Co-seismic grace gravity-based 11-layered 3-D thrust fault model for the Sumatra earthquake 2004

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MS received 24 May 2017; revised 20 April 2018; accepted 30 April 2018;
published online 19 January 2019

Our co-seismic Gravity Recovery and Climate Experiment gravity data (level 2 ‘RL_05’ data product ‘GX-OG-2-GSM’) for the Sumatra earthquake 2004 is obtained by differencing monthly gravity field average for November 2004 from that of January 2005 and band-pass filtering (17–30° and orders) in the spectral domain. Here, we propose a 11-layered three-dimensional (3-D) thrust fault gravity model based on different co-seismic rupture models in the literature. It honours co-seismic deformation of the ocean surface, ocean bottom and subsurface earth medium, different earthquake parameters and hypocentre information (~35 km below mean sea level). Our forward gravity response matches well with the observed gravity (RMS error of 0.06257 μgal (6.26%)) data and our model allowed an independent computation of rupture length, instantaneous velocity, average seismic moment and momentum, which are, respectively, 1560 km, 2.9 km/s, 4.53×10^{22} N m and 9.7×10^{17} kg m/s. These parameters fairly agree with those in the literature. The computed momentum indeed corresponds to an area pulse (9.7×10^{17} kg m/s) at ocean bottom that led to a tsunami generation. Thus, the proposed multi-layered 3-D gravity model in traditional fashion fully accounts for co-seismic gravity signal of the Sumatra earthquake 2004.

Keywords. GRACE satellite gravity; spherical harmonics; 3-D thrust fault model; co-seismic gravity anomaly.

1. Introduction

Global gravity monitoring through satellite gravity has received a big boost with the Gravity Recovery and Climate Experiment (GRACE) (Cazenave and Chen 2010). Important GRACE applications include seismic characterisation of offshore tsunami-genic mega-thrust earthquakes including 2004 Sumatra earthquake (Tapley *et al.* 2004; Han *et al.* 2006; De Viron *et al.* 2008; Heki and Matsuo 2010; Broerse *et al.* 2011; Simons *et al.* 2011; Wang *et al.* 2012a; Broerse *et al.* 2014). In mid-latitudes, the approximate spatial resolution of

GRACE works out to be a circle of radius of 300 km (Zlotnicki, private communication) and it facilitates GRACE gravity’s sensitivity to near-surface rupture processes of major offshore earthquakes.

Mega-thrust ruptures (Chile earthquake 1960 and Alaska earthquake 1964) and the associated crustal deformation at a convergent boundary leading to a mass re-distribution was initially tackled by Plafker (1972). Seismic history prior to the Sumatra earthquake 2004 was contributed by several workers (Lay *et al.* 2005; Engdahl *et al.* 2007; Katsumata 2015). Using different data sources, the co-seismic slip distribution of the Sumatra

earthquake 2004 was inferred by several workers (Ammon *et al.* 2005; Banerjee *et al.* 2005; Lay *et al.* 2005; Vigny *et al.* 2005; Subarya *et al.* 2006; Chlieh *et al.* 2007). Post-earthquake activity was also studied extensively (Araki *et al.* 2006; Briggs *et al.* 2006; Rhie *et al.* 2007; Sibuet *et al.* 2007; Konca *et al.* 2008).

The magnitude of the Sumatra earthquake 9.3 on 26 December, 2004 has resulted in a tsunami, which has caused great damage (Sumatra earthquake 2004 Wikipedia) when the Indian Plate was subducted by the Burma Plate. The epicentre of the Sumatra earthquake 2004 was 3.316°N and 95.854°E with a rupture length of 1300 km and hypocentre at 35 km (Lay *et al.* 2005; Engdahl *et al.* 2007; Katsumata 2015) below mean sea level (msl). It had a fault length more than 1300 km extending from 2° to 14°N with 10 m fault slip along the Sumatra–Andaman trough (Ammon *et al.* 2005; Lay *et al.* 2005; Vigny *et al.* 2005; Wang and Liu 2006; Chlieh *et al.* 2007; Rhie *et al.* 2007, Shearer and Bürgmann 2010). In offshore northern Sumatra, the plate motion is estimated (Socquet *et al.* 2006; Delescluse and Chamot-Rooke 2007) to be 47 mm/yr up to 004°N, and in northernmost Sumatra the motion along the right lateral Sumatra Fault is 25 mm/yr (Sieh and Natawidjaja 2000). The rupture length and slip distribution are addressed by Lay *et al.* (2005) and Poisson *et al.* (2011). Liu *et al.* (1998) have simulated the Sumatra tsunami generation and propagation.

Earlier elastic gravity modelling efforts include radially stratified earth model (Gilbert and Backus 1968), faulting on a finite rectangular plane buried in a homogeneous half-space that of Okubo (1992), dislocation in spherically symmetric earth model (Sun and Okubo 1993, 1998) and layered spherical earth involving spherical harmonic expansion (Pollitz 1997, 2011). Sun and Okubo's formulations (Sun and Okubo 1993, 1998) have initiated novel satellite gravity modelling, stress evolution and time–displacement models for simulated slip histories (Hayes *et al.* 2006; Harms *et al.* 2015). One- and two-dimensional fault models have been proposed by Wang *et al.* (2012b) and Barckhausen (2006) have dealt with the detailed magnetic anomaly map of the 2004 Sumatra earthquake. Using the sea level equation method, Broerse *et al.* (2014) have modelled the geoid undulation owing to ocean water movement initiated by co-seismic vertical and horizontal deformations of the sea floor of mega-thrust earthquakes. However, geoid

undulations can also be inferred from standard geodetic methods (Heiskanen and Moritz 1967; Lambeck 1990). Sastry and Sonker (2017) have outlined the GRACE gravity-based gravity model for 2011 Japan earthquake.

Recently, Gokula and Sastry (2015a,b) have derived the forward gravity analytic expression for a three-dimensional (3-D) vertical pyramid model and demonstrated its flexibility through several numerical experiments and a case study. The model is quite flexible enough for modelling subduction zone tectonics.

A comprehensive traditional co-seismic 3-D gravity models honouring ocean surface deformation, ocean bottom (thrust fault tectonics) upheaval and subsurface deformation duly constrained by seismological data and inferences are rare in the literature. So, we address this scientific gap through 3-D gravity modelling (11-layered thrust fault) based on a unit vertical pyramid model of Gokula and Sastry (2015a,b) for co-seismic gravity signal of the Sumatra earthquake 2004. Our model also infers rupture length, average instantaneous velocity, seismic moment and momentum of this mega-earthquake independently and these estimates match fairly well with the published literature.

We deal introduction in section 1, theory in section 2, data processing and methodology including estimation of instantaneous velocity, seismic moment and seismic momentum aspects in section 3, results in section 4, the detailed discussion in section 5 and conclusions in section 6.

2. Theory

Gravitational potential and its gradient are calculated from the spherical harmonic coefficients (90° and orders) for Sumatra earthquake 2004 from the following expressions:

$$\begin{aligned}
 V(r, \theta, \varphi) &= \frac{GM}{R+h} \sum_{n=0}^N \frac{1}{r^{n+1}} \sum_{m=0}^n [a_{nm} P_{nm} \cos \theta \cos(m\varphi) \\
 &\quad + b_{nm} P_{nm} \cos \theta \sin(m\varphi)].
 \end{aligned} \tag{1}$$

The satellite gravity is described by the following spherical harmonic expression:

$$\nabla V(r, \theta, \varphi) = \frac{GM}{(R+h)^2} \times \sum_{n=0}^N \frac{(n+1)}{r^{n+2}} \sum_{m=0}^n [a_{nm} P_{nm} \cos \theta \cos(m\varphi) + b_{nm} P_{nm} \cos \theta \sin(m\varphi)], \quad (2)$$

where $r = R/(R+h)$.

a_{nm} and b_{nm} are the spherical harmonic coefficients of degree n and order m and P_{nm} are the Legendre's spherical functions. θ and φ are the colatitude and longitude of a station, respectively, and $N = 90$.

3. Data and methodology

In the present study, we use GRACE gravity data (level 2 'RL_05' data product 'GX-OG_2-GSM') (Bettadpur 2007) in the form of spherical harmonic coefficients up to a maximum of 90° and orders. Broad steps of our methodology are as follows:

- (1) Computation of gravity potential (spherical harmonic synthesis) for pre- and post-earthquake (tsunami) using spherical harmonic coefficients available in the public domain (<http://isdc.gfz-potsdam.de>) using equation (1) and thereby the gravity anomaly distribution for both episodes.
- (2) Estimation of differential (tsunami-genic) gravity data by differencing pre- (November 2004) from post- (January 2005) gravity anomaly distribution (red contour) for different band-pass filter options of spherical harmonic coefficients.
- (3) Check for a proper band-pass filter ($17\text{--}30^\circ$ and orders) to yield co-seismic gravity signal (figure 1).
- (4) Framing a 3-D 11-layered gravity model based on available geophysical constraints in the published literature. The co-seismic top sea water layer deformation can be computed using geoidal height variation (Heiskanen and Moritz 1967; Lambeck 1990) for both pre- and post-earthquake events.
- (5) The geoidal height $N = T/\gamma$, where T and $\gamma (= GM/R^2)$ are, respectively, the disturbing potential and normal gravity on spherical earth.
- (6) A 3-D forward gravity modelling for Sumatra earthquake 2004 source region using a unit vertical pyramid model (Gokula and Sastry 2015a, b).

Figure 1. Our isolated GRACE satellite gravity map with superimposed 3-D forward gravity model response at a mean satellite height of 500 km above msl with a relevant background tectonic map of the 2004 Sumatra earthquake. The background tectonic map depicts Indian, Eurasian, Australian and Burma micro-plate boundaries. Epicentre of the 2004 Sumatra earthquake is also shown. Bold zero anomaly contour delineates the fault trace. The solid and dashed contours outline positive and negative gravity responses of our model (red) and observed (black) gravity anomalies. Gravity anomaly values are in the range -0.35 to $0.20 \mu\text{gal}$. The root-mean-square (RMS) error deviation of computed gravity response with that of the observed gravity is $0.06257 \mu\text{gal}$ (6.26%).

- (7) Estimation of instantaneous velocity, seismic moment and seismic momentum on the basis of excess mass derivable from step (4) above.

3.1 Instantaneous velocity computation

Instantaneous velocity is the speed at a given moment of a wavefront in the direction of energy propagation (Sheriff and Stickney 1984). We equate the total energy (www.alabamaquake.com/energy.html) released for an earthquake of magnitude, M9.0 is treated as kinetic energy, which in turn is related to excess mass and instantaneous velocity. The excess mass of our model can be worked out from arrived model parameters from figure 2 and using equations (3) for a unit pyramid model and equation (4), respectively. Then, one can estimate the instantaneous velocity using kinetic energy given by equation (5).

The volume, V , of a vertical pyramid of height h , with top and bottom surfaces, A_1 and A_2 , is

Figure 2. This schematic diagram explains our gravity model (dimensions and bulk densities). The unit model is a pyramid, whose analytical gravity response is provided by [Gokula and Sastry \(2015a, b\)](#). Numerals 1–5, respectively, refer to different layers in our gravity model. Dip angles $\theta_1 = 10^\circ$, $\theta_2 = 12^\circ$, $\theta_3 = 25^\circ$, $\theta_4 = 27^\circ$, $\theta_5 = 35^\circ$, $\theta_6 = 37^\circ$, $\theta_7 = 45^\circ$, $\theta_8 = 48^\circ$, $\theta_9 = 55^\circ$ and $\theta_{10} = 58^\circ$. The total forward gravity responses of layers 1, 3, 5, 7, 9 and 11 constitute the computed gravity shown in red colour in (a). The pyramid vertices in different layers are as follows: A1(−37.510, −780, 1.0×10^{-6}); A2(−37.500, −780, 0); A3(−37.490, −780, −1.222); A4(−37.470, −780, −1.226); A5(−28.470, −780, −9.226); A6(−28.460, −780, −9.332); A7(−23.455, −780, −17.232); A8(−23.440, −780, −17.239); A9(−20.440, −780, −35.000); A10(−20.430, −780, −35.010); A11(−15.430, −780, −54.998); A12(−15.420, −780, 55.000); A1'(−37.510, 780, 1.0×10^{-6}); A2'(−37.500, 780, 0); A3'(−37.490, 780, −1.222); A4'(−37.470, 780, −1.226); A5'(−28.470, 780, −9.226); A6'(−28.460, 780, −9.332); A7'(−23.455, 780, −17.232); A8'(−23.440, 780, −17.239); A9'(−20.440, 780, −35.000); A10'(−20.430, 780, −35.010); A11'(−15.430, 780, −54.998); A12'(−15.420, 780, 55.000); B1(−13.510, −780, 1.0×10^{-6}); B2(−13.500, −780, 0); B3(−13.490, −780, −1.222); B4(−13.470, −780, −1.226); B5(−5.470, −780, −9.226); B6(−5.460, −780, −9.332); B7(−3.455, −780, −17.232); B8(−3.440, −780, −17.239); B9(−2.440, −780, −35.000); B10(−2.430, −780, −35.010); B11(−1.430, −780, −54.998); B12(−1.420, −780, 55.000); B1'(−13.510, −780, 1.0×10^{-6}); B2'(−13.500, −780, 0); B3'(−13.490, −780, −1.222); B4'(−13.470, −780, −1.226); B5'(−5.470, −780, −9.226); B6'(−5.460, −780, −9.332); B7'(−3.455, −780, −17.232); B8'(−3.440, −780, −17.239); B9'(−2.440, −780, −35.000); B10'(−2.430, −780, −35.010); B11'(−1.430, −780, −54.998); B12'(−1.420, −780, 55.000); C1(−2.168, −780, 1.0×10^{-6}); C2(−2.168, −780, 0); C3(−2.168, −780, −1.222); C4(−2.168, −780, −1.226); C5(−2.168, −780, −9.226); C6(−2.168, −780, −9.332); C7(−2.168, −780, −17.232); C8(−2.168, −780, −17.239); C9(−2.168, −780, −35.000); C10(−2.168, −780, −35.010); C11(−2.168, −780, −54.998); C12(−2.168, −780, 55.000); C1'(−2.168, 780, 1.0×10^{-6}); C2'(−2.168, 780, 0); C3'(−2.168, 780, −1.222); C4'(−2.168, 780, −1.226); C5'(−2.168, 780, −9.226); C6'(−2.168, 780, −9.332); C7'(−2.168, 780, −17.232); C8'(−2.168, 780, −17.239); C9'(−2.168, 780, −35.000); C10'(−2.168, 780, −35.010); C11'(−2.168, 780, −54.998); C12'(−2.168, 780, 55.000); coordinates are expressed in km. (b) Depths of the layers: $h_1 = (1.0 \times 10^{-7})$ km; $h_2 = (-1.222)$ km; $h_3 = (-1.226)$ km; $h_4 = (-9.226)$ km; $h_5 = (-9.232)$ km; $h_6 = (-17.232)$ km; $h_7 = (-17.239)$ km; $h_8 = (-35.0000)$ km; $h_9 = (-35.010)$ km; $h_{10} = (-54.998)$ km; $h_{11} = (-55.00)$ km; assumed average satellite height from mean sea level (msl) is 500 km.

Table 1. GRACE gravity datasets used by previous authors compared to us.

Authors	GRACE datasets considered (monthly average spherical harmonic coefficients)
Our attempt	November 2004–January 2005
Broerse <i>et al.</i> (2011)	February 2003–April 2010
Cazenave and Chen (2010)	Two years before and after the earthquake (2002–2006)
De Viron <i>et al.</i> (2008)	July 2002–June 2007
Panet <i>et al.</i> (2007)	January 2004–January 2005
Chen <i>et al.</i> (2007)	January 2003–September 2006
Han <i>et al.</i> (2006)	April 2002–September 2012

given by

$$V = \frac{1}{3} \times h \times (A1 + A2 + \sqrt{A1 \times A2}). \quad (3)$$

Given the density contrast of the above pyramid to be $\Delta\sigma$, then excess mass, M , is given by

$$M = \Delta\sigma \times V. \quad (4)$$

Kinetic energy, K_e , associated with instantaneous velocity, v , is given by

$$K_e = \frac{1}{2} M v^2. \quad (5)$$

3.2 Seismic momentum estimation

As per definition, the (instantaneous) seismic momentum is a product of mass and instantaneous velocity. However, in conventional seismology, this parameter is not used. It could be of interest in offshore tsunamigenic earthquake studies.

3.3 Seismic moment estimation

The scalar seismic moment, M_0 , is defined by $M_0 = \mu S D$, where μ is the shear modulus of the rocks involved in the earthquake, S is the area (in m^2) of rupture along the geologic fault where the earthquake occurred and D (in m) is the average displacement on A (Stork *et al.* 2014).

4. Results

By using steps 1–6 of earlier mentioned methodology, we arrived at the isolated input gravity anomaly distribution (black colour contours in figure 1). Our method of isolation of anomaly is compared with alternative approaches in the literature (table 1). By considering the seismological

constraints (tables 2 and 3), steps 4 and 5 of our methodology and thrust fault model details of Sibuet *et al.* (2007), we have framed a 3-D gravity forward model (figure 2). For this gravity model, the total gravity response (red coloured contours in figure 1) has been calculated by using a unit vertical pyramid model (Gokula and Sastry 2015a, b). The detailed forward gravity responses of 11-layered gravity model are included in figure 3. The RMS error between observed and computed gravity is $0.06257 \mu\text{gal}$ (6.26%) (figure 1), reflecting a fairly good match.

The implementation of step 6 of our methodology has yielded instantaneous velocity, rupture length, scalar seismic moment and momentum of the Sumatra earthquake 2004.

Arrived seismic moment is $4.53 \times 10^{22} \text{ N m}$ for a shear modulus in the range 30 GP (Geist *et al.* 2007). The computed instantaneous velocity, rupture length and momentum for the displaced actual mass of our model are, respectively, 2.9 km/s, 1560 km and $9.7 \times 10^{17} \text{ kg m/s}$ with the inherent assumption of a total seismic moment energy $1.099 \times 10^{23} \text{ J}$ associated with an earthquake of magnitude 9.3 (www.alabamaquake.com/energy.html).

5. Discussion

5.1 General

Contrary to current practice in the literature, our entire gravity modelling is based on isolated gravity signal (as per outlined methodology) at satellite height.

In table 1, we compare the different GRACE gravity data isolation procedures adopted by various authors including us. A close scrutiny of table 1 reveals that our dataset is indeed the closest one

Table 2. Seismic constraints and parameters of our model compared to related ones in the literature with remarks.

2004 Sumatra earthquake														
Author (year)	Comparison of seismic parameters available in the literature with our adopted/computed/derived parameters										Model dimensions		Remarks	
	HS (m)	VS (m)	RS (m)	DA (deg)	HD (km)	RL (km)	IV IV	RV (km/s)	SM× 10 ²² N m	Momentum kg m/s ²	W (km)	L (km)		TD (km)
Our model	22.72	4	22.99	10	35	1560	2.9	NA	4.53	9.7 × 10 ¹⁷	1560	40	35	GRACE gravity
Shearer and Bürgmann (2010)	22.65	3.99	11–35	10	30	NA		2.0–2.8	5.6–11.7	NA	500–1500	150–300	30	Seismic and geodetic
Vallee (2007)	19.66	3.64	20	8–15	23	1300		2–2.5	5.6	NA	1500	100–200	23	data inversion Rayleigh waves (empirical Green's function method)
Banerjee <i>et al.</i> (2005)	17.85	7.58	19.4	11–35	20–30	1300		NA	7.62	NA	1300	35	30	GPS + geologic
Chlieh <i>et al.</i> (2007)	16.43	4.32	17	12–17.5	50	1500		2.0–2.7	6.93	NA	1500	150	50	GPS + geologic
Fujii and Satake (2007)	18.79	3.29	13–25	10	NA	1300		1.0–3.0	4–10	NA	1400	NA	NA	Tide gauge and satellite data
Pietrzak <i>et al.</i> (2007)	33.03*	7.62	33.9*	13	50	1100–1300		2.0–3.7	5	NA	1500	NA	50	GPS
Rhie <i>et al.</i> (2007)	34.17*	8.83	35.3*	11–18	NA	1300		1.8–2.6	7.152	NA	NA	NA	NA	Seismic + GPS
Subarya <i>et al.</i> (2006)	19.35	5.09	20	12–17.5	30	1500		NA	8.8	NA	NA	NA	30	GPS + geologic
Tanioka <i>et al.</i> (2006)	21.78	3.06	21–23	8	NA	1200		1.7	7.2	NA	1200–1300	NA	NA	GPS
Ammon <i>et al.</i> (2005)	11.02	2.90	11.4	12–17.5	20–40	1200–1300		2.5	6.5	NA	1200–1300	NA	20–40	20- to 500-s body and surface waves
Tsai <i>et al.</i> (2005)	NA	NA	NA	5.8–8.1	20–30	NA		2.2–4.1	11.7	NA	NA	NA	20–30	Centroid moment tensor analysis
Stein and Okal (2005)	13.36	1.87	12–15	8	NA	1200		NA	10	NA	NA	NA	NA	Normal modes
Vigny <i>et al.</i> (2005)	33.32*	7.69	34.2*	13	NA	NA		2.0–3.7	7	NA	NA	NA	NA	GPS

*Calculated values, HS = horizontal slip; VS = vertical slip; RS = resultant slip; DP = dip angle; HD = horizontal depth; RL = rupture length; RV = rupture velocity; IV = instantaneous velocity; SM = seismic moment; W = width; L = length; TD = total depth.

Table 3. Literature support for the choice of our gravity model parameters (density, layer thickness).

Geophysical constraints for our model	
Choice of depths and densities of our model	Published literature
Satellite height above msl*	500 km (Tapley <i>et al.</i> 2004; Cambiotti and Sabadini 2013).
Sea surface water layer above msl deformation (first layer)*	We independently computed the difference of geoid undulations ($N = T/\gamma$) for both pre- and post-earthquake GRACE datasets (Heiskanen and Moritz 1967; Lambeck 1990), where T and $\gamma (= GM/R^2)$ are the disturbing potential and normal gravity on spherical Earth, respectively. Broerse <i>et al.</i> (2011) have attempted it through a complicated sea level equation method.
Water layer from msl to ocean bottom (second layer) [†]	Using Topex (http://topex.ucsd.edu).
Average depth of ocean floor below msl*	
Deformation of the ocean floor (third layer)*	Vertical displacement using published GPS literature (Chlieh <i>et al.</i> 2007; Rhie <i>et al.</i> 2007) density (Lowrie 2007; Cambiotti and Sabadini 2013). Choice of density values of second to fifth layers are as per Cambiotti and Sabadini (2013).
Fourth layer [†]	Lowrie (2007) and Cambiotti and Sabadini (2013).
Fifth layer*	To accommodate depth-wise slip distribution in our model, a thin sandwiched layer is considered, which shares the same physical properties (densities) as that of the ninth layer (Lowrie 2007; Cambiotti and Sabadini 2013).
Sixth layer [†]	Lowrie (2007) and Cambiotti and Sabadini (2013).
Seventh layer*	To accommodate depth-wise slip distribution in our model, a thin sandwiched layer is considered, which shares the same physical properties (densities) as that of the ninth layer (Lowrie 2007; Cambiotti and Sabadini 2013).
Eighth layer [†]	Lowrie (2007) and Cambiotti and Sabadini (2013).
Ninth layer*	To accommodate depth-wise slip distribution in our model, a thin sandwiched layer is considered, which shares the same physical properties (densities) as that of the ninth layer (Lowrie 2007; Cambiotti and Sabadini 2013).
Tenth layer [†]	Lowrie (2007) and Cambiotti and Sabadini (2013).
Eleventh layer*	To accommodate depth-wise slip distribution in our model, a thin sandwiched layer is considered, which shares the same physical properties (densities) as that of the ninth layer (Lowrie 2007; Cambiotti and Sabadini 2013).

*Gravity effect included in co-seismic gravity modelling.

[†]Gravity effect not included in co-seismic gravity modelling.

to the co-seismic event. Table 2 enlists the seismic constraints and estimated parameter details from the published literature. We also include inferred seismic parameters in table 2. For our 3-D gravity, model inputs cited in table 3 were helpful. We compare our model results with relevant ones from the literature (table 4). Table 4 also contains our isolated gravity responses both at msl and satellite height and crucial co-seismic 3-D gravity response details of relevant source layers (figure 2). Thus, table 4 clarifies several issues connected with the size of co-seismic anomalies, modelling aspects and accrued results thereof.

In figure 1, we include isolated GRACE gravity anomaly, our forward gravity response and a vertical projection of our gravity model (ABFE, Down-going block, EFCD, Up-thrown block and EF, fault trace) in the background tectonic framework of the Sumatra earthquake 2004. It is to be noted that the RMS error of $0.06257 \mu\text{gal}$ (6.26%) between our forward gravity response and observed (isolated) anomaly reflects a reasonably good match. Figure 2 is a schematic explanation of our 11-layered 3-D gravity model showing a depth-wise change in dip thrust and we mention all relevant coordinates of pyramid corners

Figure 3. Gravity responses (μgal) of six layer (layers included in co-seismic forward gravity modelling) out of our 3-D 11-layered model (figure 2). (a) Water layer (gravity anomaly range $((-0.3 \text{ to } -0.05) \times 10^{-7})$ with a contour interval of -0.05×10^{-7}) is shown in blue colour. (b) Ocean floor (third layer) (gravity anomaly range $-2.5 \text{ to } -0.05$ with a contour interval of -0.05) is shown in green colour. (c) Fifth layer (gravity anomaly values $0.2\text{--}2.0$), with a contour interval of -0.2 is shown in red colour. (d) Seventh layer (gravity anomaly values $-1.2 \text{ to } -0.2$, with a contour interval of -0.2) are shown in yellow colour. (e) Ninth layer (gravity anomaly values $0.1\text{--}1.1$ with a contour interval of 0.1) is shown in light blue colour. (f) Eleventh layer (gravity anomaly values $0.05\text{--}0.40$), with a contour interval of 0.05) is shown in black colour.

and densities of different layers in the caption. In figure 3, we present the individual forward responses of all relevant six density interfaces (layers included in co-seismic forward gravity modelling) in a superimposed manner with different colours. Their combined response constitutes the total model response already shown in figure 1. Our gravity inferred average instantaneous velocity, minimum rupture length, seismic moment and momentum are, respectively, 2.9 km/s , 1560 km , $4.53 \times 10^{22} \text{ N m}$ and $9.7 \times 10^{13} \text{ kg m/s}$. These seismological parameters match well with respective values in the literature (Lay *et al.* 2005; Engdahl *et al.* 2007; Katsumata 2015). The computed momentum corresponds to an area pulse at ocean bottom that led to tsunami generation.

5.2 GRACE satellite gravity processing and our forward gravity model

It is to be noted that our current GRACE gravity data processing is similar to that of Sastry and Sonker (2017). Co-seismic gravity signal (table 1) separation is simple and straightforward and the resulting gravity anomaly pattern conforms to thrust fault environment associated with the subduction process at ocean bottom (Sibuet *et al.* 2007; Broerse *et al.* 2011). Our 3-D thrust fault gravity model (figure 2) has a rupture length of 1560 km (Sibuet *et al.* 2007) with a vertical slip of 4 m and a dip angle in the range $8\text{--}10^\circ$ at ocean bottom (Wang and Liu 2006). This simple model at sea bottom had a forward gravity response amplitude of $-2.5 \mu\text{gal}$ (figure 3 (green contours)) far exceeding the observed satellite gravity anomaly (figure 1 (black contours)). So, we have introduced an 11-layered 3-D thrust fault model, whose details are as follows:

- (1) the first layer ($1 \times 10^{-6} \text{ msl}$) km is a deformed sea surface,
- (2) the second layer (water layer) extends from msl to -1.222 km depth-wise,
- (3) the third layer ($-1.222 \text{ to } -1.226 \text{ km}$) coincides with the deformed ocean floor,
- (4) the fourth layer corresponds to the oceanic crust ($-1.226 \text{ to } -9.226 \text{ km}$),
- (5) the fifth layer (figure 2) extends from $-9.226 \text{ to } -9.232 \text{ km}$,
- (6) the sixth layer extends from $-9.232 \text{ to } -17.232 \text{ km}$,
- (7) the seventh layer extends from $-17.232 \text{ to } -17.239 \text{ km}$,
- (8) the eighth layer extends from $-17.239 \text{ to } -35.00 \text{ km}$,
- (9) the ninth layer extends from $-35.00 \text{ to } -35.01 \text{ km}$,
- (10) the tenth layer extends from $-35.01 \text{ to } -54.998 \text{ km}$ and
- (11) the eleventh layer extends from $-54.998 \text{ to } -55.000 \text{ km}$.

The gravity responses of co-seismic anomaly causative first, third, fifth, seventh, ninth and eleventh layers are included in figure 3 with respective blue, green, red, yellow, light blue and black contours in an overlapping fashion. The combined response of these (red contours) is shown in figure 1. It closely matches the observed (black contour pattern in figure 1) isolated gravity with an RMS error of $0.06257 \mu\text{gal}$ (6.26%). It may be

Table 4. Isolated satellite gravity anomaly and its modelling by our model compared to published literature.

Our isolation of co-seismic GRACE gravity signal				As per Broerse et al. (2011)		Gravity anomaly (μgal) at satellite height as per our model		Gravity response at msl (μgal)	
Observed gravity anomaly (μgal) from GRACE dataset (total gravity response at msl)	Observed gravity anomaly (μgal) from GRACE dataset (total gravity response at satellite height)	Layers included in our co-seismic gravity model	Our model (km)	(km)				Our model	Chen et al. (2007)
−8 to 6	−0.35 to 0.20	Co-seismic deformation of sea surface water (geoid height variation) (first layer)	-1×10^{-6} to 1×10^{-6} (using SH of degree and order 70)	$(-12.9 \times 10^{-6}$ to $1.6 \times 10^{-6})$		$(-0.3$ to $-0.05) \times 10^{-7}$		$(-0.3$ to $-0.05) \times 10^{-5}$	-15 to 15
		Deformation of ocean floor (third layer)	-1.222 to -1.226	NA		-2.5 to -0.05		-250 to -5	
		Fifth layer	-9.226 to -9.232	NA		0.2 – 2.0		20 – 200	
		Seventh layer	-17.232 to -17.239	NA		-1.2 to -0.2		-120 to -20	
		Ninth layer	-35.000 to -35.010	NA		0.1 – 1.1		10 – 110	
		Eleventh layer	-54.998 to -55.000	NA		0.05 – 0.40		5 – 40	

NA: not available.

mentioned that 3-D edge effect in forwarding gravity responses at satellite height is noticed only for the eighth layer (figure 2) and it is addressed by a proper singularity removal procedure (Gokula and Sastry 2015a). However, for brevity sake, we are not including those illustrations here.

Our gravity model and its gravity response, respectively, differ from conventional point deformation models (Sun and Okubo 1993) in homogeneous elastic half-space and their gravity response formulations and related studies based on them (Hayes *et al.* 2006; Cambiotti and Sabadini 2013; Han *et al.* 2013; Schultz *et al.* 2016; Harms *et al.* 2015). Rather, our forward modelling follows a more traditional approach, which involves depth-wise density variation and somewhat ‘anti-root’ concept of Isostasy (Watts 2001).

5.3 Deformation of top sea level modelling

Our simple computational method for co-seismic geoidal height deformation yields similar results as that of Broerse *et al.* (2011, 2014), which is based on a more complex sea level equation that includes the effect of ocean floor deformation.

5.4 Earlier models vis-à-vis our model

Our way of honouring co-seismic sea-surface deformation is similar to Sastry and Sonker (2017) and it avoids the complex calculations proposed by Broerse *et al.* (2011) for gravity effect of displaced ocean water mass based on earlier GRACE datasets (Han *et al.* 2006; Chen *et al.* 2007; Panet *et al.* 2007; de Linage *et al.* 2009) and earthquake slip models. We have evaluated the co-seismic deformation of the surface water layer based on geoid height computations (table 4). Further, our gravity model (figure 2) and its total response (shown in red colour contours in figure 1) fully account for the observed GRACE gravity anomaly (shown in black colour contours in figure 1). In contrast, Broerse *et al.* (2011) have only considered the gravity peak positive and negative values variation with depth.

Further, as per Broerse *et al.* (2011, 2014) horizontal slip had to be much larger in the trench region in comparison to vertical slip. As per our proposed gravity model, the horizontal, vertical and dip slips are, respectively, 22.72, 4 and 22.99 m with dip angles of 10–58° (figure 2). Our RMS error fit of 0.06257 μgal (6.26%) for computed gravity response to GRACE gravity data is novel to the best of our knowledge.

5.5 Seismic momentum

Our gravity derived instantaneous (subject to our isolation of GRACE gravity signal instants) seismic momentum ($9.7 \times 10^{17} \text{ kg m/s}$) is novel in relating it to area vector pulse of co-seismic ocean floor deformation that has been communicated to water layer overburden leading to the Sumatra 2004 tsunami. In fact, this area pulse operated like the piston of a large area. Unfortunately, such a parameter (momentum) does not figure in the seismological literature.

6. Conclusions

The GRACE satellite gravity response for the 2004 Sumatra earthquake has been isolated by the difference method. Our proposed novel 3-D (11-layered) thrust fault model extends from mean sea level to hypocentre depth and beyond (55 km). The computed forward response fully characterises the observed (isolated) co-seismic gravity signal for the Sumatra earthquake 2004 (tsunami) source. The RMS error fit of computed gravity response (0.06257 μgal (6.26%)) with isolated GRACE gravity data reflects a close match.

Our model accommodates depth-wise dip slip variation, which conforms to widely held seismological models in the literature for the Sumatra earthquake 2004. Our gravity inferred average instantaneous velocity, minimum rupture length, seismic moment and momentum are, respectively, 2.9 km/s, 1560 km, $4.53 \times 10^{22} \text{ Nm}$ and $9.7 \times 10^{17} \text{ kg m/s}$. These independent estimates of seismological parameters match well with respective ones in the literature. Further, the gravity derived instantaneous seismic momentum corresponds to an area pulse at ocean bottom that has led to tsunami generation.

Acknowledgements

We gratefully acknowledged the help received from Mr Aruj Pant, a post-graduate student under the first author in isolation of satellite gravity signal of Sumatra earthquake 2004 and Dr Anand Gokula for gravity forward modelling software of right vertical pyramid model. Mr Sonker is thankful to the Ministry of Human Resources Development (MHRD), Government of India, for financial support.

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