



# Stability of vegetation growth in a noise-induced system

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MS received 25 March 2020; revised 16 September 2020; accepted 17 September 2020

Vegetation community plays a pivotal role in various geomorphic processes. The growth of vegetation intrinsically depends on the effective shear stresses exerted by the flow of material (e.g., water or soil) along the slope. We comparatively assess the growth and decay of vegetation using linear and logistic growth model coupled with a runoff erosion model. The linear growth model predicts a sharp decrease in the non-dimensional vegetation profile from the upper reach to the lower part along the slope. However, the logistic growth model delivers a smooth gradual decrease in the vegetation extent. Additionally, we propose a stochastic model to capture the role of internal and external factors in the dynamics of vegetation growth using two Gaussian noises. The steady probability distribution functions from the stochastic model provide insights about the role of different noises on the reaction of the system and suggest that bio-environmental factors are difficult to segregate from one another.

**Keywords.** Vegetation; logistic growth model; shear stress; Gaussian noise; steady probability distribution; earth surface process.

## 1. Introduction

Vegetation community is efficient to enrich its condition through growth, decay and sustenance by virtue of the inherent physicochemical processes (Wilson and Agnew 1992). The spatio-temporal modulation in vegetation mass is greatly influenced by the coupled amalgamation of the fluvial hydrodynamic regime, hillslope configuration, climatic factors as well as soil cover that acts as a feedback mechanism to modify the geomorphic features on the earth's surface (Tucker and Bras 1999). In addition to this, the response of

vegetation to the environmental elements affecting the geomorphic variabilities is rather quite complex with inherent nonlinearities and stochasticity deep-rooted within the system.

The earliest vegetation growth model of forest cover system was elaborated by Botkin *et al.* (1972), where the environment was considered as carrying capacity limited. Subsequently, over the past few decades, there have been several significant contributions in exploring the vegetation growth models utilizing various schemes. These include linear models (Collins *et al.* 2004), exponential or logarithmic relationship between the

plant cover and biomass (Flanagan *et al.* 2007; Martinez *et al.* 2008) as well as predator-prey type models (Tanner 1975; Yoshida *et al.* 2003; Kallay and Cohen 2008). It is noteworthy to mention here that our present work draws its inspiration from the pioneering contributions by May (1976) and Meron (2015) on logistic models and nonlinear physics in ecological systems, respectively. Thornes (1985) initiated the pioneering step of introducing the concept of a coupled system for vegetation growth. This formulation involved the coupling of logistic growth of vegetation and slope dependent erosion model. The intricate details of the evolution of vegetation have been further explored using the CHILD numerical tool (Tucker *et al.* 2001) in various hydro-climatic conditions which captures certain complexities of the physical processes involved therein.

Precise deterministic modelling of many real-world phenomena are a difficult task owing to the fact that various variables and parameters of the system can behave randomly within a similar environment. Therefore, in several instances, it fails to incorporate this stochasticity of coupled biophysical systems. Noise-induced phenomena for vegetation growth and its resilience have been widely examined by various scholars in different hydro-climatic conditions. These studies include the feedback mechanism involved in between soil moisture (D'Odorico *et al.* 2005; Borgogno *et al.* 2007), water table (Ridolfi *et al.* 2006), stream flows (Camporeale and Ridolfi 2007), geomorphology (Muneepeerakul *et al.* 2007; Vesipa *et al.* 2015) and riparian vegetation.

In this paper, we are focusing on the vegetation dynamics along the fluvial system which includes the riparian vegetation, neglecting the inter-species interaction. Furthermore, we have implemented a steady-state stochastic model to analyse the bio-environmental stochasticity and their effect on the steady-state distribution of vegetation cover. Our results indicate that within a wide range of noise level, the steady distribution does not lead to a regime shift in the vegetation cover in the fluvial system.

## 2. Methods and solution scheme

### 2.1 Deterministic vegetation growth model

We follow a modelling scheme on similar lines of Tucker and Bras (1999). However, the present

formulation takes into account the vegetation proliferation as a logistic growth model that considers the growth of a particular vegetation species is dependent on the existing fractional cover of vegetation (Collins and Bras 2010).

#### 2.1.1 Logistic growth model

Our modelling scheme utilizes logistic vegetation growth (Collins *et al.* 2004) with a model of wash profile (Tucker and Bras 1999). Unlike the linear growth models, the logistic model captures both the aspects of reproduction limited as well as resource-limited condition (Thornes 1990). In such a scenario, the governing mathematical equation becomes,

$$\frac{dV_g}{dt} = K_{vg} V(1 - V), \quad (1)$$

$V_g$  is vegetation growth,  $K_{vg}$  is the rate of growth of vegetation on the unvegetated surface. Reciprocal of the vegetation regrowth rate implies the time taken by a plant community for regrowth.

In a natural system, the plant community are removed from the soil by various means. However, for our model formulation, we consider that the loss of vegetation is primarily by virtue of the channel and the riparian processes. The simplest physical process for removal of the vegetation cover will depend on the excess shear stress. This can be expressed as:

$$\frac{dV_e}{dt} = -K_{vd} V (R_f \tau - \tau_c)^n, \quad (2)$$

$V_e$  denotes the vegetation erosion,  $K_{vd}$  is the species-dependent erosion parameter,  $R_f$  is the factor of friction,  $\tau$  and  $\tau_c$  are the shear stress and the effective critical shear stress, respectively.

The effective critical shear stress ( $\tau_c$ ) is posed as a sum of the critical shear stress for pure unvegetated surface ( $\tau_{cs}$ ) and the critical shear stress under 100% vegetation cover ( $\tau_{cv}$ ).

$$\tau_c(V) = \tau_{cs} + V\tau_{cv}. \quad (3)$$

Combining the erosion and growth terms (equations 1 and 2), the governing equation yields the following form:

$$\begin{aligned} \frac{d(V_g - V_e)}{dt} &= \frac{dV}{dt} \\ &= K_{vg} V(1 - V) - K_{vd} V (R_f \tau - \tau_c)^n. \end{aligned} \quad (4)$$

### 2.1.2 Steady-state solutions

For simplicity purposes, we assume  $\eta = 1$ , i.e., the erosion law follows a linear function.  $\tau_{cs}$  is considered as zero as we have idealized that bare soil surface does not introduce any resistive shear stress. All the physical quantities, which have been taken into account to model the vegetation growth, have been converted to non-dimensional quantities for the ease of computation.

The final form of the non-dimensional steady-state equation for fractional vegetation cover (VCF) is

$$V - V^2\{1 + N_v K_{rv}(N_e^q x'^q + V)\} + N_v V^2 = 0, \quad (5)$$

$N_v$  is the vegetation number which describes the growth relative to destruction.  $K_{rv}$  signifies the friction coefficient,  $N_e$  is the erosion number that relates the shear stress with distance and  $q$  is the non-dimensional exponent that explains the non-linearity in the process involved.

The solution of equation (5) yields  $V = 0$  and the other two roots are

$$V = \frac{(N_v - N_v K_{rv} N_e^q x'^q) \pm \sqrt{(N_v - N_v K_{rv} N_e^q x'^q)^2 + 4N_v K_{rv}}}{2N_v K_{rv}}. \quad (6)$$

The first solution ( $V = 0$ ) corresponds to the specific condition, where there is no vegetation along the slope. The positive root among the other two roots has been considered for evaluation of the VCF for steady-state logistic growth model since there is no physical meaning of negative vegetation cover. The steady-state solution of linear vegetation growth model has also been evaluated for the calibration procedure. The solution is

$$V = \frac{1}{1 + N_v N_e^q x'^q} \quad \text{Tucker and Bras (1999)}. \quad (7)$$

## 2.2 Stochastic vegetation growth model

We consider two prominent sources of stochasticity in the evolution of vegetation. The inherent characteristics of the vegetation community have been coined as ‘*intrinsic*’ noise. On the other hand, the external factors, viz., inhomogeneity in precipitation amount, spatial variation of temperature, soil moisture retention capacity, ground-water table variability, aspect of slope, etc., apparently serve

as ‘*extrinsic*’ noise. In subsequent sections, we describe that the separation of the intrinsic and extrinsic noise is difficult owing to the fact of the complex interrelationship between the external and the internal factors and the system.

### 2.2.1 Formulation of stochastic model

The stochastic vegetation model with logistic growth is driven by two white Gaussian noises:  $\epsilon(t)$  and  $\Gamma(t)$  termed as additive (negative in magnitude) and multiplicative noise, respectively. One-dimensional Langevin equation with two correlated Gaussian white noises  $\epsilon(t)$  and  $\Gamma(t)$  with a non-zero correlation between the multiplicative and additive (negative in magnitude) noises leads to

$$\frac{dV}{dt} = V + C_1 V^2 + C_2 V^3 + V\epsilon(t) - \Gamma(t), \quad (8)$$

where

$$C_1 = N_v - N_v K_{rv} N_e^q x'^q - 1, \quad (9)$$

and

$$C_2 = -N_v K_{rv}. \quad (10)$$

The Gaussian noises have zero mean and are defined as:

$$\langle \epsilon(t)\epsilon(t') \rangle = 2D\delta(t - t'), \quad (11)$$

$$\langle \Gamma(t)\Gamma(t') \rangle = 2\alpha\delta(t - t'), \quad (12)$$

$$\langle \epsilon(t)\Gamma(t') \rangle = \langle \epsilon(t')\Gamma(t) \rangle = 2\lambda\sqrt{D\alpha}\delta(t - t'), \quad (13)$$

$\lambda$  denotes the degree of correlation between the noises  $\epsilon(t)$  and  $\Gamma(t)$ , while  $D$  and  $\alpha$  represent the strength of the noises  $\epsilon(t)$  and  $\Gamma(t)$ , respectively.

### 2.2.2 Steady-state analysis

We derive the Fokker–Planck equation (FPE) (Da-Jin *et al.* 1994; Ai *et al.* 2003; Li *et al.* 2015) for estimation of the steady-state of probability density function corresponding to equation (8) which is of the following form,

$$\frac{\partial P(V, t)}{\partial t} = \frac{\partial A(V)P(V, t)}{\partial V} + \frac{\partial^2 B(V)P(V, t)}{\partial V^2}, \quad (14)$$

where  $P(x, t)$  is the probability density and

$$A(V) = V + C_1 V^2 + C_2 V^3 + DV + \lambda\sqrt{D\alpha}, \quad (15)$$

$$B(V) = DV^2 + 2\lambda\sqrt{D\alpha}V + \alpha. \quad (16)$$

The stationary probability distribution of FPE is given by

$$P_{st}(V) = \frac{N}{B(V)} \exp \int_0^V \frac{A(V')}{B(V')} dV', \quad (17)$$

where  $N$  is a normalization constant. In addition, the extrema of  $P_{st}(V)$  obeys a general equation  $A(V) - \frac{dB(V)}{dV} = 0$ . It leads to

$$C_2 V^3 + C_1 V^2 + (1 - D)V - \lambda\sqrt{D\alpha} = 0. \quad (18)$$

If  $\lambda = 0$ , then there exists no correlation between the two types of noises. This shows that there is no such dependency on (negative) additive noise at the extrema position  $V = 0$  and  $V = \frac{-C_1 \pm \sqrt{C_1^2 - 4C_1(1-D)}}{2C_2}$  of the stationary probability distribution (SPD) of FPE for zero correlation ( $\lambda = 0$ ).

### 2.3 Modelling scheme and parameter estimates

It has been idealized in the present modelling scheme that the linear or logistic model for vegetation erosion holds true for the riparian vegetation (Dietrich *et al.* 2003). It is readily agreed that the  $N_v$  value cannot be negative. Additionally,  $N_v$  does not shoot up to high value as this parameter characterizes the ratio between the destruction and growth coefficient. Therefore, we simulated the non-dimensional vegetation cover considering  $N_v$  values in between 0.1 and 100 keeping the interval at 0.1. The erosion number  $N_e$  is primarily a function of the uniform rate of erosion ( $E$ ) and the coefficient of erosion ( $K$ ) (Tucker and Bras 1999). Considering homogeneity and constant critical shear stress along the slope as well as a uniform and constant erosion rate ( $E$ ), we have relaxed the effectiveness of erosion number,  $N_e$  and assumed that the value of  $N_e$  is 10. Further, it is worth mentioning that we do not provide a calibration of the model parameters, rather this study presents generalized modelling schemes to compare between the linear and logistic growth model.

The integral in equation (17) has been evaluated numerically, with the logistic growth model and varied noise parameters along with  $N_v$ . We have plotted the curves of the SPD after varying the value of one particular stochastic noise parameter among  $\lambda$ ,  $D$ ,  $\alpha$  and fixing the value of the other two

parameters. Since, in the deterministic model, the vegetation cover  $V$  is a function of the normalized position  $x'$ , therefore in the stochastic model, SPD has been considered as an implicit function of  $V$  and  $x'$ . We chose various  $N_v$  values in order to obtain the SPD. Although the higher  $N_v$  values resulted in more than half of the fractional vegetation cover destruction, we aimed to observe the effect of the different noise level in different models.

## 3. Results and discussion

### 3.1 Steady-state non-dimensional vegetation profiles and sensitivity of deterministic model parameters

Our findings imply that the linear model exhibits a rapid decline in the non-dimensional vegetation cover as the value of  $N_v$  increases. It is duly noted that for a small increase in  $N_v$  value (from 1 to 5), the non-dimensional vegetation cover declines  $\sim 30$ – $70\%$  within a non-dimensional distance of less than  $\sim 1\%$  of the total distance (figure 1). However, this rapid decline does not promote the absolute removal of vegetation along the stream. It is noteworthy to point out that the non-dimensional vegetation cover does not decline significantly after the initial rapid decline of the vegetation. Additionally, for higher  $N_v$  values ( $> 5$ ), almost more than  $\sim 90\%$  of the vegetation is removed within less than  $\sim 1\%$  of the total distance. The sensitivity of the non-dimensional vegetation portrays a significant difference in the logistic growth model. Vegetation cover displays unrealistic values, if  $N_v$  is very small (see Supplementary figure S1). The logistic growth model shows a steady decrease in the vegetation cover away from the watershed divide. The non-dimensional vegetation cover decreases  $\sim 50\%$  within almost  $\sim 40\%$  of the total distance when  $N_v$  value is set to 10 (figure 2). However, the decrease is  $\sim 70\%$ , if the  $N_v$  value is set to 50 and it does not vary considerably with an increase in the  $N_v$  value (figure 2).

The main driving force of the vegetation growth is assessed depending on the availability of moisture content, slope aspect (Stephenson 1998) or land surface temperature (Weng *et al.* 2004).  $N_v$  is the critical parameter which controls the growth as well as decay of the vegetation simultaneously and therefore it includes all of the aforementioned effects ( $N_v = \frac{K_{vd} \times \tau_{ev}}{K_{vg}}$ ) (as lump parameter).

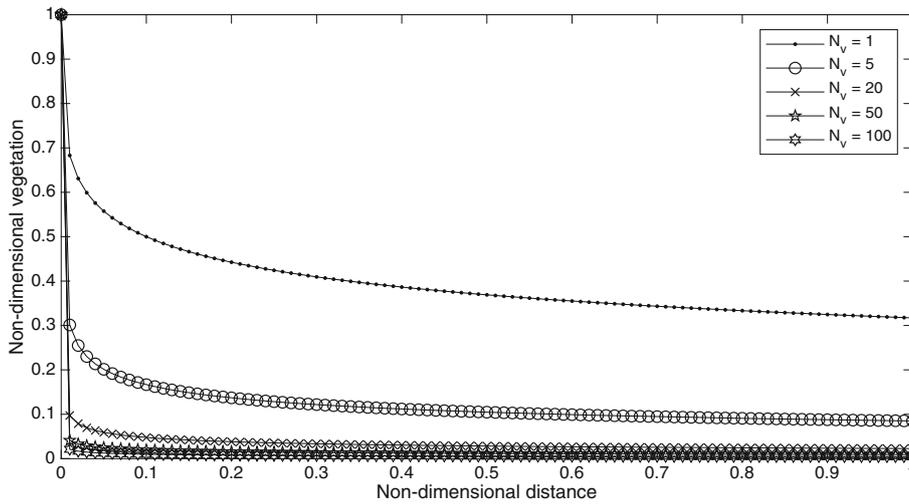


Figure 1. Non-dimensional vegetation cover along the slope with a linear growth model. Note that the fractional vegetation cover decreases at a rapid rate in the upper reach with an increase in  $N_v$  value.

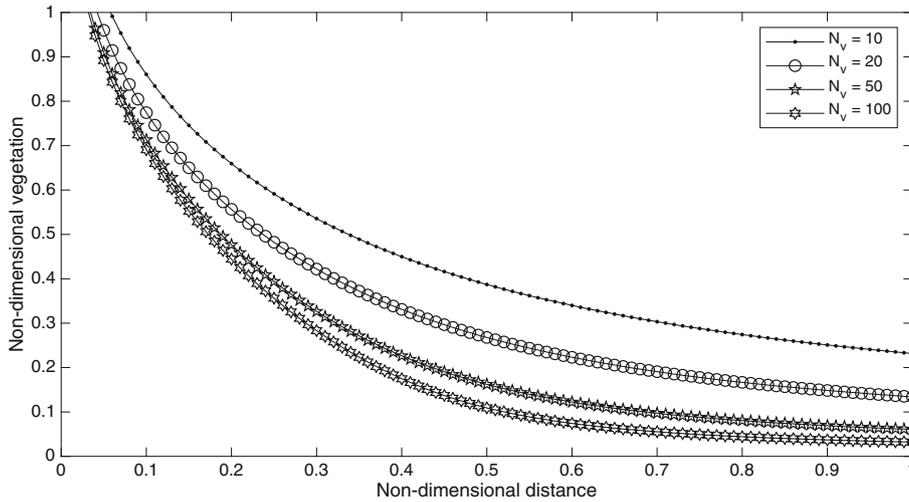


Figure 2. Non-dimensional vegetation cover along the slope for the logistic growth model for different values of  $N_v$ .

Inclusion of all of these aspects reduced the complicated problem into a single vegetation number. In our results,  $N_v$  reflects a very low vegetation number as compared to most of the model parameter values adopted in other studies (e.g., Collins *et al.* 2004). We varied  $K_{rv}$  values and observed that a range in between 0.5 and 0.8 works reasonably good for both the models (see Supplementary figure S2).

### 3.2 Role of noise-induced phenomena in vegetation distribution

We present the effect of the Gaussian noises, degree of correlation between these two noises and the vegetation number  $N_v$  parameter in figures 3–5. It is interesting to point out in this regard that in all

three cases of the noise-induced system, the peak of SPD shifts towards the left with an increase in the magnitude of  $N_v$ . This feature is universal and common because the vegetation number ( $N_v$ ) actually defines the ratio between decay and growth parameters. Therefore, as  $N_v$  increases, the vegetation cover decreases and the value of  $P_{st}(V)$  attains a peak for small vegetation cover. In other words, the overall vegetation cover disappears for a high value of  $N_v$ . However, the change in the strength of the noises with low  $N_v$  values do not affect the position of the maxima of the SPD.

Figure 3 represents the effect of the strength of the multiplicative noise ( $D$ ) that acts as a constructive force by increasing the vegetation cover. We find  $P_{st}(V)$  is weakly affected by the strength of  $D$  when the degree of  $\lambda$  and the strength of  $\alpha$  is

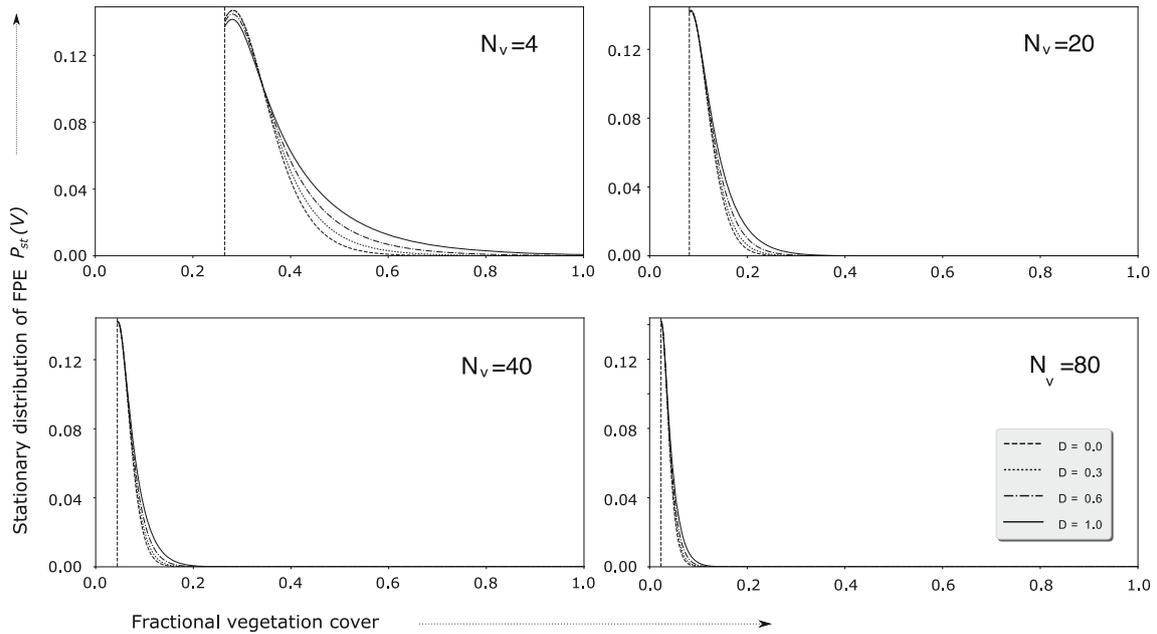


Figure 3. Plot of  $P_{st}(V)$  (probability density) and fractional vegetation cover for different values of multiplicative noise and  $N_v$ . We fixed  $\lambda = 0.1$  and  $\alpha = 0.5$ .

fixed corresponding to any fixed value of  $N_v$ . The prominent cause of the similarity of different SPD is primarily due to the normalization factor that stretches the vegetation cover from 0 to 1. The SPDs can be distinguished distinctly for a small value of vegetation number ( $N_v = 4$ ) while on the other hand, when it is increased to 80, the SPDs are barely separable and they trace out nearly the same path. At low vegetation cover ( $< \sim 0.35$ )  $P_{st}(V)$  decreases; on the contrary at high vegetation cover (in the range of  $\sim 0.4-0.8$ ), it increases as the strength of  $D$  is increased. As the value of  $N_v$  increases, the difference in  $P_{st}(V)$  is indistinguishable as the destruction of vegetation is further enhanced with higher decay coefficient. This reflects the fact that the vegetation cover along the transect is not significantly influenced by the multiplicative noise, when  $N_v$  value is quite high. This high value of  $N_v$  sets the stage for a certain extinction of vegetation. One can appreciate another fact that with increasing  $N_v$ ,  $P_{st}(V)$  for higher vegetation cover is revealed to be always high.

The role of the additive noise (negative in magnitude) strength ( $\alpha$ ) on the SPD with a fixed value of  $\lambda$  and  $D$  has been demonstrated in figure 4. With increasing the strength of  $\alpha$ , we observed that the peak of the  $P_{st}(V)$  reduces for any value of  $N_v$ . Although the magnitude of  $P_{st}(V)$  decreases for lower vegetation cover, the probability is actually

quite high for a higher vegetation cover (see figure 4). Therefore, as the strength of  $\alpha$  is increased, the magnitude of  $P_{st}(V)$  for small vegetation cover decreased while for high vegetation cover it increased. This is indicative of the fact that the additive noise (negative in magnitude) is actually equalizing the vegetation distribution along the profile by reducing the  $P_{st}(V)$  estimate at small vegetation cover. Figure 5 illustrates the effect of correlation between the two Gaussian noises on the SPD. It is evident that as the correlation strength ( $\lambda$ ) increases, the probability for the smaller vegetation cover values increases, then drops sharply around 30% of vegetation coverage. For smaller values of  $N_v$  (see for  $N_v = 4$ ),  $P_{st}(V)$  increases for lower vegetation cover ( $\sim < 0.35$ ) and then decreases when the fractional vegetation cover is higher ( $> \sim 0.4-0.5$ ). This implies that higher values of  $\lambda$  promote the destruction of the overall vegetation pattern. Interestingly, we observed that on increasing the strength of  $\lambda$  at low  $N_v$  value, the position of the peak of  $P_{st}(V)$  remains stationary.

### 3.3 Implications of the proposed model

We have compared the two end-member models of vegetation growth and observed that the logistic model of vegetation growth is more realistic in the fluvio-riparian system. Our modelling study does

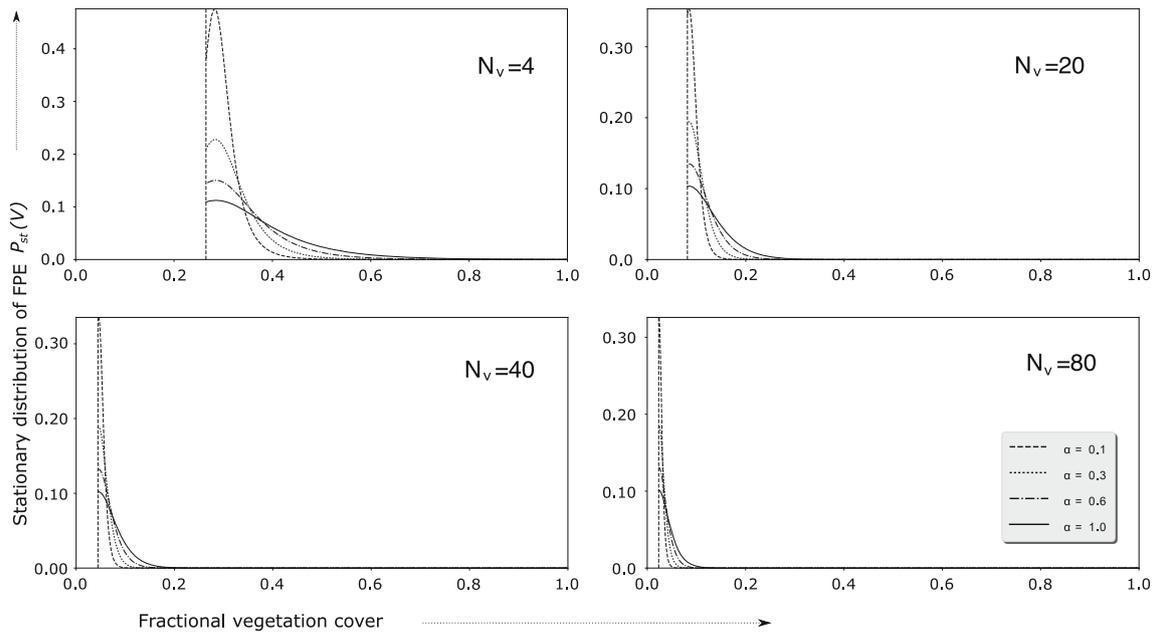


Figure 4. Plot of  $P_{st}(V)$  (probability density) and fractional vegetation cover for different values of additive noise (negative in magnitude) and  $N_v$  by fixing  $\lambda = 0.1$  and  $D = 0.4$ .

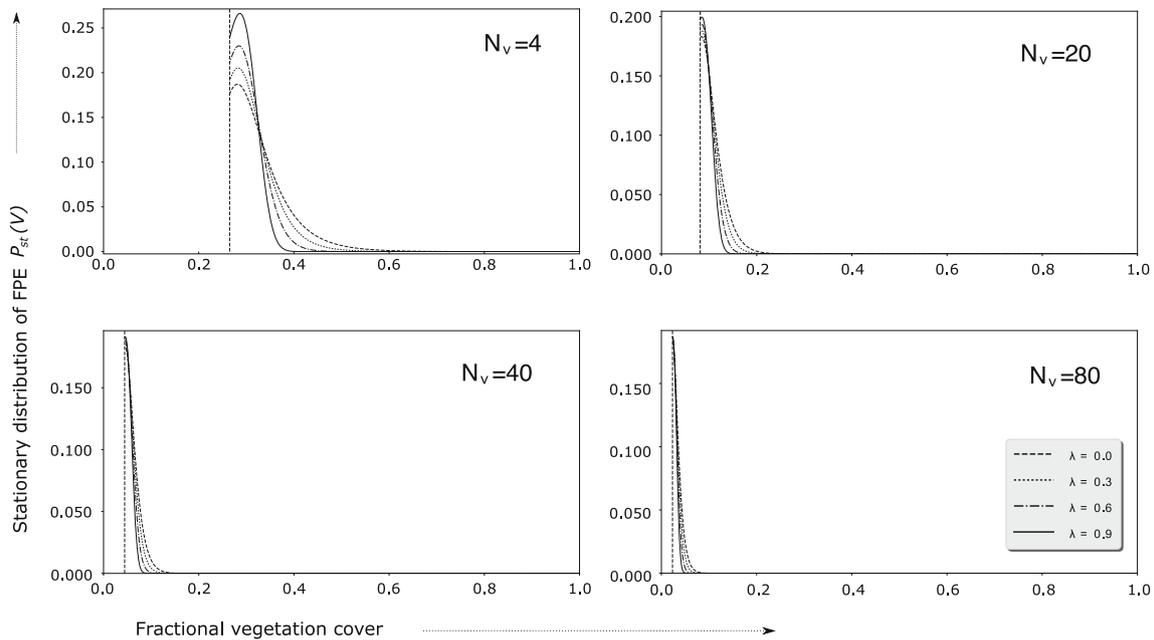


Figure 5. Plot of  $P_{st}(V)$  (probability density) and fractional vegetation cover to illustrate the effect of correlation between the two Gaussian noises and  $N_v$ , where the strengths  $D$  and  $\alpha$  have been fixed to be 0.3.

not aim to calibrate the model parameters due to unavailability of high-precision vegetation data along a slope. We suggest that the logistic growth model is a more realistic one in terms of the rate of decay of vegetation along the slope. The most suitable model parameters for the linear growth scheme underestimates the vegetation cover in the upslope region while it overestimates the vegetation cover in the downslope (see figure 1) stretch.

In fact, this model is more applicable to the region(s) with very high slope adjacent to the watershed divide and does not support to sustain the vegetation cover. Logistic growth model depicts the nature of VCF distribution to be more realistic because of its inherent property of growth in resource-limited condition. It is noteworthy to point out that the  $N_v$  value can be higher in the logistic growth model. Physically, this signifies

that despite of being continuously removed by the fluvial processes (water), the vegetation still continues to grow.

All characteristic curves of figures 3–5 indicate that the multiplicative noise does not act as a drift term unlike discussed in Ai *et al.* (2003). The vegetation community rather remains stationary with a fixed peak value of  $P_{st}(V)$ . However, one can consider the additive noise (negative in magnitude) as a diffusive term which results in a reduction of the vegetation growth and flattens the peak of  $P_{st}(V)$ . We also observed that the position of the maxima of  $P_{st}(V)$  is by no means affected by the strength of these noises. Therefore, we suggest that the intensity of different Gaussian noises does not significantly drive the system to effective growth or destruction. However, most importantly these noises effectively reshape the SPD by decreasing or increasing the magnitude of  $P_{st}(V)$  at a certain extremum of vegetation cover. Segregation of the intrinsic and the extrinsic factors in the evolution of vegetation is difficult due to their complicated intertwined behaviour. When the internal factors predominantly influence the system, it results in an increase in the vegetation growth and consequently the VCF. The external factors, on the contrary, delay the spread of the vegetation cover.

Additionally, we observed that the value of  $P_{st}(V)$  at higher vegetation cover is also higher when we increased the strength of  $\alpha$ . Our results are on similar lines as that of the observations reported by Ai *et al.* (2003). This could be attributed to the erosion model and the steady distribution of the vegetation profile. In the erosion model, the erosion rate increases from the upstream to downstream. Therefore, the rate of vegetation destruction is lower in the upstream part. Further, the amount of vegetation cover is also higher in the upper region. We suspect that the combined effect of higher vegetation cover and the lower erosion rate in the upper part results in lower sensitivity of the vegetation destruction. The deterministic vegetation growth model can further be verified using high-resolution digital elevation model (DEM) and fractional vegetation cover dataset (at least sub-meter spatial resolution) over a larger spatial scale, which may lead to the calibration of the model parameters. It can be achieved either using Lidar dataset or photogrammetric methods for DEM (Berthier *et al.* 2014; Crosby *et al.* 2020) and multi-spectral dataset for vegetation cover (Hu *et al.* 2014; Yan *et al.* 2019).

### 3.4 Revisiting the modelling assumptions

In general, channels are devoid of vegetation owing to the fact that the fluid motion does not promote any vegetation growth. The excess shear stress model, used in numerous other studies has been previously implemented as an idealized case of transport and incision process within the channel (Baldwin *et al.* 2003; Whipple 2004). However, numerical models take into consideration a single transport law for both the channel as well as the surface wash processes (Dietrich *et al.* 2003). Idealized value of  $N_e$  is another simplification of the erosion model as we do not consider the substantial change in the erodibility aspect downslope. Erodibility at a regional scale varies significantly if the landscape encounters a set of different lithology or climatic condition. Similarly, we do not consider that the friction factor  $K_{rv}$  changes substantially in order to retain the simplistic approach of our robust model. We have also kept it constant keeping in view that the scale of the transect of vegetation profile is relatively small enough to be idealized as a constant friction condition.

The major issue with the logistic formulation is that the model does not implicate  $V = 1$  at  $x = 0$ . It is duly acknowledged as a small limitation of our model formulation with the logistic growth. In spite of this, our model solution presents a more realistic non-dimensional vegetation profile than the existing linear model. Additionally, we do not intend to present sensitivity of the growth, decay, friction, lithologic and noise parameters and this is beyond the scope of the present work. However, the sensitivity analysis can shed light on the role of noise levels on the steady distribution. The most important simplification of the present model is the use of the lumped parameter  $N_v$  that incorporates growth, decay and shear stress aspects. This implies that the slope is dominated only by a particular type of vegetation and there is no interspecies interaction. We have lumped the factors of multiple species into one vegetation number and have not considered any model for intra- or inter-species competition.

We have characterized the multiplicative noise as a positive role-playing agent, while the additive noise plays a weakening role. One can argue about the character of these noises and may idealize them differently. Additionally, it is nearly impossible to segregate the internal and external factors that lead to environmental stochasticity. Factors such as solar radiation, precipitation or soil nutrients

generally augment the growth of the vegetation cover. However, anomalous amount and intensity (in case of an extreme event) of these factors may lead to probable destruction of vegetation cover as well. For example, increased rainfall can lead to a higher runoff which can eventually result in vegetation destruction. Similarly, the intrinsic character of the vegetation species can simultaneously increase or decrease the vegetation cover along a slope.

#### 4. Concluding remarks

In this study, we have proposed and presented the solutions for a logistic growth model of vegetation and a novel stochastic model with two Gaussian noises. We affirm that the logistic growth model presents a more realistic vegetation profile. The vegetation number obtained from the model needs further investigation to interpret the interaction between the growth and the decay of the vegetation community. Additionally, there is a need to investigate the model parameters with high-resolution vegetation data. Biological evolution is always regarded as a stochastic system and this provided us with the motivation for exploring the effect of random noises in the vegetation growth along a slope profile. The Gaussian noises and their correlation parameter implicate a stable change in the SPD. Additionally, in the context of the noise level, we have chosen that the vegetation growth system does not shift towards immediate sporadic growth or extinction. We observed the anomalous effect of the additive noise (negative in magnitude), which needs further investigation in the near future. On a closing note, we conclude that the effects of different intrinsic and extrinsic noises are quite difficult to separate out due to the complex interrelationship between the environment and the biological community.

#### Acknowledgement

Logistic support from Indian Institute of Technology (IIT) Gandhinagar is gratefully acknowledged.

#### Author statement

All authors have contributed equally in formulating the problem and preparation of this manuscript.

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Corresponding editor: N V CHALAPATHI RAO