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Plasma Heating in a Sheared Magnetic Field

V. Krishan *Indian Institute of Astrophysics, Bangalore 560034*

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Abstract. The mechanism of spatial resonance of Alfven waves for heating a collisionless plasma is studied in the presence of a twisted magnetic field. In addition to modifying the equilibrium condition for a cylindrical plasma, the azimuthal component of the magnetic field gives extra contribution to the energy deposition rate of the Alfven waves. This new term clearly brings out the effects associated with the finite lifetime of the Alfven waves. The theoretical system considered here conforms to the solar coronal regions.

Key words: Alfven waves—plasma heating—solar corona

1. Introduction

The magnetohydrodynamic waves are believed to be the potential candidates for heating a plasma. Since these waves are known to exist in great abundance in the solar atmosphere, they present themselves as the natural choice for heating the coronal plasma and the chromosphere-coronal transition regions. The magnetohydrodynamic waves, for example acoustic waves, Alfven waves and their other modified equivalents could be generated through the Lighthill mechanism *i.e.* direct generation by the turbulent convective motions, for example in the photospheric layers. The low frequency acoustic waves may lose major portion of their energy in the photosphere through radiative damping. On the other hand, the steepening of the wave front of the acoustic waves, as they propagate, results in shock formation, in which case, they deposit their energy over larger spatial scale. Alfven waves, as suggested by Alfven (1947), may lose their energy by ohmic losses due to the finite electrical conductivity. Piddington (1956) proposed viscous damping to be more important than Joule heating in the corona. Uchida and Kaburaki (1974) invoked the decay of slow MHD waves—produced through nonlinear coupling of Alfven waves—in order to explain the excessive heating of the active regions. The coupling of the waves has been studied using quantum field theoretical methods (Kaburaki and Uchida 1971). The coupling of Alfven waves and the fast mode waves in a stratified medium and its significance in heating the solar corona has been discussed by Melrose (1977). Plasma heating by spatial resonance between shear Alfven waves has been proposed by Chen and Hasegawa (1974). Ionson (1978) and Wentzel (1979) applied this resonance absorption mechanism for heating the coronal loops. The terms—energy deposition rate of the waves and the plasma heating rate have been used synonymously in the literature. Actually, in the fluid treatment usually performed, there is no provision for the transfer of this deposited energy to the particles. A wave-particle interaction has to be invoked to calculate the heating rate of the plasma particles. In this paper, we calculate the energy deposition rate of the Alfven waves in an inhomogeneous plasma in a twisted magnetic field. The inhomogeneous coronal density model is derived in a cylindrical geometry from the equilibrium condition in the presence of the twisted magnetic field. We find that the results in a cylindrical geometry are qualitatively different from the ones in a planar geometry considered previously. In addition to an extra contribution to the dissipation rate, we find that an arbitrarily small value of the parameter β = ratio of the gas pressure to the magnetic pressure, is not allowed in the present analysis.

2. Dissipation rate

We consider a plasma system with mass density $p(r)$ in a magnetic field $B = [0, B_\theta(r), B_z(r)]$ where $B_\theta = Bar$ and $B_z = B$, a being a constant. This represents a uniformly twisted magnetic field with the pitch of the field given by $2\pi/\alpha$. The radio evidence for the existence of the twisted magnetic fields in the corona has been presented by McLean and Sheridan (1972). The equilibrium gas pressure $P(r)$ and the magnetic field $B(r)$ satisfy the condition

$$\frac{\partial}{\partial r}\left[P(r) + \frac{B^2}{2\mu}\right] + \frac{B_\theta^2}{\mu r} = 0. \tag{1}$$

Making use of the adiabatic equation of state, one finds the spatial variation of the mass density $p(r)$ in the limit $ar \ll 1$ to be

$$\begin{aligned}\rho(r) &= \rho_0 \exp\left(-B_\theta^2/\mu P \gamma\right)\\ &= \rho_0 \exp\left(-\alpha^2 r^2/4\beta_0\gamma\right)\end{aligned} \tag{2}$$

Where, $\beta_0 = \left.\frac{\mu P}{B^2}\right|_{r=0}$

and γ is the adiabatic index. Using the standard magnetohydrodynamic equations, relaxing the conditions of incompressibility, we find the equations for the radial component of the plasma displacement vector ξ_r and the total perturbed pressure p to be:

$$\frac{d^2}{dr^2}\xi_r + \frac{d}{dr}\xi_r\left[\frac{d}{dr}\left(\frac{\alpha_1 \epsilon B^2}{\alpha_1 k_\perp^2 B^2 - \epsilon}\right) + \frac{(k_\parallel B)\, B_\theta^2\, \epsilon\, (\alpha_1 - 1)}{r\,\gamma\,\beta\,(k_\perp B)}\right]$$

$$\times \left(\frac{\alpha_1 k_\perp^2 B^2 - \epsilon}{(\alpha_1 \epsilon B^2)}\right) + \xi_r \left[\frac{d}{dr}\left(\frac{(k_\| B) B_\theta^2 \epsilon (\alpha_1 - 1)}{r \gamma \beta (k_\perp B)}\right) - \epsilon\right]\left(\frac{\alpha_1 k_\perp^2 B^2 - \epsilon}{\alpha_1 \epsilon B^2}\right) = 0 \quad (3)$$

And

$$p = -\xi_r \frac{dP}{dr} - \gamma P (\nabla \cdot \boldsymbol{\xi}) + \mathbf{b} \cdot \mathbf{B}/\mu$$

$$= \xi_r \left[\frac{B_\theta^2}{\mu r} + \frac{\gamma P k_\|^2 B^2 B_\theta^2}{r \epsilon_1 B^2}\right] + \frac{d}{dr} \xi_r [- B^2 (1 + \gamma\beta)/\mu - \gamma^2 P k_\|^2 B^2/\epsilon_1]$$

$$+ \xi_\perp [- i k_\perp B^2 (1 + \gamma\beta)/\mu - i \gamma^2 P k_\|^2 k_\perp B^2 \beta/\epsilon_1], \quad (4)$$

$$\xi_\perp = \frac{i \alpha_1 k_\perp B^2}{(\alpha_1 k_\perp B^2 - \epsilon)} \frac{d}{dr} \xi_r + \frac{i k_\| B_\theta^2 (\alpha_1 - 1)}{r \gamma \beta (\alpha_1 k_\perp^2 B^2 - \epsilon)} \xi_r \quad (5)$$

where,

$$\alpha_1 = 1 + \frac{\gamma \beta \mu \rho \omega^2}{\mu \rho \omega^2 - k_\|^2 B^2 \gamma \beta},$$

$$\epsilon = \mu \rho \omega^2 - (k_\| B)^2,$$

$$\epsilon_1 = \mu \rho \omega^2 - (k_\| B)^2 \gamma \beta,$$

$$k_\| B = k_z B_z + k_\theta B_\theta, \qquad k_\theta = \frac{l}{r},$$

$$k_\perp B = k_\theta B_z - k_z B_\theta,$$

$$\mathbf{b} = (\mathbf{B} \cdot \nabla) \boldsymbol{\xi} - \mathbf{B} (\nabla \cdot \boldsymbol{\xi}) - (\boldsymbol{\xi} \cdot \nabla) \mathbf{B} \quad (6)$$

and all perturbed quantities vary according to exp $[i(k_z z + l\theta - \omega t)]$, l is an integer. Equation (3) describes the coupling of the shear Alfven waves, the magnetosonic waves and the acoustic waves in an inhomogeneous magnetoplasma system. The singularity in the solution of equation (3), at the point where the phase velocity of a given Alfven wave propagating in the inhomogeneous plasma becomes equal to that of the local Alfven wave, has been discussed by Chen and Hasegawa (1974), Ionson (1978) and Wentzel (1979). We wish to study the effect of this particular magnetic field geometry on the singular solution and hence on the dissipation rate. An expression for the dissipation rate can be found from the relation

$$\frac{dW}{dt} = \tfrac{1}{2} \operatorname{Re} \int \mathbf{J} \cdot \mathbf{E}^* \, d^3 r$$

where

$$\mathbf{J} = \frac{\nabla \times \mathbf{b}}{\mu} \text{ and } \mathbf{E} = i\,\omega\,\boldsymbol{\xi} \times \mathbf{B}. \tag{7}$$

Substituting for all the perturbed quantities in terms of ξ_r, one finds

$$\frac{dW}{dt} = \pi\,L_z\,r_0\,\mathrm{Re}\left[\frac{i\omega}{\mu}\left\{\frac{B^2\,\epsilon}{a_1\,k_\perp^2\,B^2 - \epsilon}\,\xi_r^*\,\xi_r' + \left(\frac{(k_\parallel\,B)\,(k_\perp\,B)\,B_\theta^2\,(a_1 - 1)}{r\,\gamma\,\beta\,(a_1\,k_\perp^2\,B^2 - \epsilon)} - BB'\right)\xi_r^*\,\xi_r\right\}\right]_{r_0-\eta}^{r_0+\eta} \tag{8}$$

where L_z is the extent of the system in z direction and $\xi_r' = d\xi_r/dr$. One can solve equation (3) around the regular singular point $r = r_0$ by Frobenius' method to get

$$\xi_r = c_1\,a_0 + \sum_{n=1}^{\infty}(c_1\,a_n + c_2\,b_n)\,x^n + c_2\ln|x|\sum_{n=0}^{\infty}a_n\,x^n \tag{9}$$

Where $x = r - r_0 + i\,\delta$, such that $|r - r_0 \mid \eta| \gg |\delta|$,

$$\epsilon(r) = \left.\frac{d\epsilon_r}{dr}\right|_{r=r_0}(r - r_0) + i\,\epsilon_i$$

And

$$\delta = \frac{\epsilon_i\,(r_0)}{\left.\frac{d\,\epsilon_r}{dr}\right|_{r=r_0}}, \tag{10}$$

ϵ_r and ϵ_i being the real and imaginary parts of ϵ. Substituting equation (9) in equation (8) and evaluating at the upper and the lower limits, one finds

$$\frac{dW}{dt} = -\pi^2\,L_z\,r_0\,\frac{\omega}{\mu}\,|c_2\,a_0|^2\left\{\frac{B^2\left.\frac{d\,\epsilon_r}{dr}\right|_{r=r_0}}{(a_1\,k_\perp^2\,B^2)_{r=r_0}} + \frac{2\,B_\theta^2\,(r_0)}{r_0}\left[\frac{k_\parallel\,B}{k_\perp\,B}\,\frac{1}{(1-\gamma\,\beta)} - 1\right]\ln|\eta|\right\} \tag{11}$$

In equation (11), the first term is the familiar contribution obtained by earlier workers in a planar geometry. In arriving at the first term, though the condition that $|\eta| \gg |\delta|$ has been made use of, the quantity η does not figure explicitly. The second term, which is a direct consequence of the different equilibrium condition (1) for a cylindrical plasma in a helical magnetic field, clearly shows the finite life time effects. The presence of this term forbids the wave to approach the resonance point infinitely close and η is a measure of the closest distance of approach. Thus, whereas the first term allows the energy deposition in the limit $\eta \to 0^+$, the second term makes its contribution little away from the resonance point. This could result in giving a finite width to the absorption region.

3. Application to the solar corona

Resonant Alfven wave heating mechanism has been applied to the heating of coronal loops by Ionson (1978). In view of the recent observations, where a highly inhomogeneous nature of the chromospheric-coronal transition region and that of the dense corona has been emphasized, this mechanism could be operative over large portions of the solar corona (Feldman, Doschek and Mariska 1979). These authors, from the *skylab* data, propose that the transition zone and the corona may be confined to small structures of high plasma density. The regions in between these structures would contain a negligibly small amount of plasma relative to the plasma within these structures. In such a case, one could envisage a model where the source of Alfven waves—which could be a magnetic filament or a current sheet—lies outside these high density coronal structures. We assume a cylindrical geometry where the half cylinder is filled with the high density plasma in a helical magnetic field and the source of Alfven waves is situated outside in a relatively low density region. This facilitates to relate the constant $(c_2 a_0)$ of equation (11) to the energy density of the Alfven waves produced external to the region to be heated. To be able to do this, one has to determine the solution of equation (3) at the boundary *i.e.* at $r = 0$. Then, by satisfying the boundary conditions, one can relate the solution of equation (3) at the point $r \sim r_0$ to that at $r = 0$, which in turn is related to the external source of Alfven waves. We observe that in the present system $r = 0$ is a regular singular point and by Frobenius method, we can find a series solution only for $r > 0$ and $r < 0$. Therefore, this forbids a sharp boundary at $r = 0$ and one has to take the finite thickness of the boundary into account. In this piece of work, we make a simplifying assumption and solve equation (3) for $l = 0$ *i.e.* $k_\theta = 0$. In this case, we can find a general solution of equation (3) in the interval $0 \leqslant r < r_0$ in the form of a series

$$\xi_r = \sum_{n=0}^{\infty} d_n (r - r_1)^n \tag{12}$$

where r_1 is an ordinary point in the interval. The coefficients d_n can be determined by substituting solution (12) into equation (3) and comparing the coefficient of equal powers of r on both sides. Another simplifying assumption we make is to consider the low density medium as vacuum. We can decompose the magnetic field $\mathbf{b}$ of the wave into two components $\mathbf{b} = \mathbf{b_1} + \mathbf{b_2}$, where $\mathbf{b_1}$ corresponds to the external driving field. One can find an expression for these fields, satisfying the equations

$$\nabla \times \nabla\, b_2 = \nabla \cdot b_2 = 0,$$

$$\mathbf{b}_2 = \psi_2 \, \nabla \, e^{i\,(k_z z - i\,k_r r - \omega t)},$$

$$\mathbf{b}_1 = \psi_1 \, \nabla \, e^{i\,(k_z z + i\,k_r r - \omega t)}. \tag{13}$$

In arriving at equation (13), solution on a planar geometry has been assumed since this may be a good approximation for mode number $l = 0$. The first boundary condition is the continuity of the radial component of the magnetic field at $r = 0$, *i.e.*

$$\psi_2 = [i\,k_{||}(0)\,B(0)\,\xi_r(0) - b_{1r}(0)]/k_r. \tag{14}$$

The second boundary condition is the pressure balance condition

$$\mu\, p\,(0) = \mathbf{B}_v\,(\mathbf{b}_1 = \mathbf{b}_2)\,|_{r=0} \tag{15}$$

where $\mathbf{B}_v$ is the vacuum magnetic field. Substituting for $p(0)$ and $\mathbf{b}_1$, $\mathbf{b}_2$, one finds

$$2\,b_{1r}(0) = i\,k_{||}(0)\,B(0)\left[\xi_r(0) - \frac{1}{k_z}\,\xi_r'(0)\left(1 + \gamma\,\beta_0 + \frac{\gamma^2\,\beta_0^2}{\epsilon_1(0)}\,k_z^2\,B_z^2\right)\right]. \tag{16}$$

Matching the solutions of equation (3), we find $\xi_r(0) = -(c_2\,a_0)$ and $\xi'_r(0) = 2c_2\,a_0/r_0$ for $r_1 = r_0/2$ and $d_0 = 0$. Here we have checked that the higher order terms (for $n \geqslant 2$) in equation (12) are small. In a more general case, the treatment has to be done numerically. From equation (16), we determine $(c_2\,a_0)$ and substitute in equation (11) to get

$$\frac{dW}{dt} = \pi^2\,L_z\,\frac{|\,b_{1r}(0)\,|^2\,\omega}{k_z^2\,\mu}\,\frac{1}{\left[1 + \frac{2}{k_z\,r_0}\left(1 + \gamma\,\beta_0 + \frac{\gamma^2\,\beta_0^2}{\epsilon_1(0)}\,k_z^2\,\beta_z^2\right)\right]^2}$$
$$\times\left[\frac{2}{a_1\,\beta_0\,\gamma\left(1 - \frac{a^2\,r_0^2}{4\beta_0\gamma}\right)} + \frac{a^2\,r_0^2}{2}\left(\frac{2}{a\,r_0\,(1-\gamma\,\beta_0)} + 1\right)\ln|\,\eta\,|\right] \tag{17}$$

where $(a^2\,r_0^2)/(4\beta_0\,\gamma) < 1.$

One must remember that in the present treatment, an arbitrary small value of the parameter $(\beta_0\,\gamma)$ is not allowed because of the assumption $a^2\,r^2 < \beta_0\,\gamma$ made while deriving the plasma density profile from the equilibrium condition (1). As a matter of fact, in the solar coronal conditions this number $\gamma\beta$ is not vanishingly small. The second term, as has been mentioned earlier, brings out the finite lifetime effects. A comparison of the resonant absorption dissipation rate and that of the Joule heating rate shows, for the same energy density of the Alfven waves, the latter to be much smaller than the former. The a dependent term in equation (17) could be positive or negative depending on whether the magnetic field pitch is in the positive or in the negative direction. This will give the dissipation rate to be more for one direction of twist than for the other direction. The present treatment suffers from various assumptions and approximations. The first thing, which we plan to include in our future work is the effect of finite k_θ in the solution. In fact, the assumption of $k_\theta = 0$ may not be bad in the case of solar corona, since one desires the waves to propagate throughout the plasma, unlike in the laboratory plasma. The second improvement could be to replace the vacuum by a low density plasma, in which case the contribution from the displacement current needs to be taken into account. This investigation could be done in a nonuniformly twisted magnetic field. This would be in line with Kuperus (1965) who has stressed the need to include more realistic magnetic field configurations while studying the heating problem.

4. Conclusion

The energy dissipation rate of the Alfven waves in an inhomogeneous plasma by spatial resonance absorption mechanism is calculated in a uniformly twisted magnetic field. The physical significance of the additional contribution to the energy dissipation rate is to stress on a finite distance of the closest approach to the resonance point by the Alfven wave. In the present model, arbitrarily small values of $\gamma\beta_0$ are not allowed. The smallness of $\gamma\beta_0$ is controlled by the ratio of the azimuthal component of the magnetic field to its axial component.

References

Alfven, H. 1947, *Mon. Not. R. astr. Soc.*, **107**,211.
Chen, L., Hasegawa, A. 1974, *Physics Fluids*, **17**, 1399.
Feldman, U., Doschek, G. A. Mariska, J. T. 1979, *Astrophys. J.*, **229**, 369.
Ionson, J. A. 1978, *Astrophys. J.*, **226,** 650.
Kaburaki, O., Uchida, Y. 1971, *Publ. astr. Soc. Japan*, **23**, 405.
Kuperus, M. 1965, *The Transfer of Mechanical Energy in the Sun and the Heating of the Corona*, D. Reidel, Dordrecht.
McLean, D. J., Sheridan, K. V. 1972, *Solar Phys.*, **26**,176.
Melrose, D. B. 1977, *Austr. J. Phys.*, **30,** 495.
Piddington, J. H. 1956, *Mon. Not. R. astr. Soc.*, **116,** 314.
Uchida, Y., Kaburaki, 0.1974, *Solar Phys.*, **35**, 451.
Wentzel, D. G. 1979, *Astrophys. J.*, **233**,756.