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Comparative analyses of finite element and limit-equilibrium methods for heavily fractured rock slopes

Tümay Kadakci Koca* and M Yalçin Koca

Department of Geological Engineering, Dokuz Eylul University, 35390 İzmir, Turkey.
**Corresponding author. e-mail: tumay.koca@deu.edu.tr*

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Limit-equilibrium method (LEM) and finite element method (FEM) with shear strength reduction (SSR) technique are the most widely used analysis tools in slope stability assessment. Recently, researchers have reported that both factor of safety (FOS) values and failure surfaces obtained from LEM and FEM are generally in good agreement except in some particular cases. On the other hand, the consistency between two methods has not been adequately discussed for heavily fractured rock mass models by employing Generalized Hoek–Brown Criterion (GHBC). In this study, the FOS values and failure surfaces derived from LEM and FE-SSR based on GHBC were compared concerning static and pseudo-static conditions, various overall slope angles, geological strength index (GSI) values, and various water table levels. In this context, three homogeneous, highly fractured rock slope models with irregular geometry and different slope heights were generated by two-dimensional Slide and Phase^2 software. Limit-equilibrium (LE) analyses were performed by Bishop, Fellenius, Morgenstern–Price, and Spencer techniques. The comparisons of global minimum FOS values for 431 cases and the effects of variables on two methods were investigated by statistical analyses. Consequently, it was determined that the difference between the FOS values are statistically significant. However, if the seismic coefficient is higher than 0.1 g, slope angle is higher than 34°, and the slope is assumed to be fully saturated, Morgenstern–Price is the most well-matched technique with FE-SSR than the others. For the same cases, the failure surfaces detected by Fellenius is more similar to the ones detected by FE-SSR.

Keywords. Fractured rock mass; finite element method; limit-equilibrium methods; rock slope stability; statistical analysis.

1. Introduction

LEM is commonly used for performing deterministic slope stability analysis. LEM provides a critical FOS (global minimum) and corresponding failure surface as well as different failure surfaces with FOS values higher than the critical one (local minimum). The different LE methods (LEMs) (Fellenius, Bishop, Janbu, Spencer, Morgenstern–Price, etc.) satisfy different force and moment assumptions for equilibrium (Duncan 1996). On the other hand, the continuum based finite element method (FEM) was firstly introduced by Turner *et al.* (1956). Since the stability of a slope is evaluated by a FOS value which is the ratio of driving forces to resisting forces, Zienkiewicz *et al.* (1975) firstly used SSR technique which enables calculating FOS in FEM analysis. The FOS provided by FE-SSR is termed as strength reduction factor (SRF) and is the same as the one derived by LEM (Griffiths and Lane 1999). The

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main differences between LEM and FEM were explained by several researchers (Duncan 1996; Griffiths and Lane 1999; Hammah *et al.* 2005b; Aryal 2006; Cheng *et al.* 2007; Hammouri *et al.* 2008; Wei *et al.* 2010; Rabie 2014; Singh *et al.* 2014; Moni and Sazzad 2015; Aswathi *et al.* 2017; Vinod *et al.* 2017). They all clarified that the FEM provides the amount of deformation on the slope and requires no assumption to be made in advance about the shape or location of the failure surface, slice side forces and their directions, unlike LEM.

Many researchers used the FE-SSR in order to reveal its reliability by comparing it with LEM with the aid of various commercial software products. The studies are mainly based on comparing the FOS values, and the failure surfaces detected from two methods. Majority of these studies have examined soil slopes by using the Mohr–Coulomb Failure Criterion and the geometry of the slope models are generally simple (Moudabel 1997; Aryal 2006; Cheng *et al.* 2007; Alkasawneh *et al.* 2008; Hammouri *et al.* 2008; Wei *et al.* 2010; Ahmad 2012; Baba *et al.* 2012; Gover and Hammah 2013; Matthews *et al.* 2014; Rabie 2014; Akbas and Huvaj 2015; Alemdağ *et al.* 2015; Berisavljević *et al.* 2015; Burman *et al.* 2015; Putu Tantri and Lastiasih 2015; Liu *et al.* 2015; Moni and Sazzad 2015; Ozbay and Cabalar 2015; Tschuchnigg *et al.* 2015; Ghazaly *et al.* 2016; Neves *et al.* 2016; Sazzad *et al.* 2016; Vinod *et al.* 2017). These studies involve the comparison of two methods under seismic conditions (Aryal 2006; Alemdağ *et al.* 2015), rapid drawdown (Hammouri *et al.* 2008; Burman *et al.* 2015), various slope angles (Ghazaly *et al.* 2016) and various saturation conditions (Aryal 2006; Rabie 2014; Akbas and Huvaj 2015; Berisavljević *et al.* 2015; Putu Tantri and Lastiasih 2015; Ozbay and Cabalar 2015; Neves *et al.* 2016; Sazzad *et al.* 2016). The cases studied in the literature which have been mentioned above demonstrated that the FOS differences between FEM and LEMs are up to 40% with a mean of 7.71%, and with a standard deviation of 7.9%. The significant differences and discrepancy from case to case have been attributed to heterogeneity and complexity (seepage, consolidation, pore pressure, reinforcement, associated flow-rule) of the model.

On the other hand, there are limited studies concerning the LEM and FEM comparison for rock slopes based on GHBC (Hammah *et al.* 2005a, b; Azami *et al.* 2013; Singh *et al.* 2014) and based on Mohr–Coulomb Criterion and equivalent Mohr–Coulomb parameters (Hammah *et al.* 2004, 2005a; Aswathi *et al.* 2017). Furthermore, researchers have not examined the differences in FOS values between two methods concerning the statistical significance and have not profoundly investigated the effect of water saturation and seismic coefficients on FOS values. Moreover, the consistency between the two methods was not adequately discussed for heavily fractured rock masses obeying GHBC. For example; Hammah *et al.* (2004) employed equivalent Mohr–Coulomb parameters on a homogeneous rock slope model having 10 m height and 35.5° slope angle. They determined that Bishop and Spencer methods give slightly higher FOS values than FE-SSR (table 1). Hammah *et al.* (2005a) investigated two examples of a rock slope having simple geometry employing GHBC and equivalent Mohr–Coulomb parameters. They observed very similar FOS values obtained from different methods such as FE-SSR, Bishop, and Spencer. Hammah *et al.* (2005b) examined a variety of published and constructed slope models, including soil and rock slopes. Most of the examples are obeying Mohr–Coulomb Criterion, whereas fewer slopes are solved by GHBC. They concluded that the results of FE-SSR agree very well with the LEMs (primarily Bishop and Spencer). Such expositions are unsatisfactory since the differences were not made quantitatively and statistically considering case by case. Singh *et al.* (2014) investigated the stability of five slopes in India having slope angles between 65° and 80° with simple geometry. They employed FE-SSR and Bishop on these slopes composed of epidiorite, quartzite, phyllite, and metavolcanics considering the equivalent Mohr–Coulomb parameters. They found that Bishop yields higher FOS values ranging between 11.51% and 18.03%. Aswathi *et al.* (2017) studied a heavily fractured road cut slope having a height of 30 m and slope angle of 65° using equivalent Mohr–Coulomb parameters using FE-SSR and LEMs (Fellenius, Bishop, Generalized Janbu, Lowe–Karafiath, Corps of Engineers, Spencer, GLE/Morgenstern–Price). They determined that Morgenstern–Price predicts the minimum FOS among other LEMs and gives a better result for non-circular failure surface. In addition, the FOS value obtained from FEM is approximately 20% less than that obtained from LEM. It should be noted that employing equivalent Mohr–Coulomb parameters leads to a mean of 0.14 higher FOS values than GHBC for slope angles lower than 40°. However, it gives conservative FOS values for the slope angles higher than 40° (Kadakci 2011; Kadakci Koca and Koca 2014). A similar result was obtained by Deng

Table 1. *Summarized literature review focused on FOS comparison for rock slopes.*

References	LEM	Strength criterion	Differences in FOS (%) $[(FOS_{FE\text{-}SSR}-FOS_{LEM})/FOS_{FE\text{-}SSR}]*100$			No. of cases	Result
			min	max	mean		
Hammah *et al.* (2004)	Bishop and Spencer	Equivalent Mohr–Coulomb	−0.35 (Bishop)	−0.69 (Spencer)	–	1	FE-SSR < LEM
Hammah *et al.* (2005a)	Bishop and Spencer	Generalized Hoek–Brown	−0.17 (Spencer)	−1.68 (Bishop)	0.83	2	FE-SSR < LEM
Azami *et al.* (2013)	Spencer	Generalized Hoek–Brown	0	25.1	12.06	15	FE-SSR > LEM
Singh *et al.* (2014)	Bishop	Equivalent Mohr–Coulomb	−11.51	−18.03	−13.36	5	FE-SSR < LEM
Aswathi *et al.* (2017)	Morgenstern–Price	Equivalent Mohr–Coulomb	–	–	−20	1	FE-SSR < LEM

et al. (2016) and Li *et al.* (2008). Besides, Chen and Li (2018) argued that the FOS difference between the models based on GHBC and equivalent Mohr–Coulomb parameters is significant for the slope heights lower than 30 m. Thus, the equivalent Mohr–Coulomb criterion should be avoided for homogeneous, highly fractured slopes having a slope angle higher than 40° and slope height lower than 30 m. Azami *et al.* (2013) studied the effect of orientation of weak planes (joints, bedding planes, etc.) on the FOS values and the failure surfaces obtained from FE-SSR and compared them with LEM results. They used GHBC for the matrix and Mohr–Coulomb Criterion for the weak planes. They concluded that there is a good agreement between FE-SSR and Spencer results in cases where the weak planes aligned in the direction of the failure surface, i.e., $0° \leq \theta \leq 60°$. However, for other orientations, where the limit equilibrium approach cannot estimate an accurate safety factor, FE-SSR results are valid. The differences in FOS values obtained from LEMs and FE-SSR for rock slopes investigated in the published literature are given in table 1. It is seen from table 1, that the differences in FOS values depend on the type of LEM and strength criterion. Azami *et al.* (2013) found higher FOS values from FE-SSR when compared to Spencer. Other researchers, however, have found a converse relationship between the FOS values.

Consequently, this study aims (i) to investigate the differences in FOS and failure surfaces derived from LEM (Fellenius, Bishop, Spencer, GLE/Morgenstern–Price) and FE-SSR for homogeneous slopes with irregular geometry (with benches) considering a significant number of cases and (ii) to determine the different responses of two methods to the variation in seismic condition, GSI rating, overall slope angle, slope height and water table level. The statistical significance of the deviations in FOS values was revealed by statistical analyses performed via SPSS software (SPSS Inc. 2007).

2. Methods

2.1 *Limit equilibrium methods*

In LE analysis, FOS is the factor by which the shear strength of the soil or rock mass would have to be divided to bring the slope into a state of barely stable equilibrium (Duncan 1996). Analytical calculations are carried out for the groundmass above the pre-assumed failure surface, which is divided into a number of slices based on force and/or moment equilibrium. It is assumed that the FOS is constant along the potential failure surface. There are several methods to calculate force and/or moment equilibrium for the slices. The different LEMs assume different inter-slice and equilibrium conditions. For example, Fellenius method solves the FOS using the moment equilibrium for circular failure surfaces (Fellenius 1936). The interslice forces are neglected in this method. Matthews *et al.* (2014) stated that the Fellenius method underestimates the FOS value and is rarely used. Craig (2004) also stated that Fellenius underestimates the FOS value within the range of 5–20% when compared to more accurate methods of analysis. Bishop (1955) developed a method also satisfying moment equilibrium with additionally considering the interslice normal forces. However, there are no interslice shear forces assumed and can only be used for circular failure surfaces. Although the simplified Bishop's method does not satisfy complete static equilibrium, the procedure gives relatively accurate values for FOS values (Mansour and Kalantari 2011). Therefore, the simplified Bishop method is commonly used in slope stability analysis. Some rigorous LEMs considering both force and moment equilibrium (for example, the methods of Spencer 1967 and Morgenstern and Price 1965) have been developed. These methods can be used both for circular and non-circular failure surfaces. Duncan and Wright (1980) stated that the LEMs which satisfy all equilibrium (force and moment) conditions are more precise than the ones satisfying only one equilibrium condition. Fredlund *et al.* (1981) and Fredlund and Krahn (1977) evaluated that the FOS equations for all LEMs can be written in the same form if the moment and/or force equilibrium are explicitly satisfied except for the Fellenius method.

2.2 *Finite element method based on shear strength reduction technique*

Jointed rock masses can be simulated by continuum modeling for two cases: (1) if the rock mass is heavily fractured and can be considered as a quasi-continuous medium, the strength of the rock mass is estimated by Generalized Hoek–Brown Criterion, and rotational failure is commonly expected and (2) if a jointed rock mass is prone to discontinuity controlled failures, the continuum-interface method is used in FEM. The strength of discontinuities is given, and the rock material between the discontinuities is considered to exhibit intact rock strength. Fundamentally, case-2 requires simulation of the detachment of the discrete blocks which FEM does not allow (Jing 2003). However, FE-SSR can capture more realistic results for case-1 than case-2.

In FE analysis, the sliding body is discretized into a number of sub-elements termed as the mesh. Iterative solutions process for the nodes which link the sub-elements. The calculation of stress and deformation is performed for these nodes without any assumption of the depth or location of the failure surface. Therefore, the failure surface can be estimated from the incremental shear strain contours and plastic zones at the critical state. The critical SRF value can be increased to indicate more than one slip surfaces. The detected failure surfaces may not have a simple and regular shape as detected in LEMs. In this study, only the critical failure surface was considered.

The FOS in slope stability analysis by SSR technique is calculated based on the stress equilibrium with respect to the mechanical behaviour of the material. It is also termed as strength reduction factor (SRF). It is a factor that shear strength parameters of the rock are divided to reduce them until the iterative process does not converge. The non-convergence, therefore, means that the critical state for stability is achieved. The shear strength parameters are reduced at each step by changing SRF values until providing the condition for the critical state, and the SRF value at the critical state point is equal to the critical FOS value. Griffiths and Lane (1999) noted that this definition of the FOS is exactly the same as that used in the LEM. The reduction of Mohr–Coulomb shear strength parameters by an SRF is simple since the Mohr–Coulomb envelope is linear. On the other hand, lowering the non-linear Hoek–Brown envelope is not straightforward. The non-linear shear strength envelope of the rock mass is lowered in an automated process in the software as introduced in Hammah *et al.* (2005a).

2.3 *Statistical analyses*

The type of statistical test has a significant impact on the interpretation of the dataset. The appropriate statistical tests (parametric or non-parametric tests) for any dataset are primarily selected based on the properties of the dataset, such as distribution type, sample size, and homogeneity in variances. The parametric tests are employed on datasets showing normal distribution (mod = median = mean), homogeneous variance distribution as well as datasets having higher sample size than 30. In this study, normality tests of Shapiro–Wilk (Shapiro and Wilk 1965) were performed to decide on whether parametric or non-parametric tests are appropriate. Even though the sample size is higher than 30 (431), FOS values of each method do not show normal distribution. It is well known that parametric tests are more robust than non-parametric tests; however, they are weak to outliers in the dataset since they process on mean values. The mean value is a good estimator only if the data have zero outliers.

On the other hand, as the non-parametric tests process on the median values, they are less vulnerable to outliers than parametric tests. Consequently, non-parametric correlation (Spearman) and Kruskal–Wallis test (Kruskal and Wallis 1952) were used to investigate the effect of variables on the FOS values. Kruskal–Wallis test which is the non-parametric equivalent of one-way ANOVA test was performed to control the hypothesis that whether the FOS values significantly differ as seismic coefficient, GSI rating, slope angle, slope height, and water table level changes. In addition, the Wilcoxon test (Wilcoxon 1945) was employed to compare the FOS obtained from LEM and FE-SSR. The Wilcoxon test is the non-parametric equivalent of the paired samples *t*-test. It tests the hypothesis that if the FOS values obtained from LEM and FE-SSR are different or not.

2.4 *Slope stability model*

Even though the rock mass presumably fails through intact rock and joints, heavily fractured rock mass tends to fail along a rotational failure surface. On the other hand, the stability analysis of a heavily fractured rock slope considering all the joints in a numerical model requires intensive computing process (Aswathi *et al.* 2017). For practical purpose, the rock mass can be assumed to obey equivalent continuum, which considers isotropic, homogeneous media (Hoek *et al.* 2002). In this context, reduced rock mass strength parameters derived from GSI are used in the evaluation of the stability of heavily fractured rock mass.

In this paper, three slope models with different heights (123, 132, 135 m) were investigated considering equivalent continuum model by GHBC using

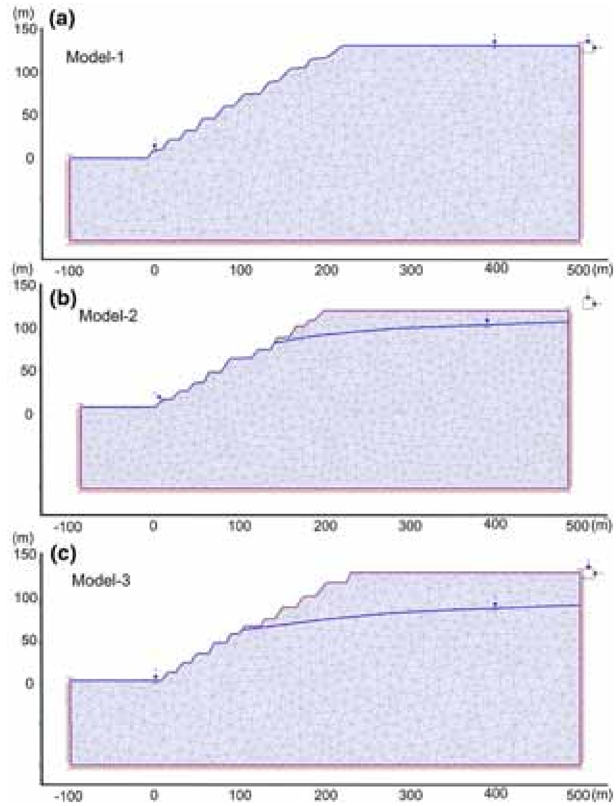

Figure 1. The Phase^2 models of three slopes with different heights adapted from Kadakci (2011) showing (**a**) fully saturated (100%) Model-1, (**b**) partially saturated (70%) Model-2, and (**c**) partially saturated (50%) Model-3.

two dimensional Phase^2 (Rocscience Inc. 2008) and Slide (Rocscience Inc. 2003) software products (figure 1). Four hundred thirty-one different cases which are formed by combinations of the various seismic coefficients, GSI ratings, slope angles, and water table levels in three rock slope models having different slope heights were generated.

In this study, elastic-perfectly plastic material behaviour for poor quality rock masses was used by considering GHBC, which expresses the shear strength with a non-linear envelope. The detailed mathematical background of the GHBC can be found in Hoek *et al.* (2002). The elastic-perfectly plastic material behaviour in FEM (the post-failure strength is the same as the peak strength) agree well with the rigid plastic material assumption made in LEM (Hammah *et al.* 2005b). In addition, non-associated flow rule (dilation parameter = 0, no volume change) was considered. The required input parameters to calculate the rock mass strength parameters are the uniaxial compressive strength (σ_{ci}) and elastic modulus of intact rock (E_i), GSI rating, Hoek–Brown's constant for intact rock material (m_i) and a disturbance factor (D) which reflects the disturbance due to

Table 2. *The rock material and rock mass properties of the investigated slopes employing Generalized Hoek–Brown Criterion.*

Rock type	Rock material properties*			Rock mass properties				
	Dilatation parameter: 0							
	Unit weight: 0.026 MN/m^3			Poisson's ratio (υ): 0.25				
	m_i	σ_{ci} (MPa)	E_i (MPa)	GSI	s	a	m_b	E_m (MPa)
Moderately weathered orthogneiss	20	27.34	33500	35	0.0001	0.516	0.562	1500.76
				40	0.0002	0.511	0.740	1950.72
				45	0.0003	0.508	0.974	2621.66
$D = 0.7$								

*All the physical and geomechanical properties of the rock material given above were taken from Kadakci (2011).

excavation method carried out in the field. The GSI rating can be estimated from qualitative and quantitative charts proposed by various researchers (Sönmez and Ulusay 1999; Marinos and Hoek 2000; Cai *et al.* 2004; Russo 2009; Hoek *et al.* 2013) and should be considered within a range of values rather than one exact GSI rating. In this study, GSI ratings of 35, 40, and 45 were introduced to the slope models. The resultant rock mass strength parameters such as elastic modulus (E_m) and Hoek–Brown's constants (s, a, m_b) of the rock mass were calculated by RocLab (Rocscience Inc. 2002) and are given in table 2. On the other hand, LEM does not require Poisson's ratio (υ) and elastic modulus of the rock mass (E_m) unlike FEM since stress-strain calculations are not processed in LEM.

Hammah *et al.* (2005b) argued that the number of mesh elements has a little impact on FOS values. However, the best approach in selecting the appropriate number of mesh elements and the domain size of the model is to find out the conditions where there is a negligible variance on FOS values. In this study, the mesh configuration was set to six noded triangular elements with a number of 2000. On the other hand, the number of slices in LEM analyses was limited to 20.

In addition to the slope models under static condition, seismic coefficients such as 0.1, 0.2, and 0.3 g regarding the different seismic zones were considered. The 50%, 70%, and 100% of water table levels correspond to the percentage of the wet part in slope height. The overall slope angles of 30°, 32°, 34°, and 36° were also simulated.

Locating the failure surface that has the lowest FOS (global minimum) is one of the essential parts of slope stability analysis, and a variety of computer techniques have been developed to automate as much of this process as possible (Duncan 1996). In this study, the grid search method for circular failure surfaces was selected to detect the critical failure surface in LEM. On the other hand, a critical failure surface was interpreted from shear strain contours demonstrated in FEM, unlike LEM.

3. Results and discussion

3.1 *Effect of variables on the FOS values*

Correlation analyses can investigate the strength and direction of the relationship; however, this relationship may not be a cause–effect relationship. In this study, correlation analyses were performed by SPSS software (SPSS Inc. 2007) to find out the strength and the direction of the relationship between the dependent and independent variables. The dependent variables are the FOS values obtained from FE-SSR, Fellenius, Bishop, Spencer, Morgenstern–Price methods. On the other hand, independent variables are seismic coefficient (0, 0.1, 0.2, 0.3 g), overall slope angle (30°, 32°, 34°, 36°), GSI rating (35, 40, 45), water table level (50%, 70%, 100%) and slope height (123, 132, 135 m). Table 3 shows that the most effective parameters on the FOS values are seismic coefficient, overall slope angle, and GSI, respectively. However, Fellenius and Spencer methods exhibit a different degree of interaction between the independent variables.

The linear relationships which confirm the correlation coefficients obtained in statistical analyses were also illustrated in figure 2. The FOS values from the 431 cases were plotted on the graphs, and the lines of best fit were constructed. It is evident from figure 2 that as the seismic coefficient, overall slope angle, and WTL increase, FOS values decrease for all methods. On the other hand, as the GSI rating increases, FOS values increase. Figure 2(d) highlights an unexpected positive proportional relationship between slope height and FOS, which essentially indicates that slight

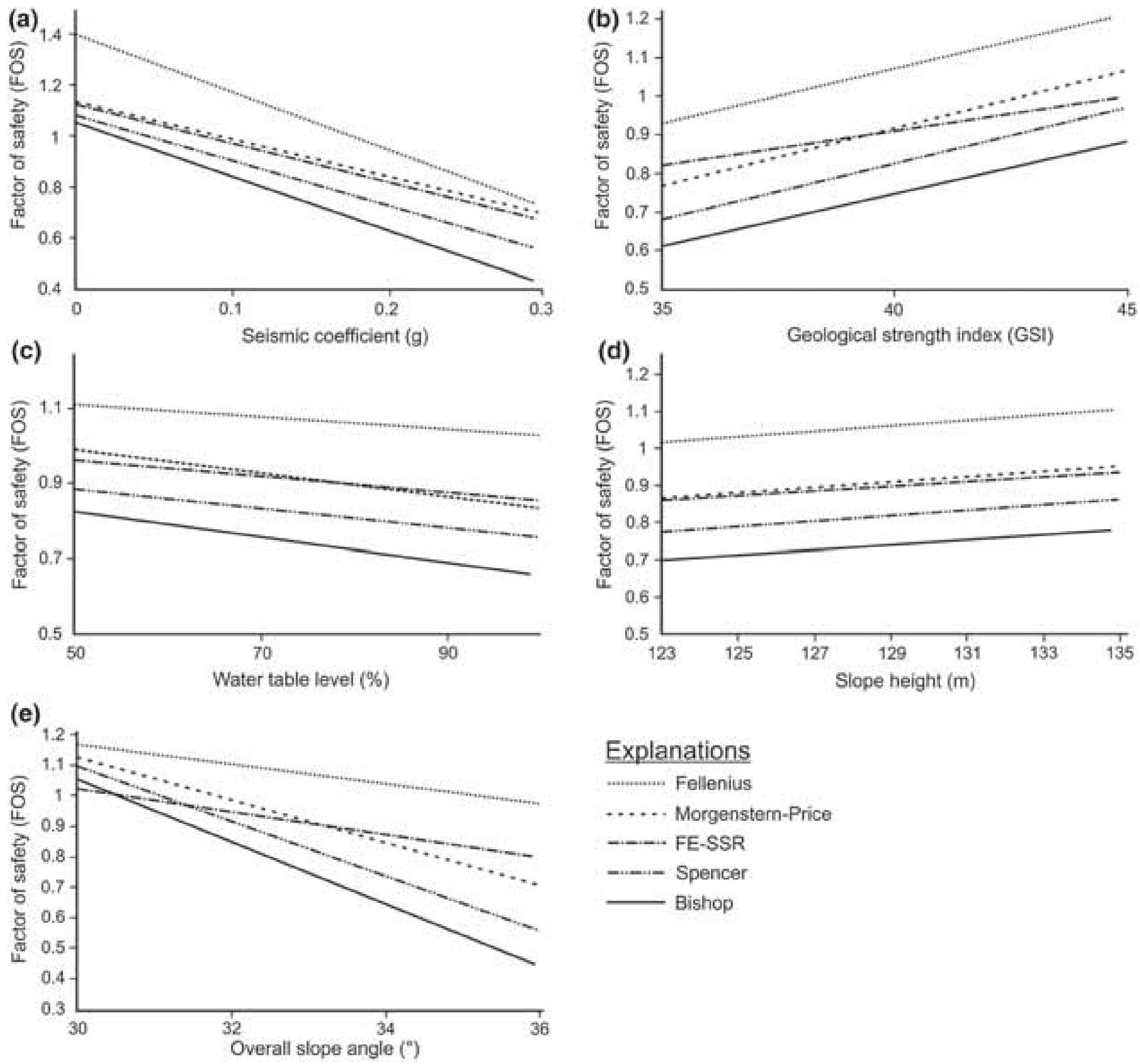

Figure 2. Variations of FOS values for different (**a**) seismic coefficients, (**b**) GSI ratings, (**c**) water table levels, (**d**) slope heights, and (**e**) overall slope angles.

Table 3. *Correlation coefficients of the relationships between independent and dependent variables and their significance.*

	Non-parametric correlation (Spearman's rho)				
	Independent variables and correlation coefficients (R)				
Dependent variables	Seismic coefficient (g)	Overall slope angle (°)	GSI	WTL (%)	Slope height (m)
FE–SSR	−0.815**	−0.344**	0.306**	−0.210**	−0.132**
Fellenius	−0.867**	−0.234**	0.343**	−0.114*	−0.002 (*p: 0.972*)
Bishop	−0.548**	−0.502**	0.243**	−0.149**	−0.149**
Spencer	−0.495**	−0.512**	0.315**	−0.277**	−0.143**
Morgenstern–Price	−0.479**	−0.475**	0.381**	−0.209**	−0.143**

**Correlation is significant at 99% confidence level ($p < 0.01$).

*Correlation is significant at 95% confidence level ($p < 0.05$).

variations in slope height are not affecting the FOS values as much as the other factors. For example; the dip angles of benches in the model-1 which has the highest slope height (135 m) are lower than the other slope models. Therefore model-1 yields higher FOS values independent from the slope height. This case leads a virtual inference that the slope height is positively proportional to the FOS values. Accordingly, it was inferred that as the slope geometry is irregular, the dip angles of the benches or overall slope angle mainly control the stability under static conditions rather than the slope height does. For this reason, the effect of slope height could not be evaluated. The increasing seismic coefficient significantly lowers the FOS values obtained from Bishop and Spencer than the other methods (figure 2a).

Furthermore, the Kruskal–Wallis test was performed to evaluate if the FOS values significantly vary as the seismic coefficient, GSI rating, slope angle, slope height, and water table level changes. It was determined that FOS values obtained from Fellenius method are not affected by the variations of water table level. However, all the mentioned methods are significantly affected by the independent variables.

3.2 *Comparison of the FOS values*

In this study, the comparisons were performed to determine the deviations in FOS values obtained from LEMs and FE-SSR. The Wilcoxon test was used to evaluate the difference between FOS values. The tests were performed to compare FE-SSR with LEMs as well as the LEMs among themselves. Besides, the tests were separately performed for all cases and individually for each seismic condition, water table level, and overall slope angle. Firstly, table 4 shows the Wilcoxon test results for all 431 cases investigated. The test results indicate that the difference between the FOS values obtained from the LEMs and FE-SSR is statistically significant ($p < 0.05$). However, as the maximum negative and positive differences were considered, Morgenstern–Price method is more consistent with FE-SSR than the other LEMs. In general trend, the most conservative method under seismic and non-seismic conditions is Bishop whereas Fellenius yields highest FOS values among the other methods after which GLE/Morgenstern–Price, FE-SSR, and Spencer are nominated, respectively. It was also determined that Fellenius only yields lower FOS values in the case of WTL is 70% or 50%, and the slope angle is lower than 34°.

In contrast to the findings of this study, Wright *et al.* (1973) determined that the FOS calculated by simplified Bishop method agrees favourably with (difference is within $\sim$5%) the FOS calculated using finite element procedures. Besides, various researchers stated that Fellenius give conservative FOS values when compared to other LEMs and FE-SSR (Whitman and Bailey 1967; Aryal 2006; Baba *et al.* 2012; Rabie *et al.* 2014; Burman *et al.* 2015; Moni and Sazzad 2015; Sazzad *et al.* 2016). The researchers related the conservative FOS values to the calculation process of the Fellenius method for omitting the shear and normal forces between interslices. Several possible explanations for obtaining higher FOS values from Fellenius in this study may be assuming total stress analysis and the complexity of the solutions based on GHBC as a controversy to the earlier findings.

Secondly, LEMs which satisfy all equilibrium conditions (Spencer and GLE/Morgenstern–Price) and satisfy the moment equilibrium (Bishop and Fellenius) were compared by Wilcoxon test among themselves. The differences in FOS values between these two pairs are statistically significant. In other

Table 4. *The differences in FOS values between the LEMs and FE-SSR based on the Wilcoxon test.*

Comparison	Max. negative difference and the number of negative cases	Max. positive difference and the number of positive cases	Mean difference and standard deviation in FOS values	Mean difference in FOS values (%)
FE-SSR-Fellenius	−1.073 (399) FE-SSR<Fellenius	0.914 (32) FE-SSR>Fellenius	0.191±0.125	21.14
FE-SSR-Bishop	−0.423 (157) FE-SSR<Bishop	0.908 (273) FE-SSR>Bishop	0.292±0.211	35.16
FE-SSR-Spencer	−0.421 (202) FE-SSR<Spencer	0.908 (228) FE-SSR>Spencer	0.237±0.168	27.88
FE-SSR-Morgenstern–Price	−0.402 (284) FE-SSR< Morgenstern–Price	0.908 (147) FE-SSR> Morgenstern–Price	0.197±0.141	22.87

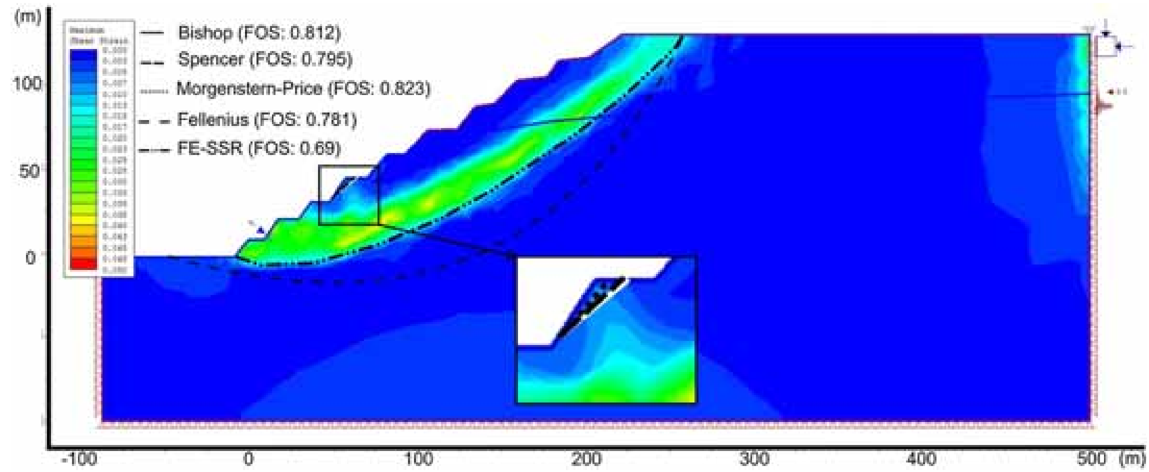

Figure 3. The failure surfaces detected by the LEMs and FE-SSR for slope model-1, while GSI: 35, slope angle: 30°, WTL: 50%, seismic coefficient: 0.3 g.

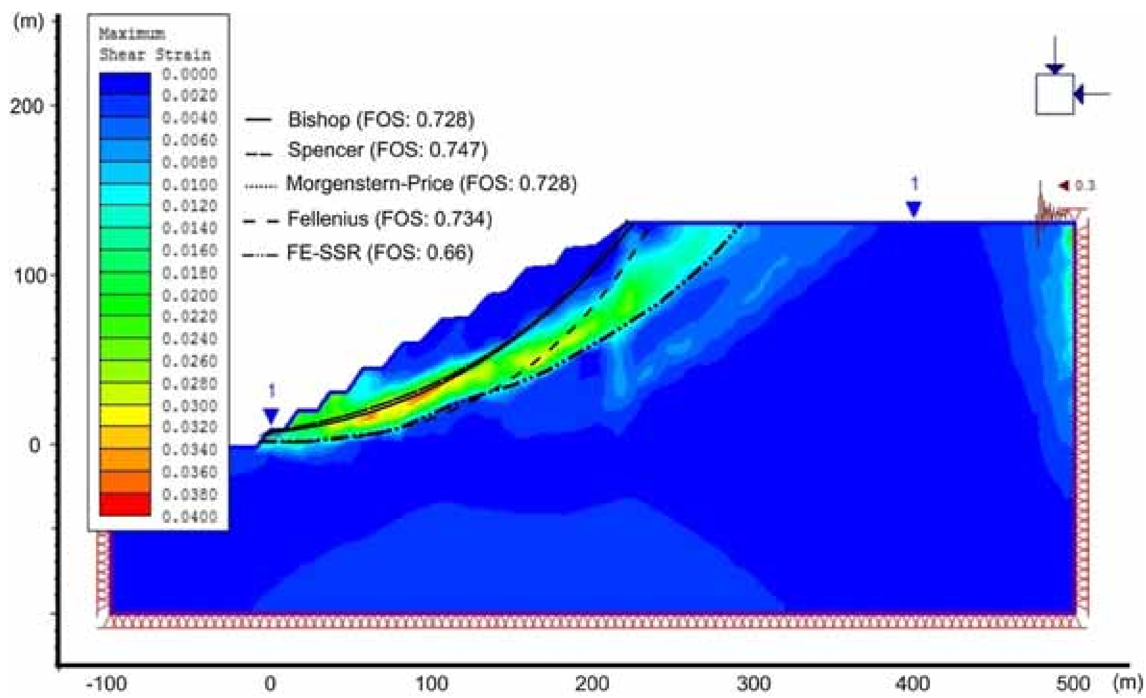

Figure 4. The failure surfaces detected by the LEMs and FE-SSR for slope model-1, while GSI: 35, slope angle: 30°, WTL: 100%, seismic coefficient: 0.3 g.

words, considering all the 431 cases, the LEMs based on the same force and moment equilibrium assumptions do not yield similar FOS values. Quantitatively, the mean percentage of difference is 13% between Spencer and Morgenstern–Price, whereas 34.77% between Bishop and Fellenius. Fredlund *et al.* (1981) explained the difference in FOS values between LEMs by the difference in the force or moment equilibrium assumptions. In addition, the type of side force function assumed or computed, results in a variation in the normal force at the base of a slice in different LEMs. Duncan (1996) also reported that the difference between different LEMs is less than 6%. According to the results in this study, it is evident that LEMs also differs from each other significantly.

Finally, the FOS values obtained from FE-SSR were compared to the ones obtained from LEMs under varying seismic coefficient, overall slope angle, and WTL. FE-SSR compares well with LEMs except for Fellenius under static conditions. On the contrary, the FOS values obtained from

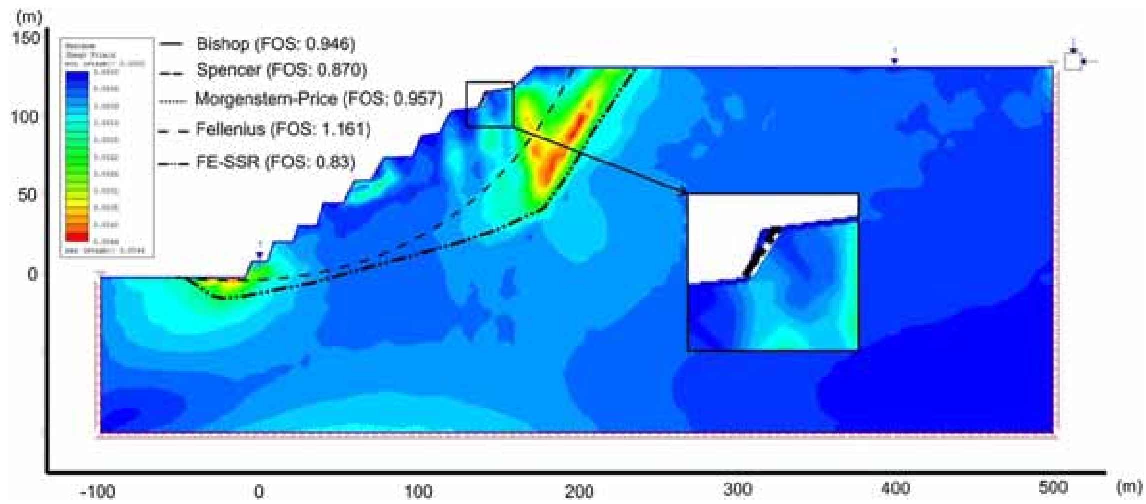

Figure 5. The failure surfaces detected by the LEMs and FE-SSR for slope model-1, while GSI: 35, slope angle: 36°, WTL: 100%, seismic coefficient: 0.

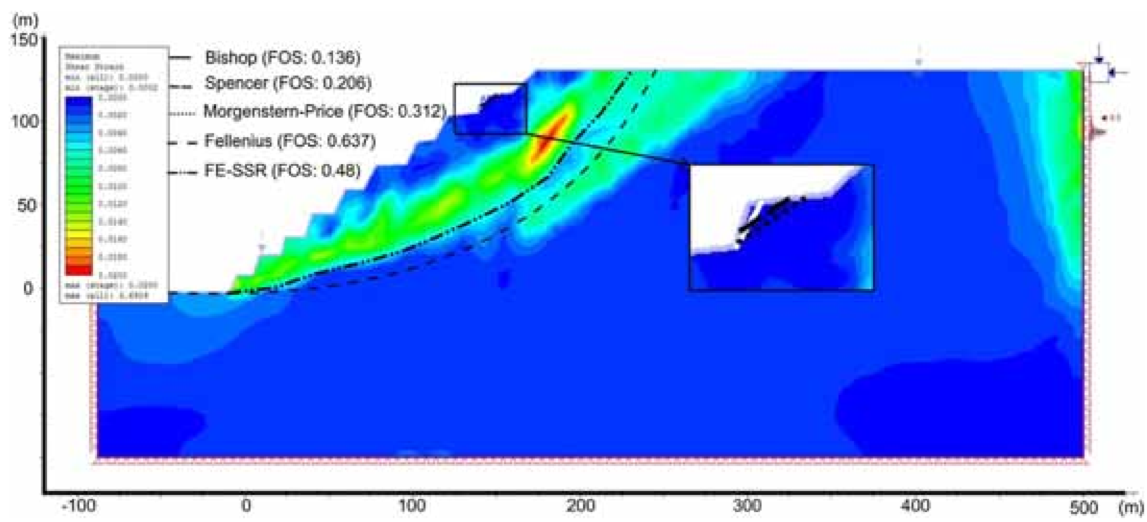

Figure 6. The failure surfaces detected by the LEMs and FE-SSR for slope model-1, while GSI: 35, slope angle: 36°, WTL: 100%, seismic coefficient: 0.3 g.

FE-SSR are most similar to the ones obtained from Spencer if only the seismic coefficient is 0.1 g and slope angle is 32°. In the case of higher seismic coefficients and slope angles, Morgenstern–Price is the only method which is in harmony with FE-SSR.

3.3 *Comparison of the failure surfaces*

The critical failure surfaces corresponding to global minimum FOS values obtained from LEM and FE-SSR were compared. The local minimum FOS values and associated failure surfaces were not considered in this study. It was determined that Fellenius detects deeper failure surfaces than the other methods for almost all cases (figures 3–7). Bishop, Spencer, and Morgenstern–Price demonstrate generally similar failure surfaces (figures 3–6). While the FOS values significantly change from case to case, Fellenius is generally not affected by the FOS value and detects similar failure surfaces. However, failure surfaces defined by Morgenstern–Price differ as the WTL and slope angle changes. Despite the drastic reduction in FOS values calculated by Bishop and Spencer under the seismic condition, the failure surfaces remain the same (figures 5 and 6). On the other hand, as the GSI rating increases, Morgenstern–Price tends to detect the overall rotational failure surface (figure 7).

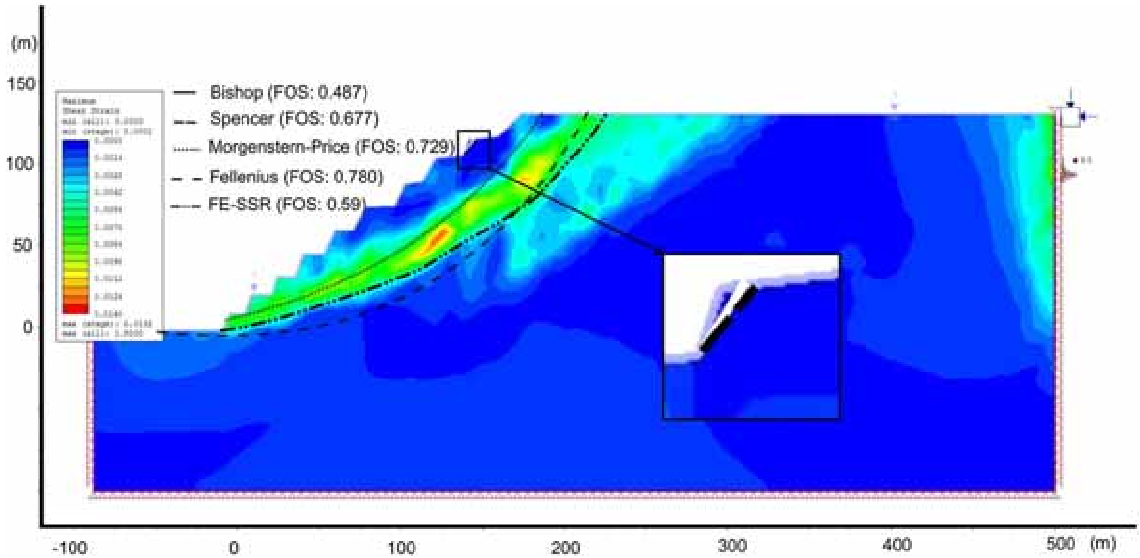

Figure 7. The failure surfaces detected by the LEMs and FE-SSR for slope model-1, while GSI: 45, slope angle: 36°, WTL: 100%, seismic coefficient: 0.3 g.

While the maximum shear strain contours obtained from FE-SSR are evaluated, the failure surface of FE-SSR is generally comparable with the failure surface detected by Fellenius. Nevertheless, the failure surfaces obtained from Fellenius are slightly deeper than the other methods.

4. Conclusions

In recent years, researchers have investigated the stability of a great number of slope models with LEM and FEM mostly for soil slopes. However, the differences in FOS values and failure surfaces have not been profoundly investigated for highly fractured rock masses under various slope conditions. Thus, it is primarily aimed to compare FOS values obtained from LEMs and FE-SSR for a variety of cases through comparative statistics. Secondly, the corresponding critical failure surfaces were examined and compared for 431 cases.

Statistical analyses indicated that, except for some cases, none of the LEMs matched with FE-SSR for FOS values. In particular cases, both methods may over- or underestimate the real case in the field. In contrast to the earlier findings, the highest FOS value was obtained from Fellenius and the lowest from Bishop. Furthermore, simplified Bishop and Spencer are very conservative in pseudo-static analyses; however, the critical failure surfaces do not change significantly although a drastic reduction in FOS occurs under seismic conditions. While the water table level is 70% and higher and/or under seismic conditions, the FOS values obtained from LEMs are not comparable among themselves as well as with FE-SSR. This result shows that the findings of various researchers that LEM and FE-SSR give similar FOS values for homogeneous slopes is not valid for heavily fractured rock slopes with high water saturation and/or under seismic conditions at lower slope angles than 36° (30–32–34). Despite all, under static conditions, all the methods agree with each other except Fellenius.

Nevertheless, Morgenstern–Price is more consistent with FE-SSR at slope angles, and seismic coefficients higher than 32° and 0.1 g, respectively. Unlike the FOS similarity, critical failure surface detected by Fellenius generally matches well with FE-SSR, especially for the cases WTL: 100%, seismic coefficient: 0.2–0.3 g, and GSI: 35. However, Fellenius is declared to be unfavourable for slope stability evaluation since it neglects the horizontal and vertical interslice forces. Fundamentally, it can be argued that the consistency between LEMs and FE-SSR in terms of FOS value decreases as the water saturation and seismic coefficient increase. It is accordingly suggested that the validity of the LEMs and FE-SSR should be supported with the field observations. Despite the lack of literature comparing LEM and FE-SSR for heavily fractured rock masses obeying GHBC, the results of this study will serve as a base for choosing appropriate methods for high slopes with benches under a variety of slope conditions.

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