism, the Rayleigh-Taylor instability, is proposed to play a role in the process of the mass take-in (Arons and Lea 1976, Elsner and Lamb 1977, Baan 1977). In their models, however, the unmagnetized plasma-blob falling across the star's field will be resisted by the inverse melon-seed effect and may lose the kinetic energy of the free fall.

An interesting situation occurs in our Case A in this context, namely, the field line reconnection can take place at the magnetic neutral ring in the equatorial plane in a very natural way. In this context, note the difference between Figs 1(a) and 1(c) which is due to the difference in the loaded mass. The, open part of the stellar magnetic flux is larger in Fig. 1(c). No dynamical or time-dependent behaviour can be argued from the sequence of the equilibrium models, but it is clear that there should be some reconnection of the field if the state is to change from that of Fig. 1(a) to that of Fig. 1(c) as the result of the increase of the loaded mass. The mass loaded on the external tube of force around the equatorial plane can be transferred to the stellar field by reconnection and can fall to the stellar surface at the edge of the polar cap area along the field lines passing through the neutral point. The reconnected part of the stellar field is now added to the open part of the stellar field. Interchange instability *in* the neutral sheet region was proposed to play an important role in the case of solar flares (Uchida and Sakurai 1977, 1981). The same mechanism may be relevant to the self-quenched mass leakage in a rapid burster type object (Lewin and Joss 1977). We consider this process in some more detail in a following paper. A more detailed discussion in this direction requires the inclusion of the effect of rotation of both the central star and of the accreted mass. The rotation of the disk introduces the pulling of the field lines into ϕ-direction and complicates the problem, but some part of the effects may be represented by $\tilde{B}$ which we ignored in the present paper for simplicity.

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