The present study investigates two important though relatively unexplored aspects of non-linear filtration through porous media. The first aspect is the influence of viscosity variation over the coefficients of the governing equations used for modelling non-linear filtration through porous media. Velocity and hydraulic gradient data obtained for a wide range of fluid viscosities (8.03E-07 to 3.72E-05 N/m²) were studied. An increase in fluid viscosity resulted in an increased pressure loss through packing which can be quantified using the coefficients of the governing equations. Coefficients of Forchheimer equation represent linearly increasing trend with the kinematic viscosity. On the other hand, coefficient of Wilkins equation represents similar values for different fluid viscosities and remained unaffected by the variation in packing properties. Obtained data were utilized to understand the nature of flow transition in porous media. Behaviour of polynomial and Power-law coefficient with variation in flow velocity were also examined. Critical Reynolds number corresponding to the deviation of flow from Darcy regime varies with the porous packing and was observed to be in the range of 0–100. Coefficients of polynomial (Forchheimer) model were observed to be independent of the range of flow velocity, whereas the Power law coefficients are extremely sensitive to the data.

**Keywords.** Post-laminar flow; porous media; viscosity variation; flow transition; Reynolds number.

1. Introduction

Flow-through porous packing can be characterized by two primary parameters, i.e., the pressure drop driving the flow and the volumetric flux resulting due to that pressure drop or vice versa. The accurate estimation of pressure drop and flow rate in porous media flow extends a great amount of significance in several natural and industrial aspects including modelling of groundwater hydrology (Ma et al. 2020), oil and gas exploration (Li and Chen 2020), design of tunnel drainage system (Zhang et al. 2020), single or multiphase flows through packed column (Kundu et al. 2019), etc. Darcy’s linear model is the most widely used approach for the modelling of flow through porous
packing. However, several experimental evidences through multiple porous packing suggest the development of non-Darcy flow condition (non-linear filtration) in higher flow velocities, where the Darcy’s linear model is not sufficient to describe the flow. It can be considered that such situations occur when the inertial drag exceeds 1% of the total pressure loss; resulting a non-linear behaviour between the pressure gradient and the superficial velocity (El-Zehairy et al. 2019). However, two of the very fundamental questions regarding the non-linear filtration through porous packing remains largely unanswered till date.

The first aspect of non-linear filtration through porous media is the unavailability of a universal flow model to describe the phenomena. The existing models for non-linear filtration are either of polynomial or Power-law form (Bear 1972). Pressure drop in polynomial model is proportional to the summation of two terms; one term includes the fluid velocity and represents the force exerted to overcome fluid viscosity, whilst the other term includes the squared value of fluid velocity and represents the force exerted to overcome fluid–medium interactions (Burcharth and Christensen 1991; Mathias and Todman 2010; Chen et al. 2015; Nezhad et al. 2019). The Forchheimer model (equation 1) is the simplest representation of this type of model (Moutsopoulos et al. 2009; Sedghi-Asl et al. 2014; Salahi et al. 2015; Li et al. 2017; van Lopik et al. 2017)

\[ i = av + bv^2, \]  

where \( a \) and \( b \) are the Darcy (s/m) and non-Darcy coefficients (s²/m²). Applicability of this equation depends on the modelling accuracy of the Darcy and non-Darcy coefficients. Extensive experimental studies have been reported in the literature on the behaviour of these coefficients with different parameters such as media size (Huang et al. 2013; Salahi et al. 2015; van Lopik et al. 2017), degree of packing (Boomsma and Poulikakos 2002; Dan et al. 2016; Houben et al. 2018), grain size distribution (van Lopik et al. 2020), convergent angle (Thiruvengadam and Kumar 1997; Venkataraman and Rao 2000; Reddy and Rao 2004, 2006; Pasupuleti et al. 2014; Banerjee and Pasupuleti 2019), pore geometry (Macini et al. 2011; Shao et al. 2020) Reynolds number (Sidiropoulou et al. 2007), maximum pore diameter (Huang et al. 2020), etc. Based on these observations, several models have been reported in the literature to estimate the values of these coefficients (Ergun 1952; Kovacs 1971; Comiti and Renaud 1989; Abbood 2009). The interior structure of porous packing is extremely complex in nature since the size of the pores differs continuously, and the flow channels are tortuous in nature. Therefore, flow inside porous packing is randomly converged and diverged, resulting in different values of these coefficients for different packing (Yu and Li 2001).

The Power-law model can be presented as (Basak 1976; Sen 1989) equation (2), whereas equation (3) is a modified form of equation (2) known as Wilkins equation.

\[ i = mv^j \]  

\[ v = C\mu^x r^y \bar{v}. \]  

The coefficient \( m \) in equation (2) accounts for all the possible influences due to packing and fluid properties. An effort was made in equation (3) to present this coefficient as a function of hydraulic radius (defined as a ratio of void fraction to the surface area per unit volume) and fluid viscosity with \( \beta \) and \( \gamma \) as their coefficients. The index \( j \) or \( y \) is considered to be a representation of the flow regime (Shi et al. 2020). The values of these exponents have significant impacts on the flow rate as well as pore pressure (Zhang et al. 2020). Reported experimental results in the literature (assuming the fluid viscosity constant) suggest a relatively non-deviating nature of the Wilkins coefficients with varying media sizes and porosities (Wilkins 1955; Garga et al. 1990; Kumar and Venkataraman 1995; Banerjee et al. 2018a, b). However, the mathematical model developed by Banerjee et al. (2019) suggests that these results depend on the flow regime.

It is very much evident from the presented discussion that understanding the behaviour of the coefficients of these models is the most crucial aspect of describing the non-linear filtration through porous media. The coefficients of these equations are essentially a function of the packing and fluid properties. Several reported studies in the literature attempted to address the behaviour of these coefficients with number of packing properties including size of the media, shape of the media, convergent angle of the flow, porosity of the packing, etc. (as presented in the previous section). However, as per the best of author’s knowledge, no experimental study has been reported in the literature, which attempts to model the behaviour of
these coefficients with the variation in fluid properties. Most of the studies reported in the literature were performed with same fluid at room temperature, thus considering the fluid viscosity as a constant parameter. The authors have identified the following reasons for the lack of effort associated with viscosity variation:

1. It is difficult and quite challenging to conduct large-scale high-velocity flow experimentation with highly viscous fluids.

2. In the case of highly viscous fluids, the velocity of flow is very low; moreover, the flow can be easily modelled using Darcy’s linear equation.

However, it is needless to mention that the fluid properties have significant impact on the pressure drop characteristics in porous media. Therefore, any model representing the non-linear flow behaviour cannot have satisfactory performance without considering the influence of fluid properties. A comprehensive model to describe the non-linear filtration through porous media should account for all the variation in packing properties as well as fluid properties. In the present study, an effort has been made to quantify the influence of viscosity variation over non-linear pressure drop in porous media. Influence of fluid viscosity over the coefficients of Forchheimer and Wilkins coefficients has been studied.

The second unanswered aspect of non-linear filtration through porous media is the identification of the onset of non-linear filtration. Based on the nature of resistive forces, gravitational flow through any saturated porous media can be primarily divided into three macroscopic flow regimes, i.e., laminar, transition and turbulent (Bdeh and Rashid 2020). It has been largely accepted that Darcy’s linear model is valid for flow in laminar regime only (Arthur 2018). However, flow transition from Darcy to non-Darcy regime in porous media is extremely difficult to identify (Burchart and Christensen 1991) primarily due to the need of expansive and sophisticated instrumentation and unavailability of a universal dimensionless number. Several definitions of dimensionless numbers such as Reynolds number and Forchheimer Number (Garrouch and Ali 2001; Zhang et al. 2018) have been used for the identification of the flow transition. However, the complexities associated with the determination of characteristic length and velocity have created a significant discrepancy in the definitions and limiting values of these numbers (Chhabra et al. 2001). Several definitions of characteristic length have been used in the literature, including the diameter of the media (Horton and Pokrajac 2009; Bu et al. 2014, 2015; Tang et al. 2020), hydraulic diameter (Takatsu and Masuoka 2005), pore diameter (Seguin et al. 1998a, b; Wood et al. 2020), pore throat radius (Thauvin and Mohanty 1998), permeability (Kundu et al. 2016), etc. Due to the discrepancies related to the definition and limiting values of Reynolds number, some researchers supported the applicability of Forchheimer number (for definition, please refer to Zhang et al. 2018) instead (Li and Engler 2001). However, similar to Reynolds number, its critical values reported in the literature (Zeng and Grigg 2006; Macini et al. 2011; Ghane et al. 2014) are found to be different for different packing. Therefore, according to Huang and Ayoub (2008), the Forchheimer number is just another representation of the Reynolds number and does not provide any better alternative (Huang and Ayoub 2008). The discussion presented in this section clearly suggests that there is still a deficiency of clear understanding on the precise nature of flow transition from Darcy to non-Darcy flow which is a prerequisite for flow modelling through porous media. The exponent of the Power-law model has been widely accepted as a representation of the flow transition in porous media. Therefore, an effort has been made in the present study to evaluate the applicability of Power-law exponent for the identification of flow transition in porous media. Addressing the issues raised in the present manuscript is a prerequisite for the calculation of pressure drop or volumetric flow rate (m³/s) in order to plan and construct any earthen and rock structures (Hansen 1992; Siddiqua et al. 2011; Ferdos et al. 2015). Furthermore, understanding of flow through porous media can be useful in solving a wide range of applications related to the estimation of discharge from aquifers (Fang and Zhu 2018; Houben et al. 2018), oil and gas exploration (Holditch and Morse 1976; Vincent et al. 1999), problems associated with packed and fluidized bed reactors (Mandal et al. 2013; Mandal 2015), flow through fractured medium (Wu 2002; Wen et al. 2006), flow through geotextiles and geomembranes (Lacey 2016), subsea tunnelling (Lai et al. 2018, 2019; Zhang et al. 2020), etc.

2. Methodology

The outcomes of the study were derived after analyzing the experimental observation from the present study and the results reported by Cheng et al. (2008). Experimentations in the present study were carried with a specially fabricated parallel flow permeameter (figure 1).
The permeameter was packed with 33.42 mm diameter regular-shaped glass spheres (Banerjee et al. 2018a). Experiments were conducted with plain water, water mixed with 10% and 20% glycerine (by weight) to model the effect of viscosity variation in non-Darcy flow through porous media. The viscosities of the fluids were measured using a viscometer. The experiments were performed at room temperature.

Extreme care has been taken during the study to ensure that any variation occurring to the coefficients of the models is due to the variation in fluid viscosity only. The complete experimentation was performed through constant packing. The media (glass sphere of 33.42 mm) was packed in the parallel flow permeameter. To avoid any reorientation in packing during experimentation, water was allowed to flow with the maximum possible volumetric flow rate for almost 2 hrs before taking the readings. The porosity of the packing was then measured by filling the entire permeameter with a known volume of water and dividing this volume with the total volume of the permeameter. The packing was kept unchanged over the entire course of the experimentation to ensure that all the variation is entirely due to the variation of fluids viscosity. For every mixture, pressure drop readings were taken for various volumetric flow rates. Around 9–11 readings were taken for each mixture. After each round of experimentation, the packing was washed with fresh water and completely dried before starting experimentation with another mixture.

Pressure tapings drilled along the length of the permeameter were connected to a manometer board which is used for the measurement of pressure difference along the length of flow. The readings were taken when a steady-state condition was achieved, which was identified by the constant fluid head in the overhead tank attached to the permeameter. A centrifugal pump at the base of the set-up was used for the circulation of fluid at a constant rate from the sump tank through the test section back to the sump tank. Flow rate was measured using volumetric method with a measuring tank attached to the experimental setup. Detailed methodology followed for the data collection is reported in Banerjee et al. (2018a).

The velocity and hydraulic gradient data reported by Cheng et al. (2008) were also analyzed to support the conclusions made in the study. Description of the experimental setup and the methodology for the results can be found in Cheng et al. (2008). The combined data allowed the authors to analyze post-laminar velocity and pressure drop characteristics over a wide range of kinematic viscosity (8.03E-07 to 3.72E-05 N/m²). Since the packing in both experiments were kept unchanged, the variations observed in the pressure drop can only be attributed to the fluid properties.

3. Results and analysis

Velocity and hydraulic gradient observed from the experimental setup for different fluid viscosities are presented in figure 2(a). Similarly, velocity and
hydraulic gradient reported by Cheng et al. (2008) are presented in figure 2(b). For a given flow velocity, pressure drop through a constant packing represents a proportionally increasing trend with an increase in fluid viscosity. As discussed in the introduction section, pressure drop in porous media can be represented either by Forchheimer or Wilkins type equations. Therefore, modelling the effect of fluid viscosity over Forchheimer and Wilkins coefficients is a pre-requisite in order to quantify its influence over the pressure drop characteristics in porous media.

3.1 Effect of fluid viscosity on the Forchheimer coefficients

Significant variation in fluid viscosity can result in a considerable difference in the pressure loss and flow velocity through porous media. Therefore, understanding and quantification of the effect of fluid viscosity is a pre-requisite for efficient modelling of non-linear porous media flow. However, efforts made in the literature to understand the effect of fluid viscosity are not very significant. The challenges associated with achieving a significant variation in fluid viscosity can be the reason for that. An attempt was made to understand the effect of fluid viscosity over the Forchheimer coefficients.

The Darcy and non-Darcy coefficients of the Forchheimer equation were calculated after fitting polynomial trend to the observed dataset (figure 2) using least square error method. Both the coefficients represent a clear increasing trend with increase in the kinematic viscosity of the fluid (figure 3a, b). The empirical relation obtained between the Darcy coefficient and the kinematic viscosity is presented as equation (4).

\[
a = 38018v + 0.040 \quad \text{with} \quad R^2 = 0.986 \quad (4)
\]

\[
b = 3 \times 10^7v + 11.54 \quad \text{with} \quad R^2 = 0.996. \quad (5)
\]

Similarly, from the results reported by Cheng et al. (2008), the Darcy and non-Darcy coefficients can be presented as functions of kinematic viscosity of the fluid in equations (6 and 7) respectively:

\[
a = 482105v + 0.0637 \quad \text{with} \quad R^2 = 0.999 \quad (6)
\]

\[
b = 3 \times 10^6v + 19.47 \quad \text{with} \quad R^2 = 0.999. \quad (7)
\]

In equations (4–7), \(a\) (m/s) and \(b\) (s²/m²) are the Darcy and non-Darcy coefficients; \(v\) is the kinematic viscosity (m²/s) of the fluid. The equations relating the Forchheimer coefficients with the kinematic viscosity are observed to be slightly different in the present study from the results obtained by Cheng et al. (2008). Such deviation may be attributed to the difference in the properties related to the media and packing such as the size of the media, porosity of the packing, surface of the media, packing geometry, etc. An empirical model addressing the deviation in the Forchheimer coefficients for different packing has been reported in Banerjee et al. (2018a). Following a similar approach, the Darcy and non-Darcy coefficients were normalized and plotted in figure 4(a, b) against the variation in kinematic viscosity. Due to the reduction in the uncertainties due to packing properties, the normalized values were observed to follow a much more predictable variation trend with \(R^2\) values 0.997 and 0.988 corresponding to Darcy and non-Darcy coefficients. Following the

Figure 2. Variation of hydraulic gradient with velocity of flow obtained from (a) present study and (b) reported by Cheng et al. (2008).
trends observed in Figure 4(a), a universal relation (equation 8) can be proposed relating the Darcy coefficients, hydraulic radius of the packing and kinematic viscosity of the fluid.

\[ a r^{-1.32} = 1.21 \times 10^9 v . \]  

(8)

The standard error of the coefficient was obtained as \(2.41 \times 10^7\); which is 0.019% of the proposed value. Similarly, following the trend observed in Figure 4(b), a relation (equation 10) can be obtained between the non-Darcy coefficient \(b\), kinematic viscosity and hydraulic radius of the packing.

\[ b r^{-1.15} = 6.68 \times 10^9 v + 38543.7 . \]  

(9)

The standard error for the slope and intercept in equation (9) was obtained as \(2.97 \times 10^8\) and 4402.21, which are 0.044% and 0.114% of their proposed values, respectively.

Equations (8 and 9) can be further modified as equations (10 and 11) to represent in terms of Reynolds number \(Re_r\), with hydraulic radius as the characteristic length.

\[ a \times Re_r = (1.21 \times 10^9) v r^{2.32} \]  

(10)

\[ b \times Re_r = (6.68 \times 10^9 v + 38543.7) \frac{vr^{2.15}}{v} \]  

(11)

After incorporating the values of Darcy and non-Darcy coefficients from equations (8 and 9) in the Forchheimer model (equation 1), the modified Forchheimer model (equation 12) can be presented as:

\[ i = (1.21 \times 10^9 v r^{-1.32}) v + (6.68 \times 10^9 v + 38543.7) r^{1.15} v^2 . \]  

(12)

The equations (10 and 11) can be further explored to find out the values of critical Reynolds number to
identify the flow regimes and flow transition. The modified form of Forchheimer equation (equation 12) can be used universally to calculate the pressure drop for a given volumetric flux.

3.2 Effect of fluid viscosity on the Wilkins coefficients

The original Wilkins equation (Wilkins 1955) represents the effect of dynamic viscosity over the velocity and hydraulic gradient relation for flow through porous media. The Wilkins equation for flow through a packing with constant media size and porosity (constant value of hydraulic radius and coefficient \( \beta \)) can be presented as:

\[
v = C_1 \mu^\gamma \dot{\nu},
\]

where \( C_1 = Cr^\beta \); the coefficient \( C_1 \) represents the Wilkins coefficient for a constant hydraulic radius and \( \gamma \) is the coefficient quantifying the effect of dynamic viscosity.

The effect of fluid viscosity on the Wilkins equation has never been thoroughly examined to the best knowledge of the authors. Therefore, the value of coefficient \( \gamma \) is not clear from the reported literature. The present section makes an effort to understand the nature of the coefficient \( \gamma \) of the Wilkins equation. The obtained results from the present study are compared with the reported results by Cheng et al. (2008).

Comparing the two similar forms presented as equation (2) with equation (13), we can conclude that

\[
\left( \frac{1}{m} \right)^{(2)} = C_1 \mu^\gamma.
\]

Equation (14) in its logarithmic form can be written as:

\[
-\left( \frac{1}{j} \right) \log m = \log C_1 + \gamma \log \mu.
\]

Equation (15) is plotted in figure 4(a, b) with \( \log \mu \) in the ‘\( x \)’ axis and \( -\left( \frac{1}{j} \right) \log m \) in the ‘\( y \)’ axis. Values of \( m \) and \( j \) are obtained after fitting a Power law equation to the velocity and hydraulic gradient plots presented in figure 2. Slope of the linear fit in figure 5 represents the value of coefficient \( \gamma \) (equation 15) which was calculated as 0.311 and 0.368 from the present study and Cheng et al. (2008), respectively. The deviation in the values of coefficient can be attributed to the difference in the pressure drop due to the irregularities in the packing. Therefore, the results obtained in the present study and Cheng et al. (2008) were combined and presented in figure 6. The \( R^2 \) value of the fit was observed to be 0.81 and the value of \( \gamma \) was achieved as 0.3 with a standard error value of 0.0585.

Similar values of coefficient \( \gamma \) obtained from both the studies point towards the applicability and validity of Wilkins equation to predict the post-laminar flow for fluids with different viscosities. The obtained results also suggest that a Wilkins type equation can be very useful to analyze and understand the flow through different hydraulic structures and natural formation with fluids of different viscosities.

3.3 Influence of test velocity range over experimental outcomes

Observation of the pressure drop for gradual variation in volumetric flux through a packing has been the traditional experimental approach for macroscopic porous media flow modelling. The range of volumetric flux or velocity considered for such studies, however, differs in the reported literature. The difference in the velocity range has never been considered to have any significant influence over the macroscopic flow models or their coefficients. Some studies, however, have reported the significance of velocity range over the proposed models (Ovalle-Villamil and Sasanakul 2018; Banerjee et al. 2019). But in large, its influence over the macroscopic models has been overlooked.

The exponent \( j \) in the Izbash equation has been traditionally considered as a representation of the flow regime. The transition of \( j = 1 \) to 2 signifies the transition of flow from laminar to turbulent regime. However, variation of this parameter with traditional dimensionless numbers for identification of flow regime is still largely undefined. The experimental data reported by Cheng et al. (2008), Thiruvengadam (2010) and Jayachandra (2006) (figure 7) present the authors with an excellent opportunity to study the flow transition in terms of \( j \) over a wide range of Reynolds number. Kovacs (1971) definition of Reynolds number (equation 16) was used in the study since it accounts for nearly all the parameters (particle size, particle shape, surface area, porosity of the packing) influencing the characteristic length and velocity through porous media.
Re = $\frac{Vd_k}{v}$, \hspace{1cm} (16) 

where $v$ is the kinematic viscosity ($m^2/s$), $d_k$ is the characteristic length defined by Kovacs as $\frac{4d}{(1-f)\pi}$, with $d$ as the particle diameter, $f$ as the porosity of the packing, and $\pi$ is the shape factor of the particle.

Increase in flow velocity through porous packing results into an increase in the value of exponent $j$ (figure 8). Variation trend observed from figure 8 is similar for all the packing under consideration. A sharp increasing trend in the value of $j$ was observed up to range of Reynolds number 1000 (approx). Referring to equation (2), such increase in the value of $j$ points to a sharp rise in the head loss. The empirical relationships observed between coefficient $j$ and the Reynolds numbers for different packing are presented in table 1. From the observed empirical relations, the values of critical Reynolds number corresponding to transition of flow from laminar regime ($j = 1$) was extrapolated. For most of the packing, the critical Reynolds number was observed to be in the range between 0 and 10 (same as mentioned by Kovacs 1971). However, the observations suggest that the Reynolds number corresponding to the flow transition from laminar regime in any random packing is purely packing specific. Therefore, it is extremely difficult to define a fixed value of Reynolds number to identify the flow transition in porous media. For, some packing (16 mm with 50% glycerine, 15.41 mm, and 28.37 mm) the critical value of Reynolds number for non-linear filtration was observed to be significantly larger. In all these cases the existence of a
It is widely accepted that the transition in the flow regime (from laminar to turbulent) takes place due to increase in inertial resistance in porous packing. The theoretical model presented by Banerjee et al. (2019) concluded that both viscous and inertial resistance increases as the flow shifts from laminar to turbulent regime. The observation can be experimentally validated from the outcomes presented in figures 9 and 10. The presented figures show clear evidence of the rising pattern of viscous and inertial resistance towards turbulent regime. However, the transition in flow regime does not have any significant impact on the behaviour of the Forchheimer coefficients (figures 11 and 12). The variation pattern of Darcy coefficient in figure 11(a) represents a decreasing trend followed by an increasing trend over the complete range of \( j \). The reason for such behaviour is not clear from the study. However, based on the observation from high viscosity fluid (figure 11b), it seems that the fluctuations are due to the error in determining the value of Darcy coefficient using the best-fit method. The non-Darcy coefficient which represents the inertial influence in porous media was observed to have more or less constant value over the complete range of \( j \) for a given packing (figure 12). The initial irregularities in the figure can be attributed to the error associated with the estimation of this coefficient using the best-fit method due to lesser amount of data points. Finally, from the observations, it can be concluded that the Darcy and non-Darcy coefficients are mostly a function of the packing and fluid properties more than the Reynolds number or flow velocity.

To understand whether the different ranges of velocity used in the study have any influence over the behaviour of the coefficients, the analysis was repeated after selecting a constant range of velocity (0.0045–0.106334 m/s) from the reported data. Similar relationship between \( j \) and Reynolds number is observed from the analysis (figure 13). The empirical equation observed from the plots and their corresponding critical Reynolds numbers are presented in table 2. Significant deviation can be observed in the extrapolated values of critical Reynolds number when the range of velocity considered for the study is altered. Similarly, variation in Darcy and non-Darcy coefficients with flow regime (value of \( j \)) were studied for a constant velocity range. The observed variation trends from the study are presented in figure 14. Slight deviation in the values of Darcy and

pre-Darcy regime was observed (where value of \( j < 1 \)). Further research needs to be carried out to understand the reason behind such deviation in the values of critical Reynolds number.

Figure 8. Variation of \( j \) with Reynolds number (Kovacs 1971) (a) over the complete range of study, (b) enlarged section up to \( \text{Re}_K =1000 \), and (c) for water mixed with 70% and 80% glycerine.
non-Darcy coefficients was observed when the range of velocity was altered, which was not so significant. Based on the observations, it can be concluded that the flow velocity range is extremely significant while analyzing non-linear filtration using Power-law models (Izbash or Wilkins

<table>
<thead>
<tr>
<th>Media size (m)</th>
<th>Porosity</th>
<th>Empirical equation ($j = ax^b$)</th>
<th>Coefficient ($a$)</th>
<th>Exponent ($b$)</th>
<th>$Re_{critical}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0167</td>
<td>0.59</td>
<td>$j = 0.835Re^{0.098}$; $R^2 = 0.97$</td>
<td>0.018</td>
<td>0.003</td>
<td>6.36</td>
</tr>
<tr>
<td>0.02</td>
<td>0.56</td>
<td>$j = 0.947Re^{0.084}$; $R^2 = 0.80$</td>
<td>0.041</td>
<td>0.006</td>
<td>1.92</td>
</tr>
<tr>
<td>0.035</td>
<td>0.54</td>
<td>$j = 0.984Re^{0.073}$; $R^2 = 0.87$</td>
<td>0.041</td>
<td>0.005</td>
<td>1.25</td>
</tr>
<tr>
<td>0.01541</td>
<td>0.42</td>
<td>$j = 0.495Re^{0.174}$; $R^2 = 0.98$</td>
<td>0.017</td>
<td>0.005</td>
<td>56.59</td>
</tr>
<tr>
<td>0.01803</td>
<td>0.42</td>
<td>$j = 1.086Re^{0.069}$; $R^2 = 0.94$</td>
<td>0.016</td>
<td>0.002</td>
<td>0.30</td>
</tr>
<tr>
<td>0.02837</td>
<td>0.425</td>
<td>$j = 0.578Re^{0.129}$; $R^2 = 0.92$</td>
<td>0.019</td>
<td>0.129</td>
<td>69.02</td>
</tr>
<tr>
<td>0.016 (0% glycerine)</td>
<td>0.50</td>
<td>$j = 0.843Re^{0.089}$; $R^2 = 0.99$</td>
<td>0.018</td>
<td>0.001</td>
<td>6.83</td>
</tr>
<tr>
<td>0.016 (50% glycerine)</td>
<td>0.50</td>
<td>$j = 0.513Re^{0.161}$; $R^2 = 0.99$</td>
<td>0.008</td>
<td>0.003</td>
<td>63.64</td>
</tr>
<tr>
<td>0.016 (70% glycerine)</td>
<td>0.50</td>
<td>$j = 0.871Re^{0.072}$; $R^2 = 0.97$</td>
<td>0.007</td>
<td>0.072</td>
<td>6.84</td>
</tr>
<tr>
<td>0.016 (80% glycerine)</td>
<td>0.50</td>
<td>$j = 0.912Re^{0.065}$; $R^2 = 0.93$</td>
<td>0.006</td>
<td>0.003</td>
<td>4.16</td>
</tr>
</tbody>
</table>

Figure 9. Variation of (a) viscous resistance and (b) inertial resistance with Reynolds number (Kovacs 1971) data from Thiruvengadam (2010) and Jayachandra (2006).

Figure 10. Variation of (a) viscous resistance and (b) inertial resistance with Reynolds number (Kovacs 1971) data from Cheng et al. (2008).
However, the velocity range and its alteration does not influence the Forchheimer coefficients which are primarily a function of packing and fluid properties.

4. Conclusions

An experimental investigation was carried out in the present study to quantify the effect of viscosity variation over the Forchheimer and Wilkins coefficients used for post-laminar flow modelling through porous media. The conclusions made from the study can be listed as follows:

(a) Experimentally obtained velocity and hydraulic gradient data suggest that for a constant velocity, increase in fluid viscosity results into an increase in head loss through porous packing.

(b) The Darcy and non-Darcy coefficients of the Forchheimer equation represent a linearly increasing variation trend with increase in the fluid viscosity. The Darcy (a) and non-Darcy coefficient (b) can be modelled as function of kinematic viscosity of the fluid and hydraulic radius of the packing as follows:

Figure 11. Variation of Darcy coefficient with $j$ for (a) glass sphere packing subjected to water and (b) higher viscosity fluid.

Figure 12. Variation of non-Darcy coefficient with $j$ for (a) glass sphere packing subjected to water and (b) higher viscosity fluid.

Figure 13. Variation of $j$ with Reynolds number (Kovacs 1971) over the selected range of velocity.
Expression of \( j^* \) in terms of Reynolds number observed from different packing and their corresponding critical Reynolds numbers for the selected velocity range.

<table>
<thead>
<tr>
<th>Media size (mm)</th>
<th>Porosity</th>
<th>Equation (( j = ax^b ))</th>
<th>Coefficient (( a ))</th>
<th>Exponent (( b ))</th>
<th>( \text{Re}_{\text{critical}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0167</td>
<td>0.59</td>
<td>( j = 1.029\text{Re}^{0.075}; R^2 = 0.91 )</td>
<td>0.033</td>
<td>0.005</td>
<td>0.68</td>
</tr>
<tr>
<td>0.02</td>
<td>0.56</td>
<td>( j = 1.201\text{Re}^{0.053}; R^2 = 0.88 )</td>
<td>0.024</td>
<td>0.003</td>
<td>0.03</td>
</tr>
<tr>
<td>0.035</td>
<td>0.54</td>
<td>( j = 0.829\text{Re}^{0.097}; R^2 = 0.91 )</td>
<td>0.045</td>
<td>0.007</td>
<td>6.87</td>
</tr>
<tr>
<td>0.01541</td>
<td>0.418</td>
<td>( j = 1.241\text{Re}^{0.056}; R^2 = 0.85 )</td>
<td>0.029</td>
<td>0.004</td>
<td>0.02</td>
</tr>
<tr>
<td>0.01803</td>
<td>0.419</td>
<td>( j = 1.239\text{Re}^{0.056}; R^2 = 0.82 )</td>
<td>0.021</td>
<td>0.003</td>
<td>0.02</td>
</tr>
<tr>
<td>0.02837</td>
<td>0.425</td>
<td>( j = 0.962\text{Re}^{0.067}; R^2 = 0.97 )</td>
<td>0.017</td>
<td>0.003</td>
<td>1.56</td>
</tr>
</tbody>
</table>

\[ ar^{-1.32} = 1.21 \times 10^9v \quad \text{with} \ R^2 = 0.997. \]

\[ br^{-1.15} = 6.68 \times 10^9v + 38543.7 \]

\[ \text{with} \ R^2 = 0.988. \]

A modified form of Forchheimer equation was proposed after incorporating the values of Darcy and non-Darcy coefficients. This equation is universally applicable and can be used to model the pressure drop and volumetric flux relationship for different media, packing and fluid properties.

The value of exponent \( x \) of the Wilkins equation which accounts for the variation in fluid viscosity was obtained as 0.3.

Exponent \( j \) of Izbash equation and coefficient \( \gamma \) of the Wilkins equation is a function of the flow velocity or Reynolds number. However, the equation relating both the parameters was observed to be different for different packing. The range of velocity used in the study was also an influencing factor to the values of coefficient \( j \) or in that matter coefficient \( \gamma \).

Transition of flow from linear to non-linear filtration is a gradual process and packing specific. Therefore, identification of a specific value for such transition is extremely difficult and can lead to inaccurate modelling. Specifying a range of the Reynolds numbers for the identification of flow transition is much more realistic.

The coefficients of Power-law models are extremely sensitive to the upper and lower limits of the experimented velocity range. However, alternation in the range of experimental data did not have any significant impact on the coefficients of the polynomial models since they are a function of packing and fluid properties.

Finally, the outcomes from the present study quantify the previously unexplored influence of fluid viscosity over the pressure loss characteristics in porous media. The Power-law models are extremely sensitive to the data. Therefore, the range of data should be clearly mentioned while characterizing the non-linear filtration using...
Power-law models. However, it is not necessary in case of polynomial models, since the model coefficients are independent of properties of the experimental data. Still, adequate precautions had to be used while utilizing the polynomial models since they have an extremely complex variation pattern with the packing properties. In such case, the characteristic length should be carefully selected and clearly defined.

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Author statement

The experimentation and data analysis has been performed by Ashes Banerjee. The experimental setup was designed by Srinivas Pasupuleti and Mritunjay Kumar Singh. The manuscript has been written by Srinivas Pasupuleti. Dandu Jagan Mohan was involved in the data analysis process.

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