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Changes in Sizes and Shapes of Spherical Galaxies in Head-on Collisions

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Abstract. Head-on collisions of two identical spherical galaxies are studied for two initial velocities (1) nearly equal to and (2) greater than the capture velocity. Orbits of about 500 representative stars are computed taking into account the effects of dynamical friction in the motion of the galaxies. From the computer studies the changes in the structure of the galaxies are deduced. The galaxies contract at closest approach and expand as they recede from each other. When the initial velocity is nearly equal to the capture velocity, the mean radius expands to almost double its size and the galaxies have a prolate structure until the closest approach with the longer axis in the direction of motion. The prolate structure is destroyed as the galaxies recede. For larger collision velocity ($V \sim 1{\cdot}5\ V_{\rm cap}$), the mean radius expands by 50 per cent and the galaxies are prolate until the closest approach and distinctly oblate after the collision. The fractional increase in the binding energy is 0·46 in the first case and 0·30 in the second case.

Key words: stellar dynamics—galaxy collisions—tidal interaction

1. Introduction

A study of the dynamics of head-on collisions of galaxies is of interest for several reasons. A head-on collision provides useful upper limits for the effects of tidal forces. The symmetry of the galaxies with respect to the direction of motion, reduces the computations. Recent cosmological N-body simulations carried out by Aarseth and Fall (1980) show that the merging of galaxies occurs mainly from marginally bound galaxies in nearly head-on collisions.

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In an earlier paper (Ahmed and Alladin 1981, hereafter referred to as Paper I) we discussed the energy transfer from the translational motion of the galaxies to the internal motions of the stars in head-on collisions of two identical spherical galaxies with the initial velocities equal to and not much greater than the capture velocity (defined as in Alladin, Potdar and Sastry 1975), assuming that the stars remain stationary during the encounter (impulsive approximation). It was noted that most of the energy transfer took place close to the position of closest approach, the effect in the second half of the orbit being double that of the first. It has been shown by other workers that for smaller velocities, dynamical friction leads to a rapid merger of the two galaxies (Toomre 1974; Van Albada and Van Gorkom 1977; White 1978). Tremaine (1981) reviews the work on galaxy mergers.

In the present paper we shall obtain deeper insight into the changes in the structure of colliding galaxies from a study of the orbits of stars. As in Paper I, we shall restrict ourselves to velocities of collisions that are not too small to result in a rapid merger of the two galaxies and yet not too large so as to produce negligible tidal effects. In particular it will be our aim to investigate how a change in the collision velocity of the galaxies would affect the sizes and shapes of initially spherical galaxies.

Because of the symmetry of the galaxies with respect to the direction of motion in a head-on collision we should expect spherical galaxies to become either prolate or oblate but not triaxial. It can be seen qualitatively from a consideration of the shapes of the zero velocity surfaces in the restricted three body problem that in the limiting case of negligible collision velocity, the spherical galaxies would first become prolate with the major axis along the direction of motion. Finally the galaxies would merge. Simulations of galaxy mergers by White (1978) and Villumsen (1980) indeed show that merger remnants resulting from head-on collisions are prolate in shape. On the other hand for large collision velocities, one can get an idea of the shapes of the galaxies after the collision from the impulsive approximation, which indicates that the velocity increment of a star over the entire collision is in a direction perpendicular to the direction of relative motion of the galaxies, and consequently after the collision, the galaxies become oblate with the minor axis along the direction of motion of the galaxies. The final shapes would be quite complicated in detail, and the description of the final structures in terms of prolateness and oblateness is therefore a rather crude one.

We obtain the equations of motion for a star in a galaxy belonging to a double galaxy system, taking into account the effects of dynamical friction in reducing the translational velocity of the galaxies and use these equations to compute the orbits of about 500 stars chosen as representative of the test galaxy. From the stellar orbits, the changes in mass distribution and shape of the galaxies during the collision are obtained. The subsequent dynamical evolution of the colliding galaxies is deduced from the virial theorem.

2. Equations of motion of a star in a system of colliding galaxies

We represent the density distribution of each of the colliding galaxies by that of a polytropic sphere of index $n=4$, and radius R, and we derive the forces between them by assuming that it is unchanged during the encounter. The potential $\mathcal{V}$ and the interaction potential energy W for these cofigurations have been given by Limber (1961) and Potdar and Ballabh (1974) respectively. We consider the tidal effects of the perturbing galaxy of mass M_1 on the test galaxy of mass M_2. Let the origin of the coordinate system be chosen at the centre of M_2. Let r (x, y, z) denote the position of the centre of M_1 and r' $(x' \; y' \; z')$ the position of a representative star in M_2. Let r″=r —r′. The motion of a star in M_2 is given by

$$\ddot{\mathbf{r}}' = -\nabla' \mathscr{V}_2 + \nabla'' \mathscr{V}_1 - \frac{1}{M_2} \nabla W \tag{1}$$

where the first term on the right hand side represents the gravitational attraction of the parent galaxy M_2 on the star while the second and third terms together give the tidal force due to the perturbing galaxy M_1 . $\mathscr{V}$ and W may be expressed in terms of the functions Φ and X respectively as in Paper I. The equations of motion of a star in Cartesian co-ordinates for a head-on collision in the z-direction hence become

$$\ddot{x}' = \frac{GM_2}{R_2^3}\left(\frac{x'}{s'}\frac{d\Phi_2}{ds'}\right) + \frac{GM_1}{\epsilon^3 R_2^3}\left(\frac{x'}{s''}\frac{d\Phi_1}{ds''}\right), \tag{2}$$

$$\ddot{y}' = \frac{GM_2}{R_2^3}\left(\frac{y'}{s'}\frac{d\Phi_2}{ds'}\right) + \frac{GM_1}{\epsilon^3 R_2^3}\left(\frac{y'}{s''}\frac{d\Phi_1}{ds''}\right), \tag{3}$$

$$\ddot{z}' = \frac{GM_2}{R_2^3}\left(\frac{z'}{s'}\frac{d\Phi_2}{ds'}\right) - \frac{GM_1}{\epsilon^3 R_2^3}\left(\frac{z - z'}{s''}\frac{d\Phi_1}{ds''}\right) + \frac{GM_1}{R_2^3}\left(\frac{z}{s}\frac{dX}{ds}\right) \tag{4}$$

where $s'=r'/R_2$, $s'' = r''/R_1$, $s = r/R_2$, and $\epsilon = R_1/R_2$.

The equation of the relative motion of the two galaxies with separation r is given by

$$V = \frac{dr}{dt} = \left[\frac{2}{\mu}\left\{E_i - W(r) - \Delta U_1(r) - \Delta U_2(r)\right\}\right]^{1/2} \tag{5}$$

where μ is the reduced mass and E_1 is the initial translational energy $\Delta U_1(r)$ and $\Delta U_2(r)$ are changes in the internal energies of the two galaxies. Equation (5) takes into account the deceleration of the translational motion of the galaxies due to dynamical friction. Equation (5) is solved as discussed in Paper I. The above treatment takes into account the changes in the force fields due to superposition effects arising from the overlap of galaxies but not due to changes in the structures of galaxies arising from tidal distortion. A more accurate treatment of the problem should also take the latter into account.

3. Initial conditions

We study head-on collisions between two identical spherical galaxies. Let M_1 approach M_2 from $z = -3R$. As shown in Paper I the integrated tidal effects of the galaxies on each other are very small beyond this distance. As in Paper I we choose the units of distance, mass and gravitational constant so that $R = 1$, $M = 1$, $G = 0{\cdot}04302$ respectively. For $M = 10^{11}$ $M_\odot$, R=10 kpc, the unit of time is 1.0×10^7 years and the unit of velocity is 1000 km s^{-1}.

Numerical calculations are performed for two values of relative velocities of the galaxies, namely 0·7 and 1·0. V_t — 0·7 is nearly the capture velocity of the galaxies, while $V_t = 1{\cdot}0$ is a case wherein the galaxies recede to infinite distance with significant change in their structure after the encounter. We shall refer to the collisions with these two initial velocities as 'slow' and 'fast' collisions, Models S and F respectively. It is assumed that initially all stars move in circular orbits and their directions of motion have circular symmetry. Representative stars are chosen at median radius $a = 0{\cdot}135$ and at distances $a = 0{\cdot}1$, 0·2, 0·3, 0·4, 0·5 and 0·6. We assign to these shell numbers 1, 2, 3, 4, 5 and 6. In our model (polytrope $n = 4$) the mass beyond 0·6 is less than 0.1 per cent. Hence $R_0 = 0{\cdot}6$ may be regarded as a reasonable outer radius of the galaxy. At each distance a we chose 72 stars. Because of the symmetry properties of a head-on collision about the direction of motion, orbits of only 25 stars at each distance a have to be computed and those of the remaining stars can be obtained from the symmetry considerations.

The positions and the velocities of the stars in the galaxy are obtained by integrating the differential equations (2), (3) and (4) numerically. We however stop the orbit computations when $z = +2$. At this stage the fractional change in the potential energy of the galaxy is nearly 0.3 which implies that the mass distribution of the galaxies can no longer be well represented by that of a polytrope of index n=4.

4. Changes in the forms of galaxies

The positions of the stars initially moving in the x–y plane at various separations of the two galaxies are shown in Figs 1 and 2, for the two collision velocities 0·7 and 1·0 respectively. It can be seen that the shape of the galaxy does not change much until the galaxies are at their closest approach, but a noticeable contraction in the galaxy is clearly discernible soon after the closest approach. The contraction is less in the case of the faster collision. The results of N-body simulations of van Albada and van Gorkom (1977), White (1978), Miller and Smith (1980) also show that galaxies momentarily contract just after the closest approach and then expand. As galaxies recede the contraction is followed by expansion, and some of the kinetic energy is converted into potential energy.

Another note-worthy point is that the various shells of stars expand at different times; the inner shells expand first and the outer shells later giving rise to fluctuations in the original density distribution of the galaxy. In Model S when $z = +1$, a zone of avoidance separating the stars originally at less than or equal to the median radius from the remaining stars is very conspicuous. When $z = +2$, the stars originally at less than or equal to 0·2 are well separated from those at 0·3 and beyond, while stars at 0·3, 0·4, 0·5 and 0·6 crowd together at the same distance from the centre. In Model F the expansion is smaller. The oscillations in the positions of the stars in the case of the slow collision are illustrated in Fig. 3. Lynds and Toomre (1976) have studied the effects of a head-on collision on a disk galaxy and have concluded that a ring galaxy such as II Hz 4 could be formed in this manner. The remarkable

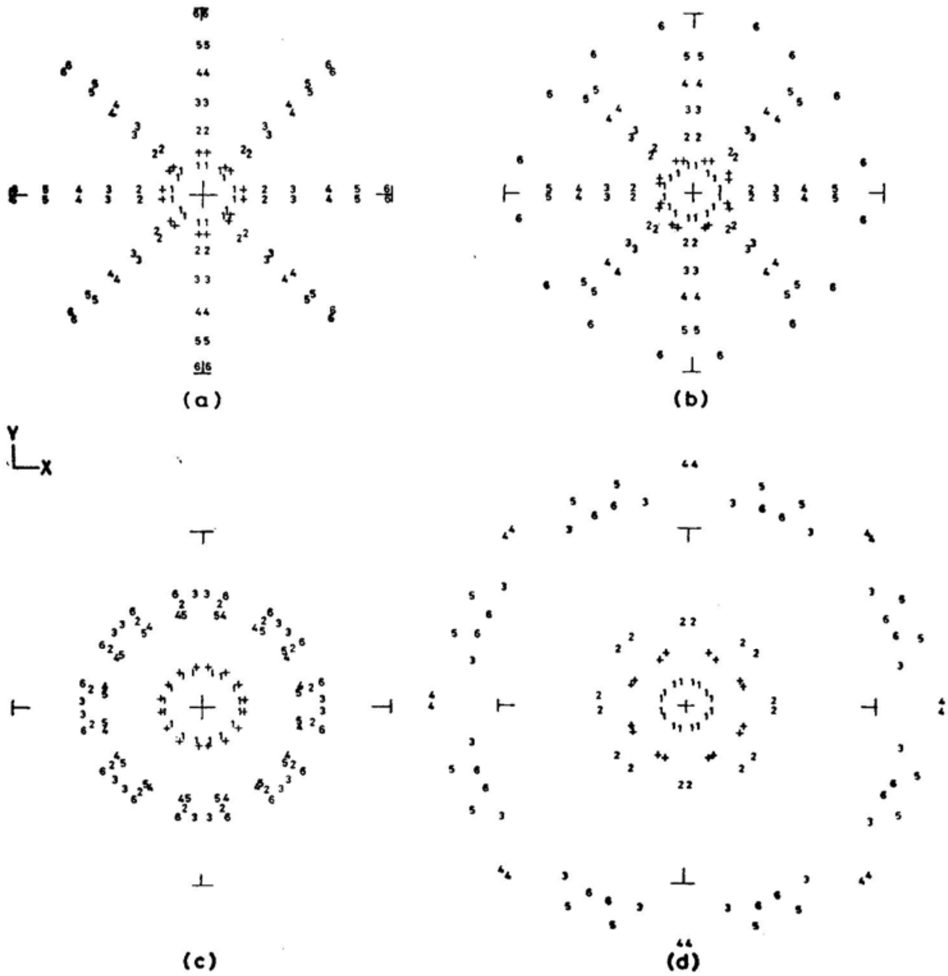

Figure 1. Positions of stars originally moving in the x–y plane in Model S (a) $z = -3$ ($t = 0$) (b) $z = 0$ (t=3·9) (c) $z = +1$ (t = 5·5) (d) $z = +2$ (t = 8·5). The numbers 1, 2 ... 6 represent stars initially at 0.1, 0·2 ... 0·6 shells respectively, and + denotes stars at median radius 0.135.

Cartwheel galaxy (A0035) was probably formed as a result of a nearly head-on collision between a disk galaxy and a spherical galaxy (Toomre 1978). In the sphere-sphere collision considered here, stars crowd together in shells (instead of rings) which are separated by regions wherein the number density of stars is small.

Figs 4 and 5 show the positions of stars originally moving in the x–z plane at various times. The positions of the stars in the y–z plane would be similar (because of the symmetry about the z-axis). In Fig. 6 we show the projections of all the stars in the x–z plane after the collision for V_t = 0·7. It may be noted that until the instant of closest approach the galaxies get elongated in the z-direction, the maximum effect being for the outermost stars. The galaxies thus become prolate. As the galaxies recede after the closest approach, they expand appreciably

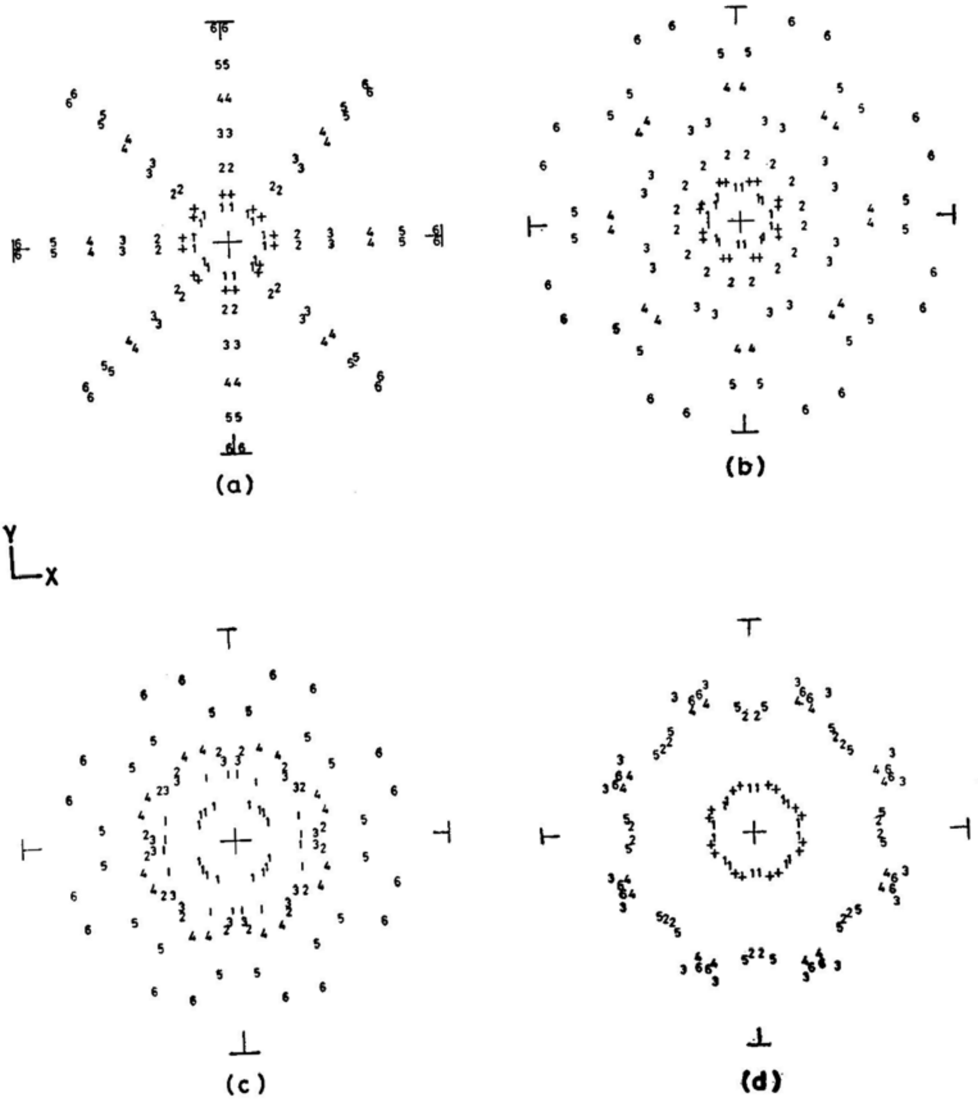

Figure 2. Positions of stars originally moving in the *x-y* plane in Model F (a) $z = -3$ ($t = 0$) (b) $z = 0$ ($t = 2{\cdot}8$) (c) $z = +1$ ($t = 3{\cdot}8$) (d) $z = +2$($t = 4.9$).

in the directions perpendicular to the z-axis, so that the prolate structure is destroyed and the galaxies tend to become oblate. Spherical galaxies undergoing a head-on collision would be prolate until the closest approach and, if they do not merge, they would generally be oblate after the collision. It appears from observations that elliptical galaxies have a preference for oblate structure (Marchant and Olson 1979).

An idea of the differential expansion of the galaxy can also be obtained by calculate-ing the average expansion of the orbits of stars at various distances from the centre. We have divided the stars into seven groups depending upon their initial distance from the centre. In Tables 1 and 2 the mean coordinates of each of these groups are given at various separations of the two galaxies for two velocities 0·7 and 1·0

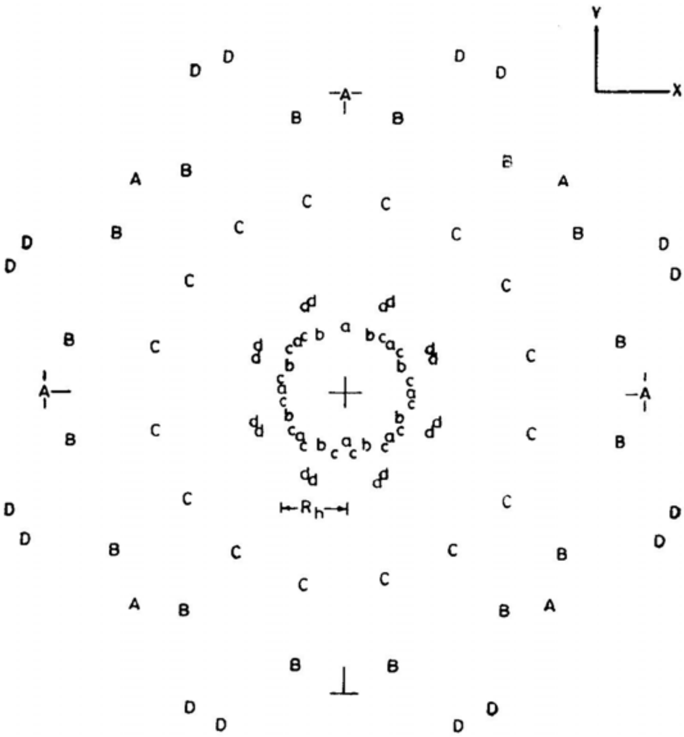

Figure 3. Positions of stars initially moving in the *x–y* plane at median radius and full radius at various separations in Model S. Capital letters, A, B, C, D denote the positions of stars initially at R_0 = 0·6 at separations z = – 3 (*t* = 0), z = 0 (*t* = 3·9), z = +1 (*t* = 5.5), *z* = + 2 (*t* = 8·5). Small letters a, b, c, d denote the corresponding positions of stars at median radius R_h.

respectively. Since $\langle \mid x' \mid \rangle = \langle \mid y' \mid \rangle$ by symmetry, only $\langle \mid x' \mid \rangle$ is listed. The mean position $\langle r' \rangle$ is a good measure of the over-all expansion of each group and the ratio $\langle |z'| \rangle \mid \langle |x'| \rangle$ is a useful measure of the departure from the spherical symmetry. Our results indicate that galaxies become prolate (or rather pear-shaped) until the closest approach. The ratios $\langle \mid z' \mid \rangle \mid \langle \mid x' \mid \rangle$ for the shell averaged over the whole system are 1·35 and 1·15 for slow and fast collisions respectively. At the end of the collision the corresponding ratios are 0.96 and 0.79. It can be seen from the table that the slow collision gives rise to rather large fluctuations in the oblateness. The results also show that the post-collision oblateness of the galaxies increases with the speed of the collision.

The prolate structure of the galaxies acquired in the first half of the collision tends to get destroyed in the second half of the collision because the *z*-component of the tidal acceleration on a star gets reversed. It can be deduced from the analysis given by Toomre (1977) that the velocity increment of a star in the *z* direction, V_{II}, would, be zero for the entire orbit.

As we go from the case of a head-on collision to the case of an off-centre collision, the prolate or the oblate structures would pass over to triaxial structures. Our slow collision case may be compared with the corresponding slow off-centre

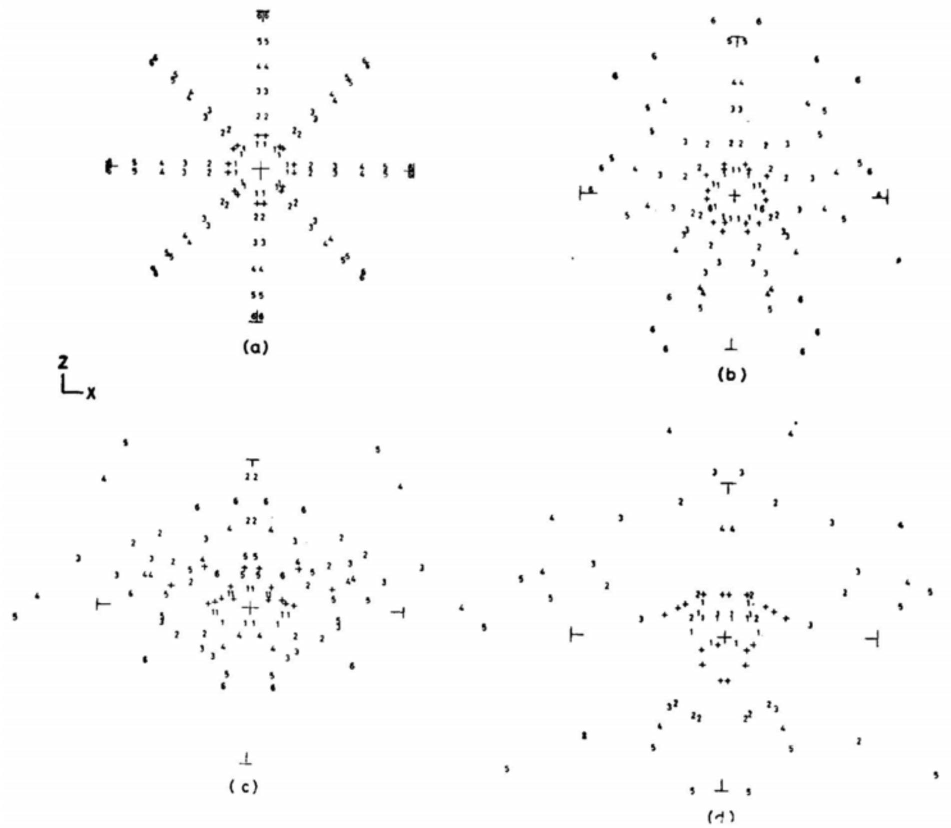

Figure 4. Positions of stars originally moving in the *x-z* plane, in Model S (a) $z = -3$ ($t = 0$), (b) $z = 0$ ($t = 3{\cdot}9$), (c) z = + 1 (t = 5.5), (d) $z = +2$ ($t = 8{\cdot}5$).

Table 1. The mean co-ordinates of stars at various separations in Model S.

z	Initial shell No.	$\langle\lvert x'\rvert\rangle$	$\langle\lvert z'\rvert\rangle$	$\langle r\rangle$	$\langle\lvert z'\rvert\rangle/\langle\lvert x'\rvert\rangle$
−3	1	0·042	0·042	0·074	1·00
	2	0·085	0·085	0·147	1·00
	3	0·128	0·128	0·220	1·00
	4	0·170	0·170	0·295	1·00
	5	0·213	0·213	0·368	1·00
	6	0·255	0·255	0·442	1·00
0	1	0·032	0·047	0·063	1·46
	2	0·064	0·092	0·126	1·44
	3	0·099	0·131	0·225	1·32
	4	0·137	0·173	0·254	1·26
	5	0·202	0·283	0·400	1·40
	6	0·231	0·254	0·438	1·10
+2	1	0·050	0·043	0·070	0·86
	2	0·141	0·204	0·281	1·45
	3	0·314	0·249	0·507	0·79
	4	0·438	0·325	0·697	0·74
	5	0·308	0·346	0·554	1·12
	6	0·378	0·299	0·610	0·79

collision case (Model D) of White (1978). In the slow head-on collision studied by us, the galaxies are neither prolate nor distinctly oblate after the collision. White's Model D also does not show much departure from spherical shape, the final

Table 2. The mean co-ordinates of stars at various separations in Model F.

z	Initial Shell No.	$\langle\lvert x'\rvert\rangle$	$\langle\lvert z'\rvert\rangle$	$\langle r\rangle$	$\langle\lvert z'\rvert\rangle/\langle\lvert x'\rvert\rangle$
	1	0·042	0·042	0·074	1·00
	2	0·085	0·085	0·147	1·00
	3	0·128	0·128	0·220	1·00
−3	4	0·170	0·170	0·295	1·00
	5	0·213	0·213	0·368	1·00
	6	0·255	0·255	0·442	1·00
	1	0·043	0·049	0·063	1·14
	2	0·083	0·128	0·167	1·54
	3	0·119	0·130	0·209	1·09
0	4	0·170	0·214	0·317	1·26
	5	0·181	0·197	0·319	1·09
	6	0·232	0·280	0·428	1·21
	1	0·050	0·049	0·077	0·98
	2	0·160	0·111	0·248	0·69
	3	0·208	0·128	0·319	0·61
+2	4	0·194	0·141	0·304	0·73
	5	0·163	0·136	0·264	0·83
	6	0·190	0·182	0·324	0·96

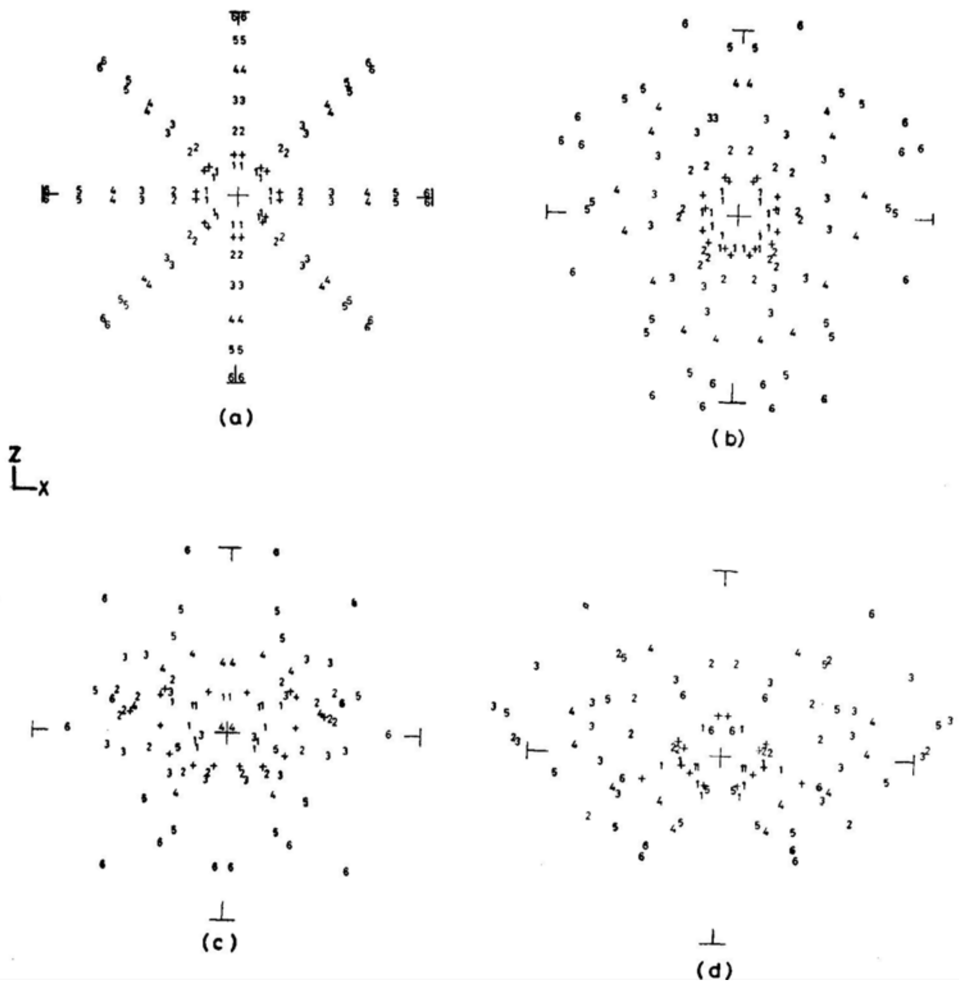

Figure 5. Positions of stars originally moving with the *x-z* plane, in Model F (a) $z = -3 (t = 0)$ (b) $z = 0$ ($t = 2{\cdot}8$), (c) z = +1 ($t = 3{\cdot}8$), (d) $z = +2$ ($t = 4.9$).

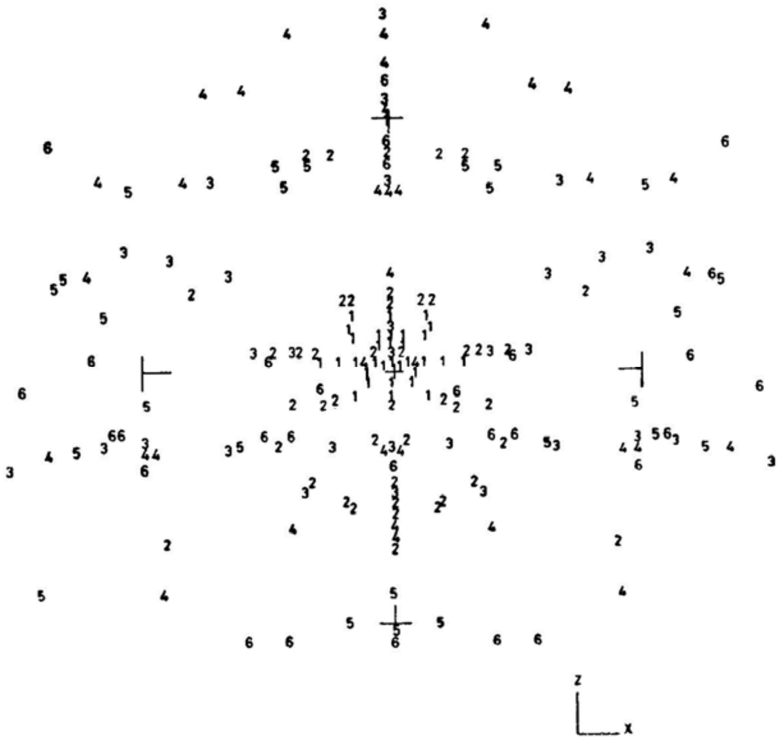

Figure 6. Projection of all the stars in the x–z plane at $z = +2$ ($t = 8{\cdot}5$) in Model S.

structures being ellipsoidal with the axes in the ratio 1:1.05:0.89 the smallest axis being perpendicular to the orbital plane and the biggest being perpendicular to the direction of the initial separation of the two galaxies in the orbital plane.

5. Changes in mass distribution during the encounter

During the encounter the distribution of stars is drastically changed as shown in Tables 3 and 4. Each star is assigned a weight equal to the mass of the shell divided by the number of stars in the shell. The mass distribution of the galaxy after the encounter is obtained from the distribution of stars shown in Tables 3 and 4. The mass within a radius 0.05 for our model is 0·0576. The results of orbit computations show that this sphere contains the same mass throughout the encounter. The mass distribution is essentially unchanged until the closest approach of the two galaxies, but there is a considerable change afterwards particularly in the outer parts. In the slow collision, the median radius changes during the collision from 0·135 to 0·160, the radius containing 75 per cent of the mass changes from 0·20 to 0·32 and the radius containing 95 per cent of the mass expands from 0·35 to 0·85. The subsequent expansion of the galaxies is discussed in Section 8.

Table 3. Distribution of star in a galaxy at z = + in model S.

Shell radii Inner — Outer	Initial Shell No. 1	2	3	4	5	6
< 0·05	—	—	—	—	—	—
0·05 — 0·15	60	—	—	—	—	—
0·15 — 0·25	12	12	—	—	—	—
0·25 — 0·35	—	44	8	—	—	—
0·35 — 0·45	—	—	4	8	—	—
0·45 — 0·55	—	12	—	—	8	—
0·55 — 0·65	—	—	20	—	4	2
0·65 — 0·75	—	4	16	—	8	30
0·75 — 0·85	—	—	—	40	32	12
> 0·85	—	—	20	12	4	12
Escaping Stars	—	—	4	12	16	16

Table 4. Distribution of stars in a galaxy at z = + 2 in Model F.

Shell radii Inner — Outer	Initial Shell No. 1	2	3	4	5	6
< 0·05	—	—	—	—	—	—
0·05 — 0·15	68	8	—	—	8	4
0·15 — 0·25	4	5	—	12	4	4
0·25 — 0·35	—	24	4	4	16	16
0·35 — 0·45	—	24	40	36	28	28
0·45 — 0·55	—	8	8	12	4	—
0·55 — 0·65	—	4	12	—	—	—
0·65 — 0·75	—	—	—	—	—	8
0·75 — 0·85	—	—	—	—	—	—
> 0·85	—	—	—	—	—	—
Escaping Stars	—	—	8	8	12	12

6. Mass escape and formation of halo

The mass escape from the galaxies during a collision is an important aspect of the study. We find that after the encounter about 1·5 per cent of the mass escapes in Model S and less than 1 per cent in Model F. Dekel, Lecar and Shaham (1980) performed N-body simulations of colliding galaxies which also includes the case of head-on collisions. Their model is less centrally concentrated than ours. Their results for identical spherical galaxies with collision velocity slightly greater than the capture velocity and with circular stellar orbits (B7 model) give 3 per cent for the fractional mass escape. This compares well with our result. Our results are also in good agreement with those of van. Albada and van Gorkom (1977), who obtained about 1 per cent of mass escape from the galaxies (represented by a polytrope n = 3) during a head-on collision with $V_i \simeq 1$ (in our units). Using isothermal model for galaxies, White (1978) also finds that only a small fraction of the total mass escapes in a head-on collision. In our model we have considered only circular motions of the stars. Models of galaxies consisting of elongated orbits of stars give greater mass loss (Gallagher and Ostriker 1972; Richstone 1975; Dekel, Lecar and Shaham 1980).

Stars having negative energies close to zero are flung into large orbits and contribute to the formation of the halo. The energy of the most loosely bound stars before the collision is – 0·036 (in our units). In Model S stars with energies in the range

0 to – 0.036 have 5·4 per cent of the total mass, while in Model F the percentage is 1·4. These stars will form the halo of the galaxy after the encounter.

7. Change in binding energy

The kinetic energy of a galaxy is given by

$$T = T_0 + \frac{M}{2} \sum_{K=1}^{S} \sum_{J=1}^{N} W_K V'^2_{JK} \tag{6}$$

where V'_{JK} is the velocity of the Jth star in the Kth shell, W_K is its weight, S and N are the total number of shells and the total number of stars in the shell respectively. T_0 is the kinetic energy of stars lying within the radius 0·05 and is given by

$$T_0 = \frac{G}{2} \int_0^{0\cdot05} \frac{M(r')}{r'} \left(\frac{dM}{dr'}\right) dr'. \tag{7}$$

This is less than 0.001, and is not changed during the encounter.

The self potential energy of a spherical stellar system is obtained from the mass distribution and is given by the relation (Chandrasekhar 1943)

$$\Omega = - G \int_0^{R} \frac{M(r')}{r'} \left(\frac{dM}{dr'}\right) dr'. \tag{8}$$

The binding energy of the galaxy is obtained from

$$U = T + \Omega. \tag{9}$$

The values of the kinetic, potential and binding energies of the galaxy initially and after the encounter ($z = +2$) are given in Table 5. The corresponding values of binding energy U_I, obtained in Paper I are also given. Dekel, Lecar and, Shaham (1980) estimate fractional change in internal energy $\Delta U/|U|$ (For B7 model) as 0·56, which agrees well with our value of 0.46 for Model S. The

Table 5. Energies of galaxies before and after the encounter

Energies	Initial $z = -3$	$z = +2$ Model S	$z = +2$ Model F
T	0·063	0·057	0·046
Ω	−0·126	−0·091	−0·090
U	−0·063	−0·034	−0·044
$(\Delta U/\|U\|)$	0	0·460	0·301
U_I	−0·063	−0·009	−0·030
$(\Delta U/\|U\|)_I$	0	0·851	0·522

comparison of these results with, those obtained in Paper I with the assumption that the stars remain stationary but the velocity of the galaxies is reduced due to dynamical friction indicates that the latter treatment overestimates the fractional increase in the internal energy by a factor of 1·8 in Model S and 1·7 in Model F. If the effect of dynamical friction in decelerating the galaxies is neglected in the impulsive approximation and the galaxies are assumed to move with constant relative velocity equal to that at the closest approach, the prediction of the impulsive approximation is quite accurate (Toomre 1977). Miller and Smith (1980) who carried out N-body simulations with 50,000 stars in each galaxy indicate that the impulsive approximation cannot be expected to be valid for velocities less than 1000 km s^{-1} for typical galaxies.

8. Subsequent evolution

It is seen from the preceeding sections that when the separation of the galaxies becomes + 2, the structure has changed considerably. Therefore, we shall not use the potentials of a polytrope $n = 4$ to compute the orbits of stars beyond this separation. A more accurate treatment of the problem would require the use of potentials changing with time. However, we shall confine ourselves to making a crude estimate of the evolution of the galaxies after the encounter, by using the virial theorem.

The initial self potential energy of the galaxy may be written (Chandrasekhar 1943) as

$$\Omega = -\frac{GM^2}{2\bar{R}}. \tag{10}$$

Since the internal potential energy of a sphere of polytrope $n = 4$ is given by

$$\Omega = -\frac{3GM^2}{R} \tag{11}$$

we get on comparing equations (10) and (11)

$$\bar{R} = R/6. \tag{12}$$

If the galaxies do not merge, in the new equilibrium established after the collision the potential energy and the binding energy are related by the virial theorem as

$$2U_f = \Omega_f, \tag{13}$$

where U_f is the binding energy of the galaxy at separation $z = +2$ which is conserved after the collision and Ω_f is the potential energy when the galaxy has attained the new equilibrium. From equations (10) and (13) we obtain

$$\bar{R}_f = \frac{GM^2}{4U_f}. \tag{14}$$

Initially the galaxies (n= 4) have an average radius $\overline{R}_t$ = 0.167. Using the values of U_f obtained from the orbit computations we find that after the collision the galaxies have radii $\overline{R}_f$= 0.316 and 0.250 for slow and fast collisions respectively. This indicates that the mean radius expands to almost double its initial value in the case of slow collision and increases by about 50 per cent in the case of fast collision. Thus the final dimensions of the galaxies depend much on the velocity of the encounter. Dekel Lecar and Shaham (1980) have obtained from N-body simulations for their B-7 model the fractional increase in the median radius $\Delta R_h/R_h$ = 1.42, which indicates that the median radius expands to more than double its initial value. This may be compared with our result for the slow collision.

We stopped the numerical integrations when the galaxies were separated by a distance equal to about 15 times the median radius. As shown in Paper I, the tidal action of the perturbing galaxy practically ceases by this time. Further evolution of the shapes of the galaxies would, therefore, be determined by the self-gravity of the galaxies. Chandrasekhar and Elbert (1972) studied dynamical evolution of stellar systems with the aid of the tensor virial theorem. Their results show that a homogeneous oblate spheroidal system with negative total energy becomes more oblate and a prolate system with negative energy becomes more prolate.

9. Conclusions

It has been the main purpose of this work to study the dependence of the sizes and shapes of post-collision galaxies on the collision velocity in a head-on collision.

If $V_t < V_{cap}$, where V_t and V_{cap} are the initial and capture velocities respectively, the galaxies perform damped oscillations about the centre of mass of the system and finally become a single system. The results of N-body simulations show that in a head-on collision the final merged product has a prolate structure with the longer axis along the direction of the relative motion of the galaxies, while off-centre collisions lead to oblate structures flattened along the initial orbital plane (Tremaine 1981).

In the case of the slow collision with V_t slightly greater than V_{cap} the mean radius expands to about twice its size. The galaxies are prolate structures before the collision with the longer axis along the direction of motion. They lose their prolateness during the collision but do not become distinctly oblate.

In the case of the fast collision, $V_t \simeq 1{\cdot}5V_{cap}$, the mean radius expands by about 50 per cent of its initial value. The galaxies are prolate before the collision and distinctly oblate after the collision. Thus the tendency of the post-collision galaxies to be oblate rather than prolate increases with the velocity of the collision. For faster collision velocities, the final mean radius of the galaxies may be deduced from the impulsive approximation as in Alladin, Sastry and Ballabh (1974).

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References

Aarseth, S. J., Fall, S. M. 1980, *Astrophys. J.,* **236**, 43.

Ahmed, F., Alladin, S. M. 1981, *Bull. astr. Soc India,* **9**, 40 (Paper I).

Alladin, S. M., Potdar, A., Sastry, K. S. 1975, in *IAU Symp. 69: Dynamics of Stellar Systems,* Ed. A. Hayli, D. Reidel, Dordrecht, p. 167.

Alladin, S. M., Sastry, K. S., Ballabh, G. M. 1974, in *Galaxies and Relativistic Astrophysics,* Eds B. Barbanis and J. D. Hadjidemetriou, Springer-Verlag, Berlin, p. 129.

Chandrasekhar, S. 1943, *Principles of Stellar Dynamics,* Dover, New York, p. 200.

Chandrasekhar, S., Elbert, D. D. 1972, *Mon. Not. R, astr. Soc,* **155**, 435.

Dekel, A., Lecar, M., Shaham, J. 1980, *Astrophys. J.,* **241**, 946.

Gallagher, J. S., Ostriker, J. P. 1972, *Astr. J.,* **77**,288.

Limber, D. N. 1961, *Astrophys. J.,* **134**, 537.

Lynds, R., Toomre, A. 1976, *Astrophys. J.,* **209**, 382.

Marchant, A. B., Olson, D. W. 1979, *Astrophys. J.,* **230**, L157.

Miller, R. H, Smith, B. F. 1980, *Astrophys. J.,* **235**, 421.

Potdar, A., Ballabh, G. M. 1974, *Astrophys. Sp. Sci,* **26**, 353.

Richstone, D. O. 1975, *Astrophys. J.,* **200**, 535.

Toomre, A. 1974, in *IAU Symp. 58: The Formation and Dynamics of Galaxies,* Ed. J. R. Shakeshaft, D. Reidel, Dordrecht, p. 347.

Toomre, A. 1977, in *The Evolution of Galaxies and Stellar Populations,* Eds B. M. Tinsley and R. B. Larson, Yale University Observatory, New Haven, p. 401.

Toomre, A. 1978, in *IAU Symp. 79: The Large Scale Structure of the Universe,* Eds M. S. Longair and J. Einasto, D. Reidel, Dordrecht, p. 109.

Tremaine, S. D. 1981, in *The Structure and Evolution of Normal Galaxies,* Eds S. M. Fall and D. Lynder-Bell, Cambridge University Press, p. 67.

van Albada, T. S., van Gorkom, J. H. 1977, *Astr. Astrophys.,* **54**,121.

Villumsen, J. V. 1980, *Preprint.*

White, S. D. M. 1978, *Mon. Not. R. astr. Soc,* **184**, 185.