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The Structure of Integrated Pulse Profiles

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Abstract. We offer two possible explanations to account for the characteristics of integrated pulse profiles, in particular their degree of complexity, their variation from pulsar to pulsar, their stability, and the tendency of complex profiles to be associated with older pulsars.

It is proposed that the pulse structure could be a reflection of surface irregularities at the polar caps, and it is shown how the surface relief can affect the number of positrons released into the magnetosphere which are subsequently responsible for the observed radio radiation. The electrons produced in the vacuum break-down in the gap carry enough energy to allow creating such a surface relief in ~ 10^6 years, and one way in which this could be achieved is discussed.

Alternatively, the presence of multipole components in the magnetic fields of older pulsars could lead to significant variations in the curvature of the field lines across the gap, and hence to structure in the integrated pulse profiles. An assessment of the two hypotheses from observed pulse profiles seems to favour the polar cap relief picture.

Key words: pulsars—integrated pulse profiles—polar cap model

1. Introduction

One of the many remarkable features of pulsar radio radiation is the integrated pulse shape. Barring mode changes which we shall not discuss here it has been well established that the integrated pulse profiles are reproducible at any time; in other words they are stable for periods of time long compared with those over which pulsars have been observed so far. Further it has been noted (Taylor and Huguenin 1971) that in general pulsars with periods less than 0·75 seconds have simple pulse profiles while those with longer periods tend to have complex pulse profiles. This correlation clearly suggests that the pulse profile becomes more complex as the pulsar ages, but no satisfactory explanation has hitherto been advanced either for the existence of such structure or its evolution with pulsar age. This is in contrast to the microstructure of pulses, drifting sub-pulses, and other short period phenomena for which a number of possible explanations have been advanced.

The observed stability of the integrated pulse structure requires that the information determining its details be stored somewhere on the pulsar. In this paper we investigate two possible ways in which this information could be stored: (a) as surface irregularities at the magnetic polar cap of the pulsar, and (b) as magnetic field variations over the polar cap. We also discuss how this information can become more complex with increasing pulsar age to account for the observed correlation referred to above. We will restrict our discussion to the framework of the RS model (Ruderman and Sutherland 1975) for pulsars and its subsequent modifications (*e.g.* Cheng and Ruderman 1980).

In the model we put forward in this paper, surface irregularities are created by the effect of sparks on local regions in the polar cap. Such irregularities build up over a long period of time and in turn affect the strength of sparks over different areas and the intensity of the associated radio radiation. The minimum scale length of irregularities created by sparks will clearly depend on the area of individual sparks. As a consequence, the maximum number of features so created within the polar cap will depend on the ratio of the polar cap area to that of a spark. The typical area of an individual spark thus plays an important role in the model, and we therefore begin in the next section by estimating it from observations of the pulse microstructure. This is followed by a discussion of the effect of height variations on spark development and the number of positrons released into the magnetosphere by sparks.

In Section 4 we argue that under the combined influence of the temporary heating under a spark and the tension in the surface layer dueto the charge on it, the surface is likely to be deformed by a small amount. It is shown that the deformation required per spark is very small and that enough energy is available to achieve such deformations.

In Section 5 we consider an alternative explanation for the pulse structure based on the presence of multipole components in the magnetic field which are expected to make their appearance with the decay of the pulsar dipole magnetic field (Flowers and Ruderman 1977). In the last section we discuss the relative merits of the two mechanisms and conclude that the presence of several components in the integrated pulse profile of some pulsars favours the polar cap relief model.

2. Microstructure and spark areas

In the RS model the current from the polar cap is interrupted due to the non-availability of positive charges (Fe^{56} ions) which are not pulled off the surface by the electric field generated by the rotation of the neutron star. This causes a vacuum gap to form above the polar cap, about 10^4 cm in both height G and radius L, in which electron-positron pairs are created by any gamma ray photons which happen to be there. These pairs in turn produce photons by curvature radiation which produce more pairs.

This process goes on until the vacuum gap is broken by an avalanche of particles which is called a ‘spark’. The positrons travel away from the star to produce the radiation that we see while the electrons go to the polar cap to complete the pulsar homopolar circuit. The gap is now reformed and breaks down again by the same process. Such sparks occur about 10^5 times per second providing positron bursts which give rise to the pulsar microstructure, and whose average characteristics are reflected in the integrated pulse profile. We shall now attempt to derive the mean area of a spark from the observations of pulsar microstructure.

Let the area of each spark be a cm^2, and let A cm^2 be the area of the polar cap. Then there are A/a independent sparking regions on the polar cap. If sparks occur at 10 μs intervals, then each area is revisited by a spark after every $A/a \times 10$ μs on the average. Thus an observer would record radiation from sparks at a given location only every $A/a \times 10 \mu$s $(=\tau_1)$. The radiation would last for a time τ_2, which, we shall discuss later. If sparks occur randomly on the polar cap, the rate of observed sparking would remain the same whether the observer is stationary at a particular longitude on the polar cap or moving across it.

If $\tau_2 < \tau_1$, radiation due to individual sparks can be resolved in each individual pulse of the pulsar, provided the observations are made with fine enough time resolution. If $\tau_2 > \tau_1$, we would only observe a continuous distribution of radiation. If $\tau_2 \approx \tau_1$, we would just resolve individual sparks, but we would interpret $\tau_2 (=\tau_1)$ as a single time scale of intensity variation. All three effects are observed in PSR 0950+08 (Hankins 1971) in which micropulses are spaced from a few hundred μs to 1 ms apart. Their duration ranges from a few hundred μs to as low as 10μs.

We associate the quasi-periodicities of $\lesssim 1$ ms in PSR 1133+16 (Ferguson *et al* 1976) and in PSR 0950+08 (Hankins 1971) and PSR 2016+28 (Cordes 1976) with the time separation τ_1 of observed sparks. For these pulsars the number of independent sparks area A/a can therefore be estimated to be of the order of $(\tau_1/10\ \mu\text{s}) \lesssim (1\ \text{ms}/10\ \mu\text{s}) = 100$. We thus conclude from the microstructure observations of pulsars that typically there are ten to hundred independent sparking areas on the polar cap. A similar conclusion was reached by Cheng and Ruderman (1977) who found from the widths of drifting subpulses that the fractional area of the polar cap occupied by a spark lies between 10^{-1} and 10^{-2}. We shall take the area of each spark to be $\approx \pi \times 10^6$ cm^2 and the radius of a spark area, assumed circular, to be $R \approx 10^3$cm.

In passing, a few remarks might be appropriate on the duration τ_2 of the observed microstructure (a few hundred microseconds) which is large compared to the supposed duration of sparks ($\approx 10\mu$s). In the RS model bunches of relativistic particles emit curvature radiation at a particular frequency from a certain extent of the magnetosphere which we will call the radiating zone for that frequency. Thus bunches travelling through the two extremities of this radiating zone would give out radiation which would reach the observer with a spread in time τ_2 which depends upon its extent. Thus τ_2 will depend on the net excess distance travelled by particles to and radiation from one extremity of the radiation zone, over that corresponding to the other.

3. Variations in the polar cap relief

We consider first the possibility that the surface of the polar cap deviates from that of a smooth sphere. It can be shown that the maximum height H of a conical hill on a base of the same material cannot exceed $3\pi\ Y\rho g$ where Y is the yield stress.

The value of H depends therefore on Y, ρ, and g all of which have uncertainties associated with them. The value of the Young's modulus for pulsar crustal material was estimated by Irvine (1978) as 10^{19} dynes cm^{-2} based on the value given by Chen, Ruderman and Sutherland (1974) for the binding energy. In view of the later work of Flowers *et al.* (1977) which indicates a decrease in the binding energy, a similar estimate would give a lower value.

If we assume that the yield stress Y is given by the Young's modulus, and lies in the range 10^{18} to 10^{19} dynes cm^{-2}, this leads to a value of H somewhere in the range 1 to 10 cm. We shall assume for the present purpose that H can have a value as high as 10 cm, and discuss later whether and how such irregularities are likely to result from the sparking process. For the moment we shall merely assume that the mean height of a spark area can differ from area to area by as much as 10 cm. The polar gap can then be likened to a parallel plate capacitor of separation G cm whose lower surface has bumps of height H cm and radius l cm. As $G \gg l \gg H$ the fractional excess electric field over the bumps would be $(\Delta\Phi/\Phi) \simeq H/l \approx 1$ per cent effective for a height $l \approx 10^3$ cm from the lower plate.

Since the gap height $G \approx 10^4$ cm, a hill of height 10 cm on the polar cap results in an average excess electric field of 0·1 per cent in the volume of the gap directly above it. We shall now show that even such a small excess electric field can significantly alter the number of charged particles produced in the gap.

Let us think of the spark discharge as developing in time 'steps' where the number of charges multiply by a constant factor at the end of each step. During the development of the discharge there is continuous leakage of charges from the gap as they reach one or other ' plate ' of the capacitor. If for some reason the number of charges effective for further multiplication in each step were increased, say by a fraction ϵ, then in each step, the number of charges participating in the avalanche breakdown would be $(1+\epsilon)$ times more; in m steps the fractional increase in the total number of charges produced would be $(1+\epsilon)^m$.

In the RS model the fraction of charges effective in the multiplication is determined by the gap height $\lambda = \lambda_e + \lambda_{ph}$ where λ_e is the mean free path for charges to radiate a photon of the appropriate energy and λ_{ph} the mean free path for this photon to produce a pair in the gap magnetic field. We now define the time step as $G/2c$, which is the mean time taken by particles presently in the gap to leave it and be replaced by the next generation. If the density of charges is assumed uniform in the gap it will be seen that the fraction that will contribute to the next generation is $\eta \approx (\mathrm{I} - \lambda/G)$; if $\lambda > G$, avalanche growth cannot occur. Now the charged particles radiate all along their path (except for the first λ_e cm) on the curved magnetic field lines even as they are accelerated by the gap electric field. An excess electric field in the gap would give the charges a little more energy at each point on their path provided that their energy is not limited by radiation reaction. This would result in an increased gamma of the charges accelerated over hills, where $\gamma = E/mc^2$. For 0·1 per cent average excess field,

$$\frac{\Delta\gamma}{\gamma} \approx 1/1000.$$

But curvature photon energy (E_{ph}) is proportional to γ^3. Therefore

$$\frac{\Delta E_{ph}}{E_{ph}} \approx \frac{3\Delta\gamma}{\gamma} \approx \frac{3}{1000}.$$

This would reduce λ_{ph}which is given by $\lambda_{ph} \propto e^{4/3\chi}$, where χ is proportional to the energy of the photon, and is typically $\approx$ 1/15(Ruderman and Sutherland 1975). We thus have

$$\frac{\Delta\chi}{\chi} \approx \frac{3}{1000},$$

$$\text{or } -\frac{\Delta\lambda_{ph}}{\lambda_{ph}} \approx 0{\cdot}06.$$

If we assume

$$\lambda_e < \lambda_{ph}, \frac{\Delta\lambda}{\lambda} \approx \frac{\Delta\lambda_{ph}}{\lambda_{ph}}$$

and we can compare η in the two cases, which differ in the values of λ in the following form:

$$\epsilon = \Delta\eta/\eta = -\frac{\Delta\lambda}{G} \times \Theta \text{ where } \Theta = \frac{1}{\left(1 - \frac{\lambda}{G}\right)}.$$

If we assume $\lambda \approx 5 \times 10^3$ cm (Ruderman and Sutherland 1975) $\Theta \approx 2$ but could be much larger if λ were closer to G. Spark discharges over a hill therefore would contain $\left(1 + \frac{2\,\Delta\lambda}{G}\right)$ times more charges at the end of each 'step'. As the time for one spark discharge $\approx 10^{-5}$ s the number of 'steps' in which the avalanche growth is completed is $\frac{10^{-5} \times 2c}{G} \approx 60$. In 60 steps the fractional increase in the total number of charges in the gap is

$$\left(1 + 2\frac{\Delta\lambda_{ph}}{G}\right) = (1 + 0.06)^{60} \approx 30.$$

As the radio emission is produced by positrons subsequent to their entering the magnetosphere the average radio intensity could well be expected to depend on the number of particles. Further because of the coherent nature of the radio radiation it is not unreasonable to expect it to be proportional to some power (larger than 1) of the number of particles that are produced in each spark. At the present state of understanding it is difficult, if not hopeless, to take into account all these considerations and to calculate the expected increase in average pulse intensity produced by sparks over hills. Our aim has been merely to show that such an increase is likely, critically dependent on the electric field in the gap, and easily capable of accounting for the observations which show a variation in intensities across the pulse window by factors of the order of $\approx 10^3$. It should be noted however that unless hills of height at least 5 cm (half the assumed value) can be supported on pulsar polar caps, the mechanism described above cannot satisfactorily account for the observations.

4. Deformation of the polar cap by sparks

In this section we will discuss the possible effect of sparks on the polar cap. In the vacuum gap both electrons and positrons are produced and are accelerated towards opposite ends of the gap. Since both types of charges are accelerated by the same electric field they gain similar energies. Therefore the electrons striking the polar cap surface must carry the same energy as the positrons which produce the observed radiation from the pulsar. As the typical total luminosity of a pulsar is $\geqslant 10^{30}$ erg s^{-1} we can expect the electrons in the spark to dump this order of energy onto the polar cap of the pulsar. Any damage done to the surface by these electrons will depend upon the structure and composition of the surface material and the details of the energy dissipation process.

We could begin by asking what are the various things that could happen to the surface as a result of a spark at a certain point on it. One possibility is what has been generally believed so far, *i.e.,* there will be local heating, consequently some excess radiation of photons and no change in the topography. At the other extreme it is conceivable that a pit is formed at the position of a spark and pieces of the crust are thrown up and scattered around the pit. A more likely possibility, however, is a very slight deformation of the crustal surface with the matter remaining bound at all stages; the energy required to create slips is very much less than the energy required to break the bonds between iron atoms.

If the energy available in each spark can effect some rearrangement of matter due to flow of material after sparking, we suggest that the result of a spark on the polar cap will be to *raise* the surface at that point by a minute amount. Although such a hypothesis might seem unlikely by analogy with the effect of meteorites and other energetic solid matter impinging on the surface of planets, we propose that it can happen in the case of pulsars. The very strong electric field at the surface of the polar cap is operative in the sense of trying to tear out ions from it against the force of gravity. The very basis of the RS model is the inability of this electric field to do so simply because of the high binding energy in the presence of strong magnetic fields. The positively charged outer crust at the magnetic polar cap may be likened to a pressurised balloon with a thin solid surface. A dimunition of the binding energy must therefore work in the sense of aiding a movement away from the centre of curvature of the crust. The duration of a spark can therefore be thought of as a short interval of time during which the cohesive energy of the lattice has been decreased, thus making it possible for the electric field to have its way and raise a tiny bump at the position of maximum heating.

The most favourable manner of raising bumps would involve stripping of atomic electrons by the intense beam of spark electrons. This would *momentarily* leave the crust under the spark with a much higher positive charge. A variation of particle density across the spark would effect a similar variation in the extra surface charge density of the softened crustal material leading to a differential upward force. If, the spark were densest in the middle, as one would expect, then a small bump on the polar cap should be left behind by a spark. We saw in the previous section that surface variations on the polar cap of the order of 10 cm could lead to the intensity variations observed across integrated pulse profiles. If an individual spark can be effective in creating a minor deformation in the polar cap, then with time the amplitude of this deformation would increase even if sparking were completely random. An upper limit for Δh, the deformation required per spark, can be obtained from the observed correlation of complex pulse profiles with pulse periods in excess of 0·75 s. The typical age of a pulsar with a period of 0·75 seconds is $\geqslant 10^6$ years and this may be taken as the time for the surface relief of the polar cap to have build up to a root mean square value of ~10 cm. Since there occurs one spark per millisecond on the average over any point on the polar cap, we have 3×10^{16} sparks corresponding to 10^6 years. On a purely random basis the peaks in the surface relief would have a height $\geqslant \sqrt{3 \times 10^6} \times \Delta h \approx 10$ cm. This leads to a value of $\Delta h \approx 10^{-7}$ cm or the incredibly small value of 10 Å.

In other words, if a single spark managed to increase the height at its centre of impact by 10 Å, then over a period of the order of a million years the surface of the polar cap must end up with variations in height of the order of 10 cm and lead to the consequences discussed in the previous section. The amount of matter under a spark area that must be rearranged to obtain a bump of height 10 Å is $\sim 10^{26}$ atoms. The question is whether enough energy can be imparted to the surface layer in a way that will make it temporarily mobile and allow the electric tension to rearrange it.

Notions regarding the stability of the crustal material have been changing since the early work of Chen, Ruderman and Sutherland (1974) who derived a value for the cohesive energy for each iron atom of ~ 10 keV. This number was revised by Flowers *et al*. (1977) according to whom the cohesive energy of iron per atom in a field of $\sim 10^{12}$ gauss is ~2 keV. If a spark can impart at least this amount of energy to each of more than say 10^{26} atoms directly under the spark, there is then the possibility of rearrangement under the effect of the electric tension. The amount of energy carried by the electrons to the surface is 10^{30} erg s^{-1}. At the rate of 10^5 sparks per second, there is thus in each spark about 10^{25} erg. The area of the spark is $\pi \times 10^6$ cm^2. Assuming the generally accepted upper limit of 10^5 g cm^{-3} for the density at the surface, there is enough energy in the spark to give ~ 2 keV per atom down to a depth of 1 cm. It is seen therefore, that if the energy is deposited in a much shorter distance, the atoms are bound to acquire much more than their cohesive energy immediately after each spark thus permitting a slight deformation within the radiation cooling time of the surface.

The heating of the surface under a spark has been discussed recently by Cheng and Ruderman (1980). According to them ‘ if a pair production discharge occurs above the polar cap an intensive flux of 10^{12} eV electrons is directed onto the surface causing strong local heating. This energy is deposited within $d \gtrsim 10$ radiation lengths (of the order of 10^{-3} cm in 10^5 g cm^{-3} of Fe) of the surface. It is largely reradiated locally from the area under the discharge.’ The temperature quoted by these authors for the surface is a quarter keV $\approx 2{\cdot}5 \times 10^6$K. This implies a total radiation loss of $\sim 10^{30}$ erg s^{-1} over the whole of the polar cap commensurate with our assumptions in the preceding paragraph.

If the energy in sparks is indeed *deposited* within 10^{-3} cm as suggested by Cheng and Ruderman (1980), then it appears to us that even allowing for the energy taken up by electrons and that radiated as photons, and errors in the various estimates, we must still have a much larger number of mobile atoms than we need. Under these circumstances an eventual modification of the polar cap relief seems inevitable.

It may be mentioned that *increase in height* of an area due to a spark on it represents a positive feed-back mechanism, since subsequent sparks over this area will carry more energy than sparks over neighbouring depressions. Although the extra energy may appear tiny, the effect is significant over the timescales required to build up a hill by a purely random process. As a result the minimum step required per spark will actually be less than the Δh = 10Å which we estimated conservatively earlier. It may also be pointed out that for a similar reason it would be harder to build up surface relief if sparks created pits instead of bumps, in which case the feed-back would be negative.

We have seen how polar cap surface modification by sparks can lead to structure in the integrated pulse profile over the timescale in which pulsars acquire such profiles. By the same token it may be noted that the profiles will remain stable over such periods. From the number of spark areas within the polar cap we can arrive at the minimum scale length for surface irregularities as of the order of 10 metres. The maximum number of hills that one can expect to find along the diameter of a typical polar cap would therefore be $\sim$ 10, and along a chord somewhat fewer. This is in good agreement with observations which show that the maximum number of major components in an integrated pulse profile is about 5. PSR 2045—16 has three major components and PSR 1237–25 has two major and three minor components. In the next section we shall discuss how similar structure in integrated profiles can result from minor magnetic field variations over the polar cap.

5. Magnetic field variations

In an earlier section we showed how the number of charges produced in a spark discharge depended critically on λ, the total path length within the gap for a charge to be accelerated, produce a photon, and for the photon to produce a pair. In that case, the variation in λ resulted mainly from the variation in λ_{ph}, the path length for a photon to produce a pair. It was also seen earlier that λ_{ph} depends critically on the energy of the photon, since $\lambda_{ph} \propto e^{4/3\chi}$, and χ is proportional to the photon energy In this expression, χ is also proportional to the strength of the perpendicular component of the magnetic field. The angles made by the field lines within the gap with the trajectory of the gamma ray photons are very small, and the perpendicular component is therefore directly proportional to this small angle. Even a small change in this angle will therefore have a large effect on λ_{ph} with a consequent change in the total number of particles produced as discussed earlier. For example, an increase of 3 parts per thousand in the small angle made by the field lines with the photon trajectory will result in the same increase of a factor $\approx$ 30 in the total number of charges, as calculated earlier for a similar increase in the photon energy.

Given a pure dipole magnetic field the curvature of the field lines will vary smoothly across the polar cap and will lead to pulse profiles that do not have a complicated or random structure. However, if pulsar fields evolve with time as discussed by Flowers and Ruderman (1977) multipole components will make their appearance in due course. Such multipole components even if present to a very small degree will significantly modify the curvature of the field lines in the polar cap. For the reasons discussed above this will then be strongly reflected in the intensity of the sparks produced at different longitudes *i.e.* structure in the integrated pulse profile.

To predict the effect of multipole components on the pulse structure is difficult because of the number of parameters involved in the addition of even a quadrupole component. Simple trial models seem to suggest that the major effect of the introduction of a quadrupole component in the gap field would be asymmetry in the pulse profile. The general result is to increase the curvature of the field linens on one side and to decrease it on the other. It seems much harder to create variations in the curvature of the field lines across the gap that could lead to a complex profile with many components as seen in several pulsars.

We note that if multipole fields in older pulsars are as inevitable and as strong as suggested by Flowers and Ruderman (1977), they must have a dominating influence on the pulse profile according to the conclusions drawn above. However, for the reasons just discussed the net result might be merely to make one part or region of the polar cap very much more effective than the rest. In such a case the observed profile would be interpreted as having a smaller duty cycle and might not be recognised as one modified by a complicated field structure in the gap. In any case we note that it multipole components can be shown to be responsible for variations in pulse structure, then this provides an observational method for determining the timescale for the evolution of the magnetic fields from the age of the pulsars which show such effects

6. Discussion and conclusions

We have outlined two possible ways in which the complex variation of the mean intensity within pulse profiles could be explained. In one case, minute local deformations of the surface due to individual sparks lead to a build-up of a surface relief pattern on the polar cap; this in turn causes small variations in the electric field intensity within the gap leading to much larger variations of the number of positrons produced by the sparks. The second possibility invokes the presenee of multipole components which modify the curvature of the magnetic field lines from point to point within the gap leading to the same conclusions as above.

Either mechanism provides general agreement with the observations in that the complicated pulse structure is correlated with older pulsars, and is stable for long periods. Also, the polarisation variation would be independent of the intensity structure within the pulse. In one case, the magnetic field structure would be unaffected by surface variations over the polar cap. For the other, multipole fields would need to modify the dipole pattern only marginally to account for the pulse intensity structure. Further, since they fall off much faster than the dipole field, their contribution in the radiating region at some distance from the pole will be greatly reduced. Hence for both mechanisms the sweep of the position angle of polarisation would be independent of the intensity variations across the pulse, as has been noted and emphasised by Manchester (1979).

To make an assessment from the observations of the importance of either of the two proposed mechanisms, one must take into account the pulse shapes obtainable in the absence of these or other such mechanisms. In magnetic pole models involving radiation into a hollow cone, both single and double pulse structures are easily explained in terms of edge and central cuts of the hollow cone by the line of sight, Many of the symmetry properties observed in pulses can be well accounted for by the circular symmetry of such a hollow cone. As pulses with complicated structure are generally from older pulsars, the radiating region would be further from the light cylinder than in younger (and faster) pulsars, and hence the asymmetry expected from sweeping back of the field lines would also be less. It is the presence of marked asymmetries and/or multiple components that cannot be accounted for in this way, and which we are attempting to explain in this paper.

We have seen earlier that a quadrupole component of the magnetic field can introduce a strong asymmetry, but is unlikely to create a complex pulse profile with many features. On the other hand, surface variations in the polar cap can produce both a large number of components and also asymmetry, since no particular symmetry will result from an essentially random process. Complex profiles, found (for example) in PSR 1237+25, seem therefore to strongly favour the polar cap relief hypothesis, whether or not the other mechanism also contributes. As mentioned earlier this is based on the assumption, which is hard to either prove or disprove at the moment, that the surface irregularities of the order of 10 cm can be supported on the polar cap. Further understanding of the structure and strength of neutron star crusts might show such an assumption to be untenable. If so, this would indicate that the alternative mechanism we have discussed is the operative one, and that the magnetic field structure of older pulsars can have at least some contributions of higher order than a quadrupole moment.

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