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The Role of General Relativity in Astronomy: Retrospect and Prospect*

S. Chandrasekhar *Laboratory for Astrophysics and Space Research, The University of Chicago, Chicago, USA*

My last attendance at a meeting of the International Astronomical Union was forty-four years ago when it met in Paris in 1935. I do not doubt that my being asked to give an invited discourse at this meeting is a personal courtesy extended to me by your distinguished President recalling, perhaps, the years when he and I were colleagues together at the University of Chicago.

I am aware that associated with my absence from these meetings for nearly half a century is the fact that during most of this period—if not all of it—my interests, at different times, have been outside whatever may have been the prevailing trends in the mainstream of astronomy. I am afraid that on this account, the point of view I shall present—retrospectively and prospectively—will not be in conformity with the trends currently prevailing. I must therefore begin by asking for your patience and for your forbearance.

Dr Blaauw, when he invited me to give one of the three discourses at this meeting, suggested that in selecting a topic I might wish to take into account the fact that this year is the centennial of Einstein's birth. The subject of my discourse is in accordance with that suggestion.

The general theory of relativity was conceived by Einstein in the faith that laws appropriate to the different domains of the physical sciences must be mutually and harmoniously consistent with one another. Since Newton's laws of gravitation are based on the notion of instantaneous action at a distance, it is discordant with the precepts of the special theory of relativity derived, in the first instance, from Maxwell's laws governing electrodynamics. Therefore, argued Einstein, the Newtonian laws of gravitation must be modified to eliminate this discordance by allowing for the finiteness of the velocity of light.

Besides, at the base of the Newtonian theory was the enigmatic fact of the equality of the inertial and the gravitational mass—an empirically found equality to which Newton gave its deserved prominence by stating it in the opening sentences of his *Principia.* Einstein wished to eliminate this element of magic in the Newtonian theory by some general principle. The general principle is, of course, his principle of equivalence.

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Even with the principle of equivalence as a base and the special theory of relativity as a guide, the formulation of a consistent theory of gravitation is fraught with ambiguities. But Einstein succeeded in formulating his general theory of relativity by a combination of physical reasoning and mathematical arguments of simplicity and elegance. It was, as Weyl stated, a triumph of speculative thought. And the fact that Einstein was able to arrive at a complete and a coherent physical theory by such speculative thought is the reason why, when we follow his thoughts, we feel as though a 'wall obscuring truth has collapsed'—quoting Weyl once again.

The element of controversy and doubt, that have continued to shroud the general theory of relativity to this day, derives precisely from this fact, namely, that in the formulation of his theory Einstein incorporates aesthetic criteria; and every critic feels that he is entitled to his own differing aesthetic and philosophic criteria. Let me simply say that I do not share these doubts; and I shall leave it at that.

The general theory of relativity is a theory of gravitation; and like the Newtonian theory of gravitation, which it refines and broadens, its natural home is astronomy. It is, therefore, not surprising that the early interest in the general theory was related to the verification of the small departures from the Newtonian theory which it predicts in the astronomical domain.

As is well known, the three classical tests relate to:

(i) the dependence of the rate of a clock on the value of the gravitational potential at its location;

(ii) the deflection that light must experience as it traverses a gravitational field; and

(iii) the slow precession which the Kepler orbit described by a planet must experience.

Of these three tests, the first is not really a test of the particular form of Einstein's equations; it is rather a test of the principle of equivalence.

Perhaps the best way to explain what particular features of the theory are verified by the different tests is to consider the coefficients of the metric to which they refer.

We all know that in space, free of any gravitational field, the appropriate geometry is that of special relativity, associated with the Minkowskian metric,

$$ds^2 = -c^2dt^2 + (dx^2 + dy^2 + dz^2), \tag{1}$$

where c denotes the velocity of light. The principle of equivalence requires that we must replace this metric by

$$ds^2 = -c^2dt^2\,(1 - 2U/c^2) + (dx^2 + dy^2 + dz^2), \tag{2}$$

where U *is* the gravitational potential determined by Poisson's equation:

$$\nabla^2 U = -4\pi G\rho. \tag{3}$$

By this generalisation, the intervals of the proper time, $d\tau_1$ and $d\tau_2$, at locations where the gravitational potentials are U_1 and U_2, are in the ratio

$$d\tau_1/d\tau_2 = [(1 - 2U_1/c^2) \,/\, (1 - 2U_2/c^2)]^{1/2} = 1 - (U_1 - U_2)/c^2 \tag{4}$$

—a relation which implies the slowing down of a clock as it is transported to regions of higher gravitational potential. This predicted slowing down of a clock has, as is known, been experimentally confirmed by the experiments of Pound and Rebka.

If one supposes that the metric (2) is adequate to determine the deflection of a light ray as it traverses a gravitational field, then one would find, as Einstein found in 1911, that a light ray grazing the solar disc must be deflected by 0·83". Indeed, Einstein thought in 1911 that 0·83" was the amount of the deflection to be expected; and an expedition which had set out to verify this early prediction was aborted by the beginning of World War I.

The full theory of Einstein requires that for the purposes of predicting the deflection of light, the metric must be modified to the form

$$ds^2 = -c^2(1 - 2U/c^2)dt^2 + (1 + 2U/c^2)(dx^2 + dy^2 + dz^2). \tag{5}$$

The factor $(1+2U/c^2)$ in front of the spatial element $(dx^2 + dy^2 + dz^2)$ represents the curvature of space and is a genuine general relativistic effect. It is only when this additional modification of the metric is taken into account, do we find that the theory predicts a deflection of 1·7" for a light ray grazing the sun's disc.

As is well known, this latter value for the deflection of light, rather than 0·87″ or 3·4″, was confirmed by the British eclipse expeditions of 1919 under circumstances which catapulted Einstein to world renown.

In recent times this prediction concerning the deflection of light has received much more precise confirmation from long base-line interferometric radio observations of the two quasars 3C 273 and 3C 279 which are 9° apart and which are occulted by the sun every year in October. The latest analysis of these observations confirms the predictions of the general theory of relativity to a fraction of a per cent. A related test consists in measuring the round-trip radar travel-time in the solar system; and the results of these experiments by Shapiro and his associates again confirm the predictions of the general theory (based on the metric (5)) to within experimental errors.

The last of the major tests relates to the precession of a Keplerian orbit. This effect depends on completing the metric (4) so that it may provide consistent equations of motion allowing for all first-order departures from the Newtonian theory. The appropriate form of the metric then takes the form

$$\begin{aligned} ds^2 = &-[1 - 2U/c^2 + (\ldots)/c^4]\,(dx^\circ)^2 + (P_\alpha/c^3)dx^\circ\, dx^\alpha \\ &+ (1 + 2U/c^2)\,[(dx^1)^2 + (dx^2)^2 + (dx^3)^2], \\ &(x^\circ = ct,\ x^1 = x,\ x^2 = y,\ x^3 = z) \end{aligned} \tag{6}$$

where the terms of order c^{-4} and c^{-3} depend on the distribution of matter and motion in the central mass. In particular, P_α depends only on the presence of internal motions and we may neglect it in evaluating the rate of precession of the Keplerian orbit of a planet round the sun since its rotation is slow and hardly makes any contribution.

The observed precession of 43″ per century of the orbit of Mercury was shown, already by Einstein in his first publication, to be in perfect accord with the predictions of his theory.

More recently, in the binary pulsar discovered by Hulse and Taylor, the rate of precession of the orbit, as determined by these authors, is 4·2° per year. And this observed rate would lead one to infer that the total mass of the binary pulsar is 2.83 ⊙. I should however point out that the recent observations by Crane, Nelson, and Tyson suggest that the companion of Hulse and Taylor's pulsar may have been optically detected; and if the indicated presence of an optical companion should be confirmed, then inferences that have been drawn concerning this pulsar require a re-evaluation.

May I at this point emphasise that the confirmations of the general theory of relativity of which I have spoken relate only to the first non-trivial departures from the Newtonian theory predicted by the general theory and as such they refer only to two or at most three of the coefficients of the terms of the lowest order in a post-Newtonian expansion of the metric beyond those required by the principle of equivalence. It is clear that there is no hope, in the foreseeable future, to verify the coefficients of the terms of higher order that are present in approximations beyond the post-Newtonian. On this account, the principal question which is currently debated, in the context of the elaborate and the expensive experiments conducted to verify the general theory, is not whether the predictions of the general theory are confirmed; it is, rather, the viability or otherwise of the innumerable theories, alternative to Einstein's, that continue to proliferate. Before the present precise confirmations of the particular predictions of the general theory, to which I have made reference, were available, one had to contend with theories which were *ad hoc* on. all accounts. Now, the only difference is that authors of alternative theories must arrange that they agree with Einstein's theory in the lowest order *i.e.* with the two or three coefficients in the post-Newtonian expansion of the metric which are involved. When I view these theories, I am reminded of what E. A. Milne once told me: Given a finite set of observational constraints, ' It cannot be beyond the wit of man to construct theories which will meet those constraints.' And certainly there has been no lack of wit in constructing theories alternative to Einstein's. I hope you will forgive me if I recall in this connection an admonition of Eddington's, 'The off chance that posterity may find wisdom in our words is no reason for making meaningless noises.'

I now turn to the role which general relativity plays in our understanding of the largescale structure of the universe *i.e.* in the realm of cosmology. I think it can be fairly said that the only aspect of general relativity with which most astronomers were concerned, up until the early sixties, is in the cosmological models it provides for interpreting the results of observations. To a large extent the role, as perceived, was an exaggerated one; and for the following reasons.

The basic facts one was concerned with in cosmology before the discovery of the microwave background-radiation are the following:

(i) In a first approximation the distribution of the extragalactic nebulae is locally homogeneous and isotropic.

(ii) The galaxies are receding from us and one another with velocities which are proportional to their mutual distances—relations codified in Hubble's law.

As was pointed out and emphasised by E. A. Milne, these facts have a very simple interpretation which requires no special appeal to any particular theory. The facts imply that all the nebulae, we now observe, must have been, at one time, collected together in a small volume of space. This inference is no more than the common inference which one would draw from a swarm of bees flying radially outward from a tree, namely, that there was a beehive in the tree! However, (again, as Milne emphasised) a homogeneous swarm of particles radially expanding from one of the particles has a remarkable property. A simple application of the parallelogram of velocities shows that the description will be the same with respect to any other particle in the swarm provided one does not go too near the boundary. In such a system, every local observer can, in the words of Eddington, consider himself as the plague-spot of the universe! In other words, a cosmological principle must prevail. This is a straightforward interpretation of very simple facts. What is remarkable about the application of general relativity to the cosmological problem is that requirements of homogeneity and isotropy do not allow a static universe: it can only be an expanding (or a contracting) one in which, locally, a Hubble relation must obtain. In making this assertion, I am, of course, supposing that the cosmical constant is zero. It will be recalled that Einstein introduced the cosmical constant with the sole purpose of permitting a static homogeneous universe in the framework of his theory. With a nonvanishing cosmical constant, one has an additional adjustable parameter. If the cosmical constant had not been invented, much of the game in which cosmologists, both theoretical and observational, have indulged themselves would have been spoiled.

I need not at this point go over the facts which must be familiar to all of you, namely, that the postulates of homogeneity and isotropy yield, in the framework of general relativity, the Friedmann models associated with the Robertson-Walker metric.

The Friedmann model commonly described as the ' big-bang ' model for the universe, strictly interpreted, implies that the universe began in an initial singularity and, if the universe should be a closed one, will end in a future singularity. The fundamental question in this context is how seriously one should take the predicted singularities. One could well be sceptical of extrapolating the Friedmann models backwards to the time when the radius of the universe was zero, and in the case of the closed models, forward to the time when the radius of the universe will again tend to zero. For, the Friedmann models assume *strict* spherical symmetry, *strict* homogeneity, and *strict* isotropy; and none of these assumptions is strictly realised or can be realised. One can therefore argue that some slight inhomogeneity, some slight anisotropy, and some slight departures from exact spherical and strict radial flow will replace the singularity by a state of high mean density which need not transcend any ' reasonable' limit which we may wish to impose. This scepticism was widespread during the fifties and the early sixties; and it was maintained by some of the most perceptive cosmologists of the time, McCrea and Lifshitz and Khalatnikov, for example. Thus, in a survey of cosmological theories written by McCrea in 1962, we find the statement: ' There, is no known feature of the universe that gives any indication of its ever having been in a state of extreme congestion as required by the Friedmann models.'

One's views with respect to the occurrence of singularities in solutions describing the evolution of gravitating physical systems in general relativity changed radically in 1965 when Roger Penrose proved that, so long as matter obeys certain very reasonable conditions (such as that the energy density as measured by an observer, in a frame of reference in which he is at rest, is always positive), singularities are inevitable once a process of collapse has started and a point of no return has been reached. (Subsequent theorems by Penrose and Hawking have succeeded in relaxing the original conditions of Penrose.)

The essential reason for the occurrence of singularities in general relativity is that every force which operates against collapsing to a singularity in the Newtonian theory (such as pressure or rotation) only adds to the inertia of the system and enhances the very gravitational force which is the cause of the collapse.

I shall return to the problem of gravitational collapse presently, but the point I wish to emphasise now is that the deduction that the universe very likely did begin in an initially singular state is not based solely on the discovery of Penzias and Wilson as is commonly believed. What their discovery does imply is that the present observed homogeneity and isotropy of the universe can be extrapolated backwards to a time when the radius of the universe was a 1000–2000 times smaller than it is at present and the physical conditions of density and temperature were such that the recombination of protons and electrons into hydrogen atoms took place, radiation and matter got decoupled, and the present matter dominated era began.

With the assurance that the present homogeneity and isotropy prevailed at the time of the electron-proton recombination, the singularity theorems of Penrose and Hawking give us the needed confidence to extrapolate further back in time. As is well known, this extrapolation backwards to three minutes from the initial singularity allows us to account for the cosmic abundance of helium and deuterium provided—and this is an important proviso—we incorporate in the calculations the empirically determined ratio of 10^9 for the number of photons to the number of baryons in the universe.

The densities and temperatures at which nucleosynthesis took place and the present abundance of helium was established are by no means extreme: the densities are comparable to, in fact much less than, what they are in atomic nuclei. But if the universe was in that state, the singularity theorems of Penrose and Hawking place no limit on the densities and temperatures to which we may—indeed must—extrapolate backward.

It is generally thought that quantal effects will require modifications of the general theory of relativity when we wish to discuss phenomena which may occur (rather will occur) in regions whose linear dimensions are of the order of the Planck length $(\hbar G/c^3)^{1/2} \sim 1{\cdot}6\times10^{-33}$cm and in intervals of time of the order of Planck length/velocity of light ($\sim 5{\cdot}3 \times 10^{-44}$s). It appears that at these extreme densities and temperatures there will be spontaneous creation of particles even as they occur at the horizons of black holes as Hawking radiation. One of the most interesting aspects of general relativity, in its applications to cosmology, is this interface which it provides for a unification of general relativity, quantum theory, and elementary particle physics. Regardless of the final outcome of these investigations, it would appear that the occurrence of cosmological singularities in the framework of general relativity raises some of the deepest questions in current physical thought.

I now turn to what has become one of the central problems of astronomy, namely, that of gravitational collapse.

While one can readily concede that much of the detailed calculations carried out at present depend on information concerning the physical state of dense nuclear matter derived in recent years, the basic reason for considering gravitational collapse in the context of the late evolution of stars was clearly recognised and stated more than forty-five years ago.

As is well known, stars with masses exceeding a certain limit do not have finite equilibrium states determined by electron degeneracy. This limiting mass is 14 ⊙ if a mean molecular weight of 2 per electron is assumed. The existence of this limiting mass in turn implies that in the final stages of the evolution of stars, their contraction cannot be arrested by the zero-point degeneracy pressure of the electrons. This result appeared so secure that statements such as these were made at the time.

‘ *Given an enclosure containing electrons and atomic nuclei* (*total charge zero*), *what happens if we go on compressing the material indefinitely* ?’ (1932).

‘ The life history of a star of small mass must be essentially different from the life history of a star of large mass. For a star of small mass the natural white-dwarf stage is an initial step towards complete extinction. A star of large mass cannot pass into the white-dwarf stage and one is left speculating on other possibilities.’ (1934)

Eddington clearly recognised that the existence of this upper limit to the mass of completely degenerate configurations, *if accepted*, implied inevitably the occurrence of black holes as the end products of the evolution of massive stars at least in some instances. He thus stated in January 1935:

‘ The star apparently has to go on radiating and radiating and contracting and contracting until, I suppose, it gets down to a few kilometers radius when gravity becomes strong enough to hold the radiation and the star can at last find peace.’

But he went on to say:

‘ I felt driven to the conclusion that this was almost a *reductio ad absurdum* of the relativistic degeneracy formula. Various accidents may intervene to save the star, but I want more protection than that. I think that there should be a law of nature to prevent the star from behaving in this absurd way.’

Indeed, since Eddington considered the conclusion derived from the Fermi degeneracy of electrons, allowing for the effects of special relativity, as leading to a *reductio ad absurdum*, he modified the relativistic degeneracy formula so that finite equilibrium states will be possible for all masses.

It is difficult to understand why Eddington, who was one of the early enthusiasts and staunchest advocates of general relativity, should have found the conclusion that black holes may be formed during the course of the evolution of stars so unacceptable. But the fact is that Eddington’s supreme authority in those years effectively delayed the development of fruitful ideas along these lines for some thirty years.

I hope you will forgive me if I recall on this occasion that at the last meeting of the International Astronomical Union in 1935 that I attended, Eddington stated his views, that I have quoted, in no uncertain terms in a report he gave at a meeting of the Commission on the Internal Constitution of the Stars. I passed on a note to Henry Norris Russell, who was presiding on that occasion, requesting that I be allowed to express views contrary to Eddington's. But I was denied that request.

The question whether the contraction of a massive star in excess of the critical mass can be arrested at a stage when matter attains nuclear density of 10^{13} g cm^{-3} was in part answered by an investigation of Oppenheimer and Volkoff in the late thirties on the equilibrium states of neutron stars. This investigation required, of course, the application of the equations of hydrostatic equilibrium in the framework of general relativity. But it is an entirely general consequence of the relativistic equations of equilibrium that the range of allowed masses must necessarily have an upper limit. I shall explain the origin of this upper limit a little later. The actual value of the maximum mass of a neutron star depends on the exact form of the equation of state; but upper limits to the attainable maximum can be set as Hartle and others have shown. But regardless of what this upper limit may be, it is clear that sufficiently massive stars must, in some cases at least, find that peace which Eddington considered as a *reductio ad absurdum.*

There are many different groups of investigators who are now exploring the mechanics of gravitational collapse to answer questions such as these. What is the nature of the remnants that will be left behind during such collapse ? To what extent is it justifiable to consider such remnants as the remnants of supernova explosions? What may be the amount of energy radiated as gravitational waves during such collapse? These and other related questions are being discussed at various sessions at this meeting. I shall say no more about them, but pass onto what the general theory of relativity provides as solutions appropriate for black holes.

A solution describing a stationary black hole must have the following properties. It must partition space into two regions: an inner region bounded by a smooth surface which is the envelope of null geodesies; and an outer region which becomes asymptotically flat *i.e.* it becomes the familiar spacetime of the special theory of relativity. The bounding surface separating the two regions defines the horizon of the black hole; and it is a necessary consequence of the definition that the space interior to the horizon is incommunicable to the space outside.

It is a startling fact that with these simple and necessary restrictions on a solution to describe a black hole, the general theory of relativity allows only a single unique two-parameter family of solutions. This is the Kerr family of solutions in which the two parameters are the mass and the angular momentum of the black hole. It includes Schwarzschild's solution as a limiting case appropriate for zero angular momentum.

Karl Schwarzschild derived his solution in December 1915 within a month of the publication of Einstein's series of four short papers outlining his theory. Schwarzschild sent his paper to Einstein for communicating it to the Berlin Academy. In acknowledging the manuscript, Einstein wrote, 'I had not expected that the exact solution to the problem could be formulated. Your analytical treatment of the problem appears to me splendid.'

Roy Kerr derived his solution in 1962. I should include this discovery of Kerr as among the most important astronomical discoveries of our time. It is, in my judgement, the only discovery in astronomy comparable to the discovery of an elementary particle in physics.

I shall now briefly consider the nature of the spacetimes around black holes described by the Schwarzschild and the Kerr solutions. The best way to visualize them is to exhibit the 'light-cone structure' in the manner of Roger Penrose.

Imagine that at a point in space, a flash of light is emitted. Consider the position of the wave front of the emitted flash of light at a fixed short interval of time later. In field-free space, the wave front will be a sphere about the point of emission. But in a strong gravitational field, this will not be the case. The sphere will be distorted by the curvature of spacetime about the point of emission.

Fig. 1 displays these wave fronts at various distances from the centre of symmetry of the Schwarzschild black hole. The section of the wave fronts by a plane through the centre of symmetry is illustrated. One observes that the sections of the wave fronts are circles far from the centre as one should expect; they are, however, progressively displaced asymmetrically inward as one approaches the centre. And on the *horizon* the wave front is directed entirely inward towards the centre with the point of emission on the wave front—the wave front has become tangential to the horizon. This is clearly the reason why light emitted from the horizon of a black hole does not escape to infinity. The situation in the interior of the horizon is even more remarkable. The wave front does not include the point of emission:

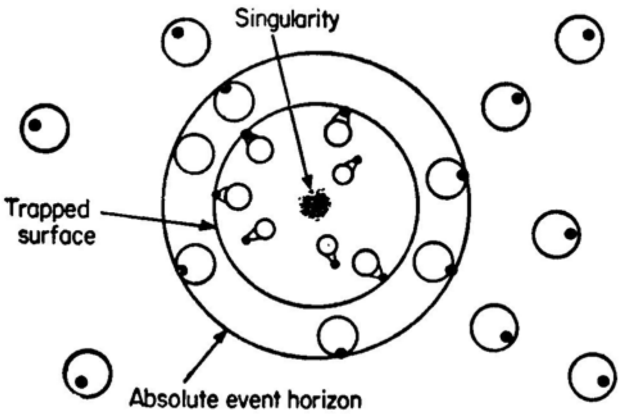

Figure 1. Effect of the curvature of spacetime on the propagation of light from points in the neighbourhood of a Schwarzschild, (nonrotating) black hole.

the wave front has detached itself. And since no observer can travel with a speed faster than that of light, it follows that there can be no stationary observers within the horizon—the inexorable propulsion of every material particle towards the singularity at the centre cannot be avoided.

Turning next to the geometry of the spacetime in Kerr geometry, we illustrate in Fig. 2 sections of the wave fronts of light emitted at various points on the equatorial plane of the Kerr black hole. The singularity in this case is a ring around the centre in the equatorial plane. In contrast to the Schwarzschild geometry, we have to distinguish, besides the horizon—where the wave front is entirely inside the horizon—a second surface where the wave front just manages to be attached to the source of emission. This second surface describes what has been called the ergosphere. In the region between the ergosphere and the horizon, while the wave front has detached itself from the point of emission, it is still possible for a particle, with a sufficient velocity suitably directed, to escape to infinity. The importance of this intermediate region is that it is possible for a particle entering this region from infinity to break up in two in such a way that one of the fragments is absorbed by the black hole, while the other escapes to infinity with an energy which is in excess of that of the incident particle. This is the so-called Penrose process for extracting the rotational energy of the Kerr black hole. An analogous phenomenon occurs when electromagnetic or gravitational waves of sufficiently small frequencies are incident on the black hole in suitable directions. In these cases, the reflection coefficient for such incident waves exceeds unity and is called super-radiance.

One of the questions that has been extensively studied in the context of black holes concerns the amount of energy that can be radiated as gravitational waves when objects fall into black holes or, indeed, when two black holes collide. It appears from the investigations of Detweiler that substantial energies can be radiated away only in collisions in which the object circles the black hole, at least once, before it falls in. I shall not discuss these questions any further. And I shall not also discuss, for lack of time, the many astrophysical questions relating to the physical phenomena, such as the emission of Xrays, which may be manifested during the accretion of matter around black holes; nor shall I consider the interesting dynamical questions relating to the altered distribution of stars around massive black holes that are presumed to occur at the centres of active galaxies.

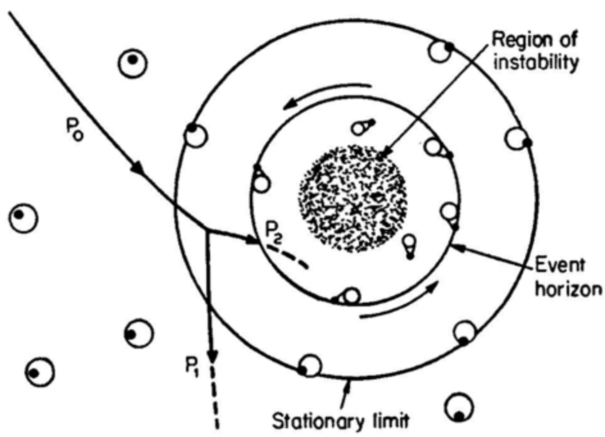

Figure 2. Equatorial cross-section of a Kerr (rotating) black hole. The positions of the wave fronts of light signals emitted at various points should be contrasted with those shown for the Schwarzschild black hole in Fig. 1. The rotational energy of the Kerr black hole can be extracted by a particle (P_0) that crosses thestationary limit from outside: the particle divides into two particles, one of which (P_2) falls into the black hole while the other (P_1) escapes from the ergosphere with more mass energy than the original particle (P_0).

I now turn to consider what the prospects may be for general relativity in astronomy. I am aware that it is a dangerous pastime for anyone to put on the cloak of a prophet; and it is certainly not my intention to play that part. What I do wish to say is more in the nature of reflections. I hope you will forgive me if they appear to you no more than a reflection of my own personal attitudes.

The general theory of relativity is a theory of gravitation and as I said at the outset, its natural home is in astronomy in the sense that its manifestations, whatever they may be, must be in the realm of astronomy. On this account one may be safe, I think, in expecting that the true role of general relativity in astronomy will be in providing as a basis for our understanding, its consequences under well-defined conditions, consequences so secure that we may incorporate them, on an equal footing, with other established facts of observation. In making this statement I am envisaging a role for theory in astronomy which is largely unrecognised and largely not practised. Since this is the case, I should like to clarify what I mean in the context of certain consequences which follow from the general theory of relativity.

Eddington once told me that his interest in the internal constitution of the stars arose from his interest in developing a pulsation theory for Cepheid variability; and how this interest led him to a study of the radial oscillations of gaseous stars in radiative equilibrium. It is therefore natural that one of the first problems in relativistic astrophysics which was considered in the early sixties was precisely the problem of the radial oscillations of a gaseous star in the framework of general relativity. This happens to be a particularly simple problem. Indeed, Eddington could have solved it in 1918; and certainly in 1934. The solution to this problem at once led to a qualitative difference in the criterion for the dynamical stability which follows from the Newtonian theory and from the general theory of relativity. Allow me to take a little time to explain the nature of this difference.

It is well known that in the framework of the Newtonian theory, the condition for the dynamical instability of a star derived from radial perturbations is that the effective ratio of the specific heats γ or more precisely some average of it, is less than 4/3; and dynamical stability is guaranteed if γ, or some average of it, is in excess of 4/3. But this result is changed in the framework of general relativity. A star with a ratio of specific heats γ, no matter how high, will become unstable if its radius falls below a certain determinate multiple of the Schwarzschild radius. It is this fact which accounts for the existence of a maximum mass for a neutron star to which I referred to earlier; and, indeed, for the instability of *all* equilibrium configurations as they approach the value 9/8 of the Schwarzschild radius. (For a solar mass, the Schwarzschild radius is 2½ km; and it increases linearly with the mass.)

I may parenthetically note here that this important result that configurations in stable hydrostatic equilibrium in general relativity must have radii in excess of 9/8 of the Schwarzschild radius, was established by Karl Schwarzschild in February 1916 in his second paper devoted to general relativity and published just three months before he died.

This instability of relativistic origin has never been directly observed. Yet, one can be so confident of the predicted instability that we can incorporate it along with other more conventionally established facts in our attempts to understand astrophysical phenomena. Let me give an example.

As I have said, instability of relativistic origin will set in whenever the ratio of the specific heats is close to four thirds. This is the case for degenerate configuretions near the limiting mass; and the application of the relativistic criterion shows that they become dynamically unstable before the limiting mass is reached. Precisely what happens is the following. On the Newtonian theory, it can be shown that the period of radial pulsation decreases monotonically to zero as we approach the limiting mass; but in the framework of general relativity, because of the instability it causes, the period attains a minimum, just prior to the limiting mass and before the sequence becomes unstable. In other words, while general relativity does not modify to any appreciable extent the structure derived from the Newtonian theory, it changes qualitatively the period mass relation; it exhibits a minimum period that was absent in the Newtonian theory. This minimum period is about seven-tenths of a second. Since pulsars are known to have periods much shorter than this minimum value, the possibility of their being white dwarf configurations was ruled out; and this was a deciding factor in our concluding that pulsars are neutron stars.

I should add that instabilities of relativistic origin occur also in stars clusters; and it is clear that these theoretically predicted instabilities must be included in any discussion pertaining to the evolution of large agglomerations of mass, be they individual stars, clusters of stars, or clouds of gas.

I should next like to consider a second example in the same genre.

It is, I believe, a matter of common knowledge, that at some point along the Maclaurin sequence of rotating homogeneous oblate spheroidal masses, the triaxial Jacobian sequence branches off. And as Kelvin pointed out, already in 1883, the Maclaurin spheroid, while not dynamically unstable, is nevertheless secularly unstable at the point of bifurcation in the sense that any dissipative mechanism that may be operative will induce an instability and propel it along the Jacobian sequence. Thus if the rotating mass is viscous, then the e-folding time of the secular instability varies inversely as the coefficient of viscosity so that it becomes infinite in the limit of zero viscosity. While all these are relatively well known, it was not generally known, until recently, that at the point of bifurcation of the Maclaurin sequence, there are, in fact, two alternative sequences along which evolution may proceed: besides the Jacobin sequence, there is a second congruent Dedekind sequence. These ellipsoids of Dedekind, unlike the ellipsoids of Jacobi, are stationary in the inertial frame and owe their triaxial figures to internal vortical motions. Dedekind discovered these ellipsoids in 1860; but they were forgotten and ignored, along with a beautiful theorem which he discovered in this context, for more than a hundred years.

The relevance of the Dedekind sequence for astronomy emerged only recently in the context of general relativity. In general relativity, a dissipative mechanism is built into the theory in the sense that a non-axisymmetric perturbation which induces a variable quadrupole, or higher, moment will dissipate energy and angular momentum as a result of radiation-reaction and emission of gravitational radiation. On examination, it was found that this source of dissipation will induce a secular instability of the Maclaurin spheroid at the point of bifurcation and propel it along the Dedekind sequence. This was an unexpected result. But a far greater surprise was yet to come. In a paper of remarkable power and insight, John Friedman has proved that all rotating objects, I mean, *all* rotating objects, are unstable in the framework of general relativity by virtue of the same radiation-reaction. In my judgement, next to the discovery of Kerr's solution, Friedman's theorem is of the most far-reaching significance that has been proved in general relativity in the realm of astronomy.

To emphasise the generality of Friedman's theorem, let me only point out that our rotating earth is unstable in the framework of general relativity. The reason why, in spite of its instability, the earth has endured over a billion years is simply that the *e*-folding time of the relativistic instability is to be measured in billions of billions of years. But the security we on the earth have enjoyed cannot be shared by objects which during a process of gravitational collapse, or some other cause, attain radii of a few times their Schwarzschild radii. Clearly the instability predicted by Friedman must be taken into account in all discussions pertaining to gravitational collapse or the early formation of galaxies and stars.

In my attempt to clarify my views on the prospective role of general relativity in astronomy, I have left myself no time to discuss questions in the domain of general relativity which are currently actively being pursued. I am referring in particular to the continuing efforts, both theoretical and experimental, concerned with the detection of gravitational waves from astronomical sources. These efforts are not only in the building of detectors of greater sensitivity, but also in estimating the wavelengths and the amounts of gravitational radiation which we may expect from likely astronomical sources. And one dreams that eventually we may be able to detect a universal background of gravitational radiation similar to the universal background of microwave radiation. These are all specific questions which require specific answers. I have not dealt with them since there are others in the audience who can address themselves to these questions with a competence which I do not have. I have chosen instead to address myself to the nature of the larger role which general relativity may play in astronomy.

I began this discourse by stating that the real home of general relativity is astronomy. May I conclude by suggesting the likelihood that, in time, some of the fundamental aspects of astronomy will find their natural home in the general theory of relativity.