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Groundwater contamination in mega cities with finite sources

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Groundwater contamination due to multiple sources occurring in mega cities was modelled. One constant source contamination was considered at the source boundary, whereas other sources may join in between at various locations at different times. Initially, the aquifer was contamination-free in mega cities and was subsequently contaminated by means of different sources in due course of time. One-dimensional ADE (Advection Dispersion Equation) for modelling groundwater contamination was used and solved analytically in the semi-infinite aquifer domain for a finite number of point sources. A numerical solution was also obtained for two sources to compare analytical solutions. Results were examined for different velocity profiles to show the maximum contaminant concentration level with distance. This may be helpful to model the maximum possible number of point sources of contamination (i.e., it represents approximately what happens in the field situation). Some remedial measures may be taken to overcome these kinds of contamination problems in mega cities by treating the sources so that recharge of the aquifer is not affected.

Keywords. Groundwater; contamination; mega cities; finite sources.

List of symbols

x' and T'	Distance [L] and time [T]
$D_{x'}$	Dispersion along positive x' direction $[L^2T^{-1}]$
D_0	Initial dispersion coefficient $[L^2T^{-1}]$
$u(T')$	Groundwater velocity profile $[LT^{-1}]$
u_0	Initial velocity of groundwater $[LT^{-1}]$
C_i, c_0, c_0''	Constant concentration $[ML^{-3}]$
$C'(x', T')$	Contaminant concentration for first source $[ML^{-3}]$
x_0 and t_0	Fixed distance [L] and fixed time [T]
$C_1(x', T')$	Contaminant concentration for second source $[ML^{-3}]$
C_i'	Concentration at (x_0, t_0) of first source $[ML^{-3}]$
C_{n0}	Contaminant concentration of nth source $[ML^{-3}]$
C_{ni}	Initial contaminant concentration for the nth source $[ML^{-3}]$

1. Introduction

Groundwater has been one of the most valuable sources for water supply in mega cities in India. Mega cities of different size have been formed, depending upon the migration from rural areas and

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growth of population and industries. Rapid growth of industries has been responsible for increasing the risk of pollution in-and-around the cities. Industrial wastes have been and are dumped without recycling. River Yamuna near Delhi is one of the glaring examples of how badly the pollutants of mega city have affected the entire ecosystem. Similarly, groundwater bodies in-and-around mega cities have been severely polluted because of pollutant effluents. Groundwater usually assimilates the contaminants at a certain rate, but if the rate of mixing of pollutants is greater than the rate of assimilation then the aquifer becomes more contaminated in due course of time. In mega cities, groundwater has been contaminated in several ways, such as industrial pollutants, municipal waste, mine residues, etc., that normally occur.

During the past few decades, a multitude of studies have been carried out on groundwater contamination at local as well as global levels. For example, the pollution in mega cities like Delhi situated on the banks of River Yamuna may affect the nearby aquifer. Misra (2010) described that the Yamuna has turned into a small watercourse mainly because of pollution. There has been a constant effort made by the Central Groundwater Board, New Delhi, to reduce pollutant loads and protect the river. According to Christopher *et al.* (2012), the presence of heavy metals degrades the water quality of River Yamuna in Delhi. At the entry to Delhi, the river satisfies the water quality standards (Dissolved Oxygen (DO) and Biochemical Oxygen Demand (BOD)), although during its exit, the quality of the water becomes low and becomes unusable.

The Yamuna River water quality assessment near Delhi using index mapping was attempted by Katyal *et al.* (2012) who developed a methodology using GIS to integrate the Water Quality Index (WQI) for effective interpretation of the status of river water quality. Yamuna's water quality near Delhi was also examined by Dhillon *et al.* (2013) who concluded that Yamuna was one of the most polluted rivers in India largely because of rapid growth of industries and high density of population. Delhi dumps about 58% of its treated or partially treated waste into the river that is directly connected to the aquifer. Heavy metals and other pollutants mix with groundwater and contaminates rapidly. If we concentrate on the River Yamuna, then the surrounding aquifer has been contaminated with heavy metals. It may be said that there is a direct impact of the pollutants of surface water bodies on the groundwater in mega cities. To overcome groundwater contamination in mega cities has now become a challenging task in India.

Various groundwater contamination problems have been modelled to investigate the level of contamination in groundwater. Analytical solutions of various types of advection dispersion equations were derived by Gershon and Nir (1969), Gelher and Collins (1971), Sim and Chrysikopoulos (1999), Smedt (2006), Srinivasan and Clement (2008), among others. Groundwater flow and radionuclide transport in single fractures with diffusion was discussed by Saied and Khalifa (2002) and Khalifa (2003). Singh *et al.* (2014) and Singh and Kumari (2014) considered time dependent source condition with longitudinal dispersion in a semi-infinite aquifer with unsteady groundwater flow, where velocity and dispersion were considered as functions of time. With the use of arbitrary inlet boundary condition, Chen and Liu (2011) and Chen *et al.* (2012a, b) obtained a generalized analytical solution. Zamani and Bombardelli (2013) analytically solved non-linear variable-parameter transport equations to verify the numerical solvers. One-dimensional advection dispersion equation was solved numerically by Ahmad *et al.* (1999), Kumar *et al.* (2007), and Diwa *et al.* (2001). In view of numerical solution, a semi-analytical solution was presented by Zhang *et al.* (2012) for conservative and non-reactive tracer. Comparison of numerical and analytical solutions was made by van Genuchten *et al.* (1982) and Ataie-Ashtini *et al.* (1996).

In this study, assuming simplified mega city contamination, one-dimensional advection–dispersion equation was employed where a finite number of sources (i.e., n) were acting one after another and mixing in the aquifer after a certain time (see figure 1). The aquifer was initially contaminated with some constant level which may be applicable for big cities where various leakages in high drains and industrial wastes are mixed with the aquifer and contaminate groundwater. Most of the investigations carried out in the past considered input sources either at the origin or at any intermediate point, which may not always be the case for big cities. However, these two cases may occur at the same time. A single point source may not have so much of an impact on groundwater contamination. In this paper, a semi-infinite aquifer was considered so that many sources may mix after certain distances and certain times. A single source is not enough to model

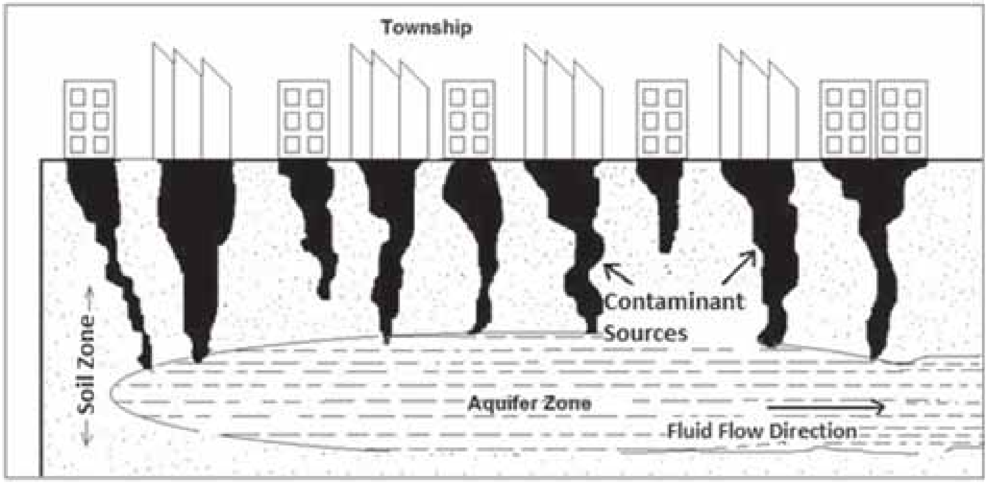

Figure 1. Various point sources acting one after another in the aquifer.

this kind of pollution in big cities in developing countries like India. A finite number of sources (n) which mix with one after another in the aquifer were considered to analytically model contaminant transport. The velocity of groundwater was considered unsteady and solute transport occurred only in the positive course of x-axis, i.e., not against the direction of the flow. The Laplace Transform Technique (LTT) was used to solve the ADE analytically and then an explicit finite difference method was used to validate the solution. Groundwater quality in mega cities was discussed by Foster *et al.* (1999), Foster (2001), Onodera *et al.* (2008) and Umezawa *et al.* (2008), but most of these investigators dealt with the management and distribution of groundwater. To the best of our knowledge, no mathematical model is available for groundwater contamination in maga cities. The aim of this work, therefore, is to set up a simple but useful mathematical model for mega cities. In a mega city, different types of sources is mixed with the aquifer at various places with increasing time. This situation can be explained by using the governing advection dispersion equation subject to different sources at various places. That may be useful for further research on groundwater contamination in mega cities with varying conditions and geometry.

In this study, our main objective was to bridge the gap between point- and non-point sources which have been studied independently to deal with various types of groundwater contaminations. The finite number of point sources, i.e., n, were considered to model solute transport. If these sources were distributed throughout the domain, then they would be like a non-point source of contamination. This may be a newer approach to solve groundwater contamination problems in mega cities and other areas as well using simple mathematical models.

2. Case analysis and mathematical formulation

2.1 *Case analysis*

The soil zone is heterogynous, depending upon space and geometry. To predict groundwater from contaminant transport through porous media, the advection dispersion equation is often used. In this present study, we want to model contaminant concentration in mega city using multiple sources. In order to incorporate heterogeneity, we considered that dispersion ($D(x,t)$) and velocity ($u(x,t)$) were functions of space as well as time. Source/sink terms, which must be functions of both space and time ($g(x,t)$), may be incorporated in the ADE. Thus, the model equation can be written as follows:

$$\frac{\partial C}{\partial t} = D(x,t)\frac{\partial^2 C}{\partial x^2} - u(x,t)\frac{\partial C}{\partial x} \pm g(x,t). \quad (1)$$

Now the next question arises about the initial condition of the aquifer. Two cases may be present in a real life scenario: (i) contaminant-free and (ii) contaminated. If the aquifer is contaminated initially, then two probable cases arise: (i) constant amount and (ii) space dependent. So, the general initial condition may be written as:

$$C(x,t) = C_i + P(x), \quad t = 0,\ x \geq 0. \quad (2)$$

Now if we look for the source condition then it is clear from the real life phenomenon that only two types of source conditions may be present: (i) constant in nature and (ii) time-dependent in nature. The general source condition can then be written as:

$$C(x,t) = c_0 + F(t), \quad x = 0,\ t \geq 0. \tag{3}$$

For outlet boundary we considered zero flux condition, as it is quite significant in real life situations.

In the present problem, we considered that 'n' number of sources were acting one after another in a certain interval so the considered ADE must be the same for all sources. As we are talking about cities we have some facilities that we want to impose on the derived equation:

1. Generally a city or mega city is formed in the same region, i.e., over the same geological formation. That means the heterogeneity is not that much impactful in the case of cities/ mega-cities so one can remove the space dependency of dispersion and velocity from the presented ADE, i.e., now the dispersion and velocity depend on time only.
2. We considered the situation from the beginning of the city so there was no chance for additional source or sink present in the aquifer-aquitard system. Hence, we can consider that $g(x,t) = 0$.

Equation (1) changes to:

$$\frac{\partial C}{\partial t} = D(t)\frac{\partial^2 C}{\partial x^2} - u(t)\frac{\partial C}{\partial x}. \tag{4}$$

We used equation (4) to model contaminant transport in porous media.

As in most of the cases, the geological formation of a city or mega city is the same, so space dependency did not have much impact on groundwater contamination. So we can model the situation by using constant term for the initial condition.

Since the movement of groundwater is slow with respect to time, very few specific functions are often used to model the system mathematically. If we look at the series expansion of functions, we see that the major contribution is coming from the constant source and a very small impact is coming from the time dependent part, so one can ignore the time-dependent part and can consider constant source only. Although it is quite simple in nature, this model is applicable in real life to model groundwater contamination in mega cities.

2.2 *Mathematical modelling*

It was considered that initially there may be a one large industry in a mega city and the industry dumped its contaminants in soil which reached the aquifer and contaminated groundwater. The ADE was introduced to predict the contaminant concentration in the aquifer. If we look back, we see that the treatment of pollutants was not that much strict in India so initially the industries dumped their garbage without treating them. Therefore, we may be able to consider that the first industry which was placed in the maga city dumped his garbage in a specific area. On each day, a constant amount of contaminants or pollutants comes and joins the dumped region. To model the situation mathematically, we considered that the constant source of contamination was occurring at the origin (the place of dumping). After a certain time, when the number of industries increased then the number of sources also increased. Similarly, we considered the constant source of contamination for other industries too. One thing remained is about the impact of the first source on the second and so on. To overcome this problem, we considered the average impact of all the sources acting one after another on groundwater contamination.

Studies have indicated that the contaminant level becomes zero after some certain distance after some certain time. The zero flux type boundary condition was considered to model the system mathematically, considering the aquifer to be semi-infinite. We considered the initial time when the first source mixed with the aquifer. However, if we think closely we can say that before the first industry comes into the picture there must be some contaminant present in the aquifer system. But the amount of that contaminant concentration is not much and does not depend on time and space. To represent the situation mathematically, the aquifer was assumed to be contaminated with an initial background concentration as a constant input source say, C_i.

Then, contaminant transport was mathematically formulated as:

$$\frac{\partial C'}{\partial T'} = D_{x'}\frac{\partial^2 C'}{\partial x'^2} - u(T')\frac{\partial C'}{\partial x'}, \tag{5}$$

with the following conditions:

$$C'(x',T') = C_i, \quad T' = 0,\ x' \geq 0, \tag{6}$$

$$C'(x',T') = c_0, \quad x' = 0,\ T' \geq 0, \tag{7}$$

$$\frac{\partial C'}{\partial x'} = 0, \quad \text{as } x' \to \infty. \tag{8}$$

where $C'(x', T')$ is the contaminant concentration for the first source, C_i, c_0 are the constant concentrations, $D_{x'}$ is the dispersion along positive x' direction, and $u(T')$ is the groundwater velocity profile.

Next we consider another constant source of contaminant concentration (c_0'') after a certain distance and time, i.e., x_0 and t_0, respectively. The effect of the first source of contamination in the aquifer obtained by the solution of the first system at (x_0, t_0) was treated as an initial concentration (C_i') for the second source. The flux type boundary condition at the semi-infinite extent of the system was considered for the second source. The mathematical formulation of the second system was written as follows:

$$\frac{\partial C_1}{\partial T'} = D_{x'} \frac{\partial^2 C_1}{\partial x'^2} - u(T') \frac{\partial C_1}{\partial x'}, \tag{9}$$

with the following conditions:

$$C_1(x', T') = C_i', \quad T' = t_0, \; x \geq x_0, \tag{10}$$

$$C_1(x', T') = c_0'', \quad x' = x_0, \; T' > t_0, \tag{11}$$

$$\frac{\partial C_1}{\partial x'} = 0, \quad \text{as } x' \to \infty. \tag{12}$$

We now have to find the value of C_1 to determine the complete solution for the system.

In a similar manner, we can model the system with initial and boundary conditions for third, fourth, and fifth sources, and so on. Therefore, the model for the nth source can be written as:

$$\frac{\partial C_{n-1}}{\partial T'} = D_{x'} \frac{\partial^2 C_{n-1}}{\partial x'^2} - u(T') \frac{\partial C_{n-1}}{\partial x'}, \tag{13}$$

with the following conditions:

$$C_{n-1}(x', T') = C_{ni}, \quad T' = t_{n-2}, \; x \geq x_{n-2}, \tag{14}$$

$$C_{n-1}(x', T') = c_{n0}, \quad x' = x_{n-2}, \; T' > t_{n-2}, \tag{15}$$

$$\frac{\partial C_{n-1}}{\partial x'} = 0, \quad \text{as } x' \to \infty. \tag{16}$$

$C_{n-1}(x', T')$ is the contaminant concentration for the second source, C_{n0} is the value of the nth source, and C_{ni} is the value of contaminant concentration when the nth source mixes.

3. Analytical solution

We first introduce the coordinate system for clarity. Authors consider a semi analytical domain in the distance and consider that the first source mixes at the origin and then the sources mix one after another after some time and distance. During the solution process our coordinate system is fixed, although we shift the origin when another source mixes with the aquifer so our variable changes; the initial situation is as given in figure 1. We assume that $u = u_0 f(T')$ and $D = au$. Here $f(T')$ is either exponential or sinusoidal. For discussion, we take $f(T') = 1 - \sin(mT')$, which is a sinusoidal form of velocity.

Consider, $T = \int_0^{T'} f(T') dT'$.

Taking $x = \frac{x' u_0}{D_0}$, $C = \frac{C'}{c_0}$, and $t = \frac{u_0^2 T}{D_0}$, equation (5) can be written as:

$$\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial x^2} - \frac{\partial C}{\partial x}. \tag{17}$$

The initial and boundary conditions, represented by equations (6–8), can also be written as:

$$C(x, t) = \frac{c_i}{c_0}, \quad x \geq 0, t = 0, \tag{18}$$

$$C(x, t) = 1, \quad t \geq 0, x = 0, \tag{19}$$

$$\frac{\partial C}{\partial x} = 0, \quad x \to \infty. \tag{20}$$

Now we consider that $C(x, t) = K(x, t) e^{(x/2 - t/4)}$ and by substituting in equations (17–20) and using LTT, we get:

$$C(x, t) = C_{ii} + e^{(x/2 - t/4)} L^{-1} \left[\frac{c_0'}{s - \frac{1}{4}} e^{-x\sqrt{s}} \right]. \tag{21}$$

With reference to the new coordinates system, we consider another source with constant contamination after time t_0 and at point x_0. Then, the transformed equations can be written as follows:

$$\frac{\partial C_1}{\partial T'} = D_{x'} \frac{\partial^2 C_1}{\partial x'^2} - u(T') \frac{\partial C_1}{\partial x'}, \tag{22}$$

$$C_1(x', T') = C_i', \quad T' = t_0, \; x \geq x_0, \tag{23}$$

$$C_1(x', T') = C_0'', \quad x' = x_0, \; T' > t_0, \tag{24}$$

$$\frac{\partial C_1}{\partial x'} = 0, \quad \text{as } x' \to \infty. \tag{25}$$

where $C_i' = C(x_0, t_0)$.

Since we want to get the ADE with constant coefficient, we consider that $T=\int f(T')dT'$ to change the time dependent coefficient to a constant one.

Also, using $x=\frac{x'u_0}{D_0}$, $C_1'=\frac{C_1}{C_i'}$, and $t=\frac{u_0^2T}{D_0}$, equation (22) can be transformed as:

$$\frac{\partial C_1'}{\partial t}=\frac{\partial^2 C_1'}{\partial x^2}-\frac{\partial C_1'}{\partial x}, \tag{26}$$

$$C_1'(x,t)=1, \quad t=t_0', x_1\geq x_0', \tag{27}$$

$$C_1'(x,t)=\frac{C_0''}{C_i'}, \quad x=x_0', t_1>t_0', \tag{28}$$

$$\frac{\partial C_1'}{\partial x}=0, \quad x\to\infty. \tag{29}$$

Considering $T_1=(t-t_0')$ and $X_1=(x-x_0')$, the governing equations and boundary conditions change as follows:

$$\frac{\partial C_1'}{\partial T_1}=\frac{\partial^2 C_1'}{\partial X_1^2}-\frac{\partial C_1'}{\partial X_1}, \tag{30}$$

$$C_1'(X_1,T_1)=1, \quad T_1=0, X_1\geq 0, \tag{31}$$

$$C_1'(X_1,T_1)=\frac{C_0''}{C_i'}, \quad X_1=0, T_1>0, \tag{32}$$

$$\frac{\partial C_1'}{\partial X_1}=0, \quad X_1\to\infty. \tag{33}$$

For solving these equations, we take

$$C_1'(X_1,T_1)=K_1(X_1,T_1)e^{(X_1/2-T_1/4)},$$

where $K_1(X_1,T_1)$ is a function such that if we multiply $e^{(X_1/2-T_1/4)}$ with this we get back the contaminant concentration ($C_1'(X_1,T_1)$).

Now taking the Laplace transform and solving equations (30–33) (Singh *et al.* 2014; Singh and Kumari 2014; Singh and Das 2015), the final solution can be written as follows:

$$C_1'(X_1,T_1)=\frac{C_0''}{2C_i'}\left[\operatorname{erfc}\left(\frac{X_1}{2\sqrt{T_1}}-\frac{\sqrt{T_1}}{2}\right)+\exp(X_1)\operatorname{erfc}\left(\frac{X_1}{2\sqrt{T_1}}+\frac{\sqrt{T_1}}{2}\right)\right]+\frac{1}{C_i'}. \tag{34}$$

Proceeding in a similar manner we were able to get the contaminant concentration after the third source mixes after time t_0' at distance x_0' from the origin as follows:

$$C_2'(X_2,T_2)=\frac{C_0'''}{2C_i''}\left[\operatorname{erfc}\left(\frac{X_2}{2\sqrt{T_2}}-\frac{\sqrt{T_2}}{2}\right)+\exp(X_2)\operatorname{erfc}\left(\frac{X_2}{2\sqrt{T_2}}+\frac{\sqrt{T_2}}{2}\right)\right]+\frac{1}{C_i''}. \tag{35}$$

where C_0''' is the value of the third source which mixes with the aquifer and C_i'' is the value of contamination when the third source mixes with the aquifer system. Also $X_2=X_1-x_1$ and $T_2=T_1-t_1$.

Proceeding in a similar manner we may able to find the solution when the fourth source mixes with the system and so on. The contaminant concentration after the nth source mixes with the aquifer was obtained as follows:

$$C_{n-1}'(X_n,T_n)=\frac{C_{n0}}{2C_{ni}}\left[\operatorname{erfc}\left(\frac{X_n}{2\sqrt{T_n}}-\frac{\sqrt{T_n}}{2}\right)+\exp(x_n)\operatorname{erfc}\left(\frac{X_n}{2\sqrt{T_n}}+\frac{\sqrt{T_n}}{2}\right)\right]+\frac{1}{C_{ni}}, \tag{36}$$

where C_{n0} is the value of the nth source, C_{ni} is the value of contaminant concentration when the nth source mixes, $X_n=X_{n-1}-x_{n-1}$, and $T_n=T_{n-1}-t_{n-1}$.

4. Numerical solution

For numerical solution, we considered $X'=1-e^{-x}$ to change the semi-infinite domain into a bounded domain [0, 1]. Equations (17–20) are now transformed to:

$$\frac{\partial C}{\partial t}=(1-X')^2\frac{\partial^2 C}{\partial X'^2}-2(1-X')\frac{\partial C}{\partial X'}. \tag{37}$$

The corresponding initial and boundary conditions is as follows:

$$C(X',t)=\frac{c_i}{c_0}, \quad X'\geq 0, t=0, \tag{38}$$

$$C(X',t)=1, \quad t\geq 0, X'=0, \tag{39}$$

$$\frac{\partial C}{\partial X'}=0, \quad X'\to 1. \tag{40}$$

We take $X'_i = X'_{i-1} + \Delta X'$ with $X'_0 = 0, \Delta X' = 0.01$ and $i = 1, 2, \ldots, M$ and $t_j = t_{j-1} + \Delta t$ with $t_0 = 0, \Delta t = 0.001$, where $j = 1, 2, \ldots, I$.

Using forward time and central space, equation (37) changes to:

$$\frac{C_{i,j+1} - C_{i,j}}{\Delta t} = (1 - X'_i)^2 \frac{(C_{i+1,j} - 2C_{i,j} + C_{i-1,j})}{\Delta x^2} - 2(1 - X'_i) \frac{(C_{i+1,j} - C_{i-1,j})}{2\Delta x}, \tag{41}$$

and the initial and boundary conditions change to:

$$C_{i,0} = \frac{c_i}{c_0}, \quad i > 0, \tag{42}$$

$$C_{0,j} = 0, \quad j > 0, \tag{43}$$

$$C_{M,j} = C_{M-1,j}, \quad j > 0. \tag{44}$$

The stability condition for the size of time step was obtained as follows:

$$0 < \Delta t \leq \frac{1}{2\left(\frac{D_0}{(\Delta X')^2} + \frac{u_0}{2\Delta X'}\right)}. \tag{45}$$

Similarly, we proceeded for the second source, the third source, and so on to obtain the final contaminant concentration where $\Delta X'_0$ took the value of 0.5 and the stability criterion remained the same.

5. Results and discussion

The contaminant concentration of two sources was investigated analytically and numerically. Rather than plotting n number of sources we show, for simplicity, two sources to discuss contaminant transport. We considered that the aquifer was initially contamination free and the contaminant concentration of the first source was 0.5. Let the first source mix in the aquifer in the month of July, i.e., the time of rainy season so the velocity of groundwater was more, say 0.07 km/year, $d_0 = 0.25$ km^2/year, and time was taken in years.

In figure 2, the sinusoidal velocity and dispersion were considered. The contaminant concentration decreased, with a peak value of 0.5 and the contaminant concentration was fixed with a distance of 1. At point x=0.5 just after a few months, the second source mixed with the aquifer with a constant contamination of 1.0. Since it was not a rainy season, the velocity of groundwater was considered as 0.05. The initial contamination for the second source was not zero and depended on the first source, here it was approximately 0.45. For the second source, the peak value became 1.0 and decreased up to a distance of 1 and then the contaminant concentration became fixed at almost 0.8 after the distance of 1. This case is possible for any aquifer. Here we compared the analytical solution with the numerical solution, which allowed to show that case was realistic and the data which were taken matched the real situation. In figure 3, the exponentially decreasing velocity and dispersion were considered in which red lines show the numerical solution and the black lines show the analytical solution and both of them were in good agreement.

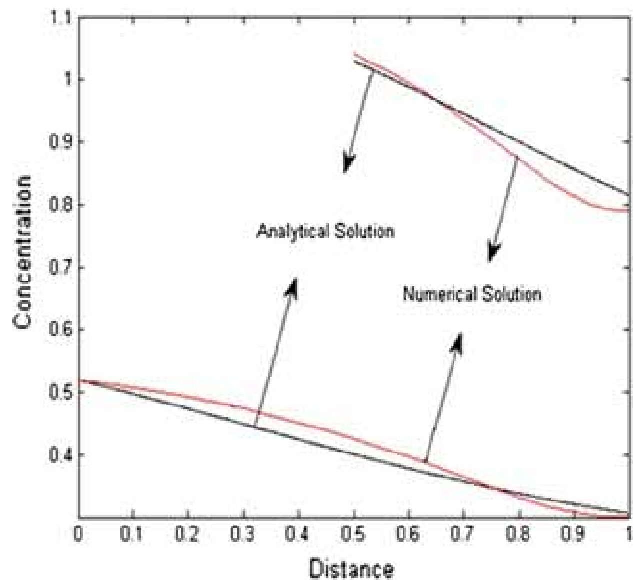

Figure 2. Contaminant concentration distribution with distance for sinusoidal velocity pattern.

At first, the contaminant concentration decreased for some time and then after a certain length of the aquifer, it stabilized. When the second source was mixed, a sudden jump in contaminant concentration was observed and after some instance the concentration decreased and neutralized. Similarly, if the contamination profile were plotted for the third, fourth, fifth sources and so on, similar phenomena would be observed.

For example, we consider only big cities or mega cities. There are some industries that dump their waste into the soil and this waste may mix with the aquifer and after some distance there may be a leak in drains and municipal garbage or mine spoils may be there to contaminate the aquifer. So there is enough scope to interpret the results as an n point

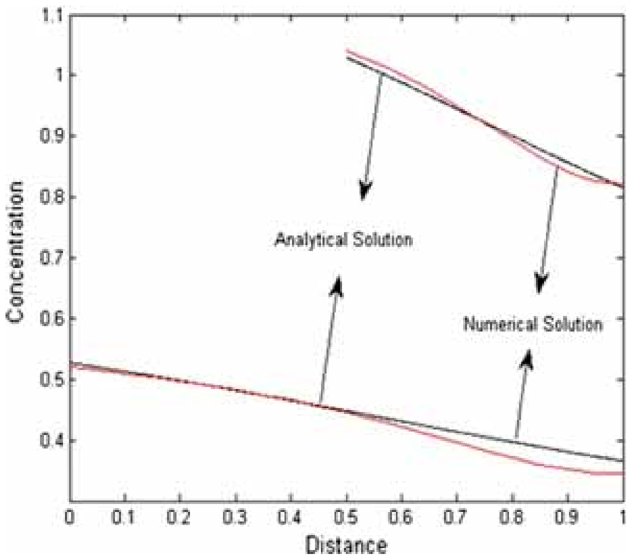

Figure 3. Contaminant concentration with distance for exponentially decreasing velocity.

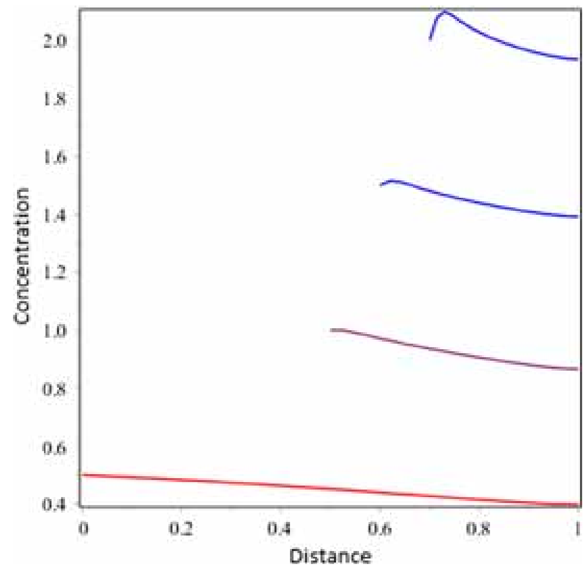

Figure 4. Contaminant concentration with distance for different sources acting one after another at x=0, x=0.5, x=0.6 and x=0.7 depicted, respectively, in (1), (2), (3) and (4).

source contamination where basically each source mixes with the aquifer after a certain time and certain distance. Now if we minimize the distance between two sources to a smaller length and do the same for n sources, then n point sources of contamination may behave like a distributed non-point line source after some time.

We used maple to show the effect on the aquifer of four sources mixing at distances x=0, x=0.5, x=0.6, and x=0.7, respectively, one after another in between one year. The Du Fort Frankel scheme was used to solve the system of equations numerically at a fixed time. Du Fort Frankel scheme has been designed to overcome the stability problem of simple algorithms. Du Fort Frankel method is a trick which exploits the unconditional stability of the intrinsic method for simple differential equations. When von Neumann stability analysis was implemented, we observed that the method was unconditionally stable but the accuracy of this method was not much high. Here we want to show how the contamination in the aquifer increases rapidly with the mixing of sources after some distance and certain time. From figure 4, we see that the contamination level increases when the number of sources increases.

6. Validation of the mathematical model

Groundwater water quality in maga cities has been discussed by many researchers, but most of them deal with the management and distribution of groundwater. According to Foster (2001), pollution by nitrate compound and level of salinity increase day by day. Foster *et al.* (1999) have discussed that solubility of iron and other metals increases because of organic carbon. Umezawa *et al.* (2008) showed that in the aquifers of Metro Manila, Bangkok, and Jakarta, NO^{3-} and NH^{4+} contamination is not excessive at present but the increment of nitrogen load and Gross Domestic Product (GDP) may affect the contamination level in near future. Onodera *et al.* (2008) discussed the effect of intensive urbanization on the intrusion of shallow groundwater into deep groundwater in the context of Bangkok and Jakarta. In West Bengal, the state government (2016) reports various forms of pollutants in Kolkata and nearby cities. The number of affecting regions increases day by day. Similarly, Kumar *et al.* (2009) discussed that 64% of the groundwater in Delhi transformed to saline in 2003, although the percentage was 45% in 2000. Haque *et al.* (2013) described how nitrate and chloride concentrations in groundwater increased with increasing time. In this context, we demonstrate how the contaminant concentration in maga cities increases with time. From our research we can conclude that day by day groundwater contamination increases for maga cities up to saturation level. If our analytical model produces the same results then it can be said to be also valid for groundwater contamination in mega cities.

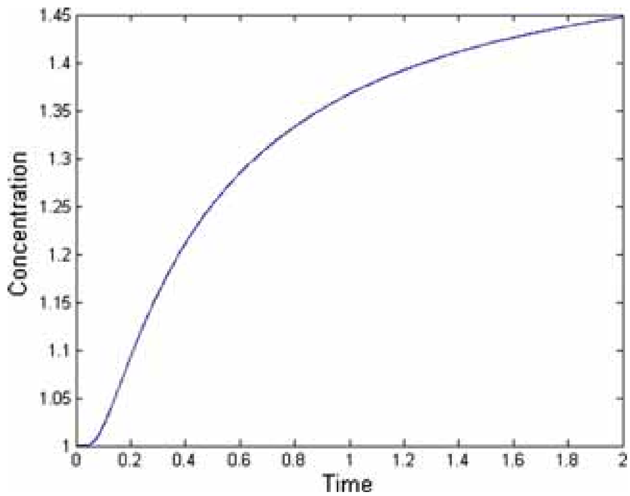

Figure 5. Contaminant concentration with time for fixed value of x=1 unit.

Using a simple mathematical model, we can interpret groundwater contamination in mega cities. We depict through graphical representation that the contaminant concentration level increases up to 1.5 with time. We use the nth source value as 0.5 and initial contamination for nth source as 1 and therefore, it is clear that the value of contaminant concentration increases with time at a rapid rate which is quite similar with the real-life situation as stated above and depicted in figure 5.

Both models show different approaches to predict groundwater contamination, but the goal of these two models is same. In both models we want to predict contaminant concentration after some certain time by different approaches. In this case we are not comparing models but the results of these models with our model. So this comparison is considered as validation.

7. Conclusions

The maximum contaminant concentration level reduced with distance and it may be further reduced to a constant value for a single source. The mixing of every source after a certain distance may affect the level of contaminant concentration. The contaminant concentration increases rapidly with distance when the number of sources increases and cannot be controlled so easily without preventing the sources. The contaminant concentration level in the aquifer is dependent on the number of sources; when the number of sources increases in a small time then the contaminant concentration level also increases. Therefore, the sources may be treated by taking some remedial measures, such as (1) residue of the industries and household may be treated properly before releasing to the soil and (2) maintain a certain distance between two industries or households for reducing the risk of groundwater pollution. The n number of point sources taken in this study behaves like a non-point line source in due course of time and may help model the real life situation of mega cities. The numerical and analytical results are compared and are found to be in good agreement.

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State of Environment report West Bengal 2016.

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