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Comments on the Source Function Equality in (Zeeman)-Multiplets

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Abstract. The conditions for the source functions of a multiplet to be equal are studied for plasmas with and without magnetic fields. It is found that source function equality holds—in addition to the case of collisional predominance—only when the redistribution functions are all identical and no interlocking with other lines occurs. When magnetic fields are present, the assumption of source function equality leads to a violation of the invariance conditions of the scattering matrix and should therefore not be made.

Key words: radiative transfer—non-LTE—Zeeman effect—polarization

Non-LTE calculations for the radiative transfer of multiplet lines have recently been studied by several authors for plasmas without magnetic fields (*cf.* Mihalas 1978) and with magnetic fields (see *e.g.* Landi Degl'Innocenti 1975; Stenholm and Stenflo 1978; Auer, Heasley and House 1978). Many of these calculations, in particular those for Zeeman lines, are based on the assumption that the source functions of the lines are frequency-independent and equal. For the field-free case it is shown by Jefferies (1968), Athay (1972), Mihalas (1978) and others that source function equality occurs when the collision rates between the fine structure levels of the upper term are much larger than the collision rates between these levels and the ground state. In the general case equality is only possible when the mean intensities

$$\bar{J}_{ij} = \tfrac{1}{2} \int_{-\infty}^{\infty} \int_{-1}^{+1} \phi_{vi} (\tau, \mu, v)\, I (\tau, \mu, v)\, d\mu dv$$

scale as

$$(B_{12}\,\bar{J}_{12} + C_{12}) : (B_{13}\,\bar{J}_{13} + C_{13}) : (B_{14}\,\bar{J}_{14} + C_{14}) =$$

$$(C_{21} + C_{23} + C_{24} - C_{32} - C_{42} + A_{21}):$$

$$(C_{31} + C_{34} + C_{32} - C_{23} - C_{43} + A_{31}): \qquad (1)$$

$$(C_{41} + C_{42} + C_{43} - C_{24} - C_{34} + A_{41}).$$

(A_{ij}, B_{ij} and C_{ij} are the Einstein A and B coefficients and the collision rates for transitions $i \to j$). Equation (1) is easily derived from the statistical equations (*e.g.* Mihalas 1978) under the assumption that the upper levels are equally populated and that the induced emission can be neglected.

It is evident that equation (1) is satisfied—except for singular cases—only if no line of the multiplet is blended with other lines and the redistribution functions are identical. In stellar atmospheres or gas clouds with magnetic fields the last condition is not satisfied, since the absorption coefficients for the Zeeman components have a quite different angle dependence (*cf. e.g.* Stenholm and Stenflo 1978 ; see also equation (6) below). Obviously this may introduce very large deviations from an equal population of the upper levels, in particular for low density configurations (collision rates small!) of spherical geometry (finite optical depth).

In addition, the basic invariants of the scattering matrix (Abhyankar and Fymat 1969) are not conserved, when identical source functions are used. This can be seen as follows: Let us start with the redistribution matrix F (Stenholm and Stenflo 1978)

$$\mathbf{F} = \begin{pmatrix} K_I\,K'_I & K_I\,K'_Q & K_I\,K'_U & K_I\,K'_V \\ K_Q\,K'_I & K_Q\,K'_Q & K_Q\,K'_U & K_Q\,K'_V \\ K_U\,K'_I & K_U\,K'_Q & K_U\,K'_U & K_U\,K'_V \\ K_V\,K'_I & K_V\,K'_Q & K_V\,K'_U & K_V\,K'_V \end{pmatrix}, \qquad (2)$$

K'_x refers to the absorption for the Stokes vector x=(I, Q, U, V) and K_x is the corresponding quantity for re-emission). For this F, the calculation of the invariants leads to 9 equations between K_x and K_x' (Abhyankar and Fymat 1969), of which only two are different from each other:

$$K_I^2 = K_Q^2 + K_U^2 + K_V^2, \qquad K_I'^2 = K_Q'^2 + K_U'^2 + K_V'^2. \qquad (3)$$

When K_x = K'_x these equations reduce to one. Under the assumption of source function equality the K_x are defined as sums over the three components so that

$$K_I = \tfrac{1}{2}\,[\phi_0 - \tfrac{1}{2}\,(\phi_+ + \phi_-)]\ (1-\mu^2) + \tfrac{1}{2}\,(\phi_+ + \phi_-),$$
$$K_Q = \tfrac{1}{2}\,[\phi_0 - \tfrac{1}{2}\,(\phi_+ + \phi_-)]\ (1-\mu^2)\cos 2X, \tag{4}$$
$$K_U = \tfrac{1}{2}\,[\phi_0 - \tfrac{1}{2}\,(\phi_+ + \phi_-)]\ (1-\mu^2)\sin 2X,$$
$$K_V = \tfrac{1}{2}\,(\phi_- - \phi_+)\,\mu.$$

ϕ are the profiles of the components, χ is the azimuthal angle and μ=cos θ, where θ is the inclination angle relative to the radius vector. For simplicity we have chosen the magnetic field to be radially directed. Inserting these expressions into equation (3) we obtain

$$-\phi_0 = \frac{2\phi_+\,\phi_-\,\mu^2}{(\phi_+ + \phi_-)\,(1-\mu^2)}, \tag{5}$$

which cannot be fulfilled.

Equation (5) shows that for magnetized plasmas even when the upper levels are equally populated the assumption of source function equality leads to results that are not physically realistic. Consequently, whenever scattering is important in plasmas containing magnetic fields such an assumption should not be used. It is therefore unavoidable that we treat the components as separate but interacting analogous to the calculation for fluorescence lines (*cf. e.g.* Weyman and Williams 1969): For this purpose we introduce three redistribution matrices $\mathbf{F_0}$, $\mathbf{F_+}$, $\mathbf{F_-}$ with elements

$$\begin{aligned}
K_{0I} &= \tfrac{3}{8}\,\phi_0\,(1-\mu^2) & K_{\pm I} &= \tfrac{3}{4}\,\phi_\pm\,(1+\mu^2)\\
K_{0Q} &= \tfrac{3}{2}\,\phi_0\,(1-\mu^2)\cos 2X & K_{\pm Q} &= \tfrac{3}{4}\,\phi_\pm\,(1-\mu^2)\cos 2X\\
K_{0Q} &= \tfrac{3}{2}\,\phi_0\,(1-\mu^2)\sin 2X & K_{\pm U} &= \tfrac{3}{4}\,\phi_\pm\,(1-\mu^2)\sin 2X\\
K_{0V} &= 0 & K_{\pm V} &= \mp\,\tfrac{3}{2}\,\phi_\pm\,\mu.
\end{aligned} \tag{6}$$

They are normalized so that

$$\frac{1}{4\pi}\int_0^{2\pi}\int_{-1}^{+1} K_I\,d\mu\,d\kappa = 1$$

i.e. the number of photons is conserved in scattering processes. By elementary calculations it can now be shown that the invariance relations for $\mathbf{F_0}$, $\mathbf{F_+}$ and $\mathbf{F_-}$ are fulfilled identically.

In order to be consistent the transfer equation has now to be written

$$\frac{dI_\nu}{dz} = \kappa_0\,\mathbf{F_0}\,\mathbf{1}\,S_0 + \kappa_+\,\mathbf{F_+}\,\mathbf{1}\,S_+ + \kappa_-\,\mathbf{F_-}\,\mathbf{1}\,S_-$$
$$- (\kappa_0\,\mathbf{F_0} + \kappa_+\,\mathbf{F_+} + \kappa_-\,\mathbf{F_-})\,I_\nu. \tag{7}$$

with

$$(\mathbf{1})_{ij} = \begin{cases} 1 \text{ for } i = j = 1 \\ 0 \text{ for all other } i, j, \end{cases}$$

$$\kappa_0 = (u_1 B_{13} - u_3 B_{31}) \frac{h\nu}{4\pi} ,$$

$$S_0 = n_3 A_{31}/(n_1 B_{13} - n_3 B_{31}),$$ and

$k_\pm$ and $S_\pm$ are defined in an analogous way.

We note that in these equations the random phase approximation is used. This is justified for thermal plasmas, when the magnetic field is so high that the components are separated in the reference system of the atom.

In conclusion we state that for non-LTE plasmas containing magnetic field the source functions for the lines of a Zeeman triplet are *not equal* and calculations based on this assumption should be considered with caution. For radiative transfer calculations it is therefore necessary to consider the lines as being separate, but blended. The corresponding equations, which fulfill the invariance and normalization conditions, are given above.

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