Linear regression models for estimating true subsurface resistivity from apparent resistivity data

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Simple linear regression (SLR) models for rapid estimation of true subsurface resistivity from apparent resistivity measurements are developed and assessed in this study. The objective is to minimize the processing time and computer memory required to carry out inversion with conventional algorithms. The arrays considered are Wenner, Wenner–Schlumberger and dipole–dipole. The parameters investigated are apparent resistivity (ρ_a) and true resistivity (ρ_t) as independent and dependent variables, respectively. For the fact that subsurface resistivity is nonlinear, the datasets were first transformed into logarithmic scale to satisfy the basic regression assumptions. Three models, one each for the three array types, are thus developed based on simple linear relationships between the dependent and independent variables. The generated SLR coefficients were used to estimate ρ_t for different ρ_a datasets for validation. Accuracy of the models was assessed using coefficient of determination (R^2), F -test, standard error (SE) and weighted mean absolute percentage error ($wMAPE$). The model calibration R^2 and F -value are obtained as 0.75 and 2286, 0.63 and 1097, and 0.47 and 446 for the Wenner, Wenner–Schlumberger and dipole–dipole array models, respectively. The SE for calibration and validation are obtained as 0.12 and 0.13, 0.16 and 0.25, and 0.21 and 0.24 for the Wenner, Wenner–Schlumberger and dipole–dipole array models, respectively. Similarly, the $wMAPE$ for calibration and validation are estimated as 3.27 and 3.49%, 3.88 and 5.72%, and 5.35 and 6.07% for the three array models, respectively. When compared with standard constraint least-squares (SCLS) inversion and Incomplete Gauss–Newton (IGN) algorithms, the SLR models were found to reduce about 80–96.5% of the processing time and memory space required to carry out the inversion with the SCLS algorithm. It is concluded that the SLR models can rapidly estimate ρ_t for the various arrays accurately.

Keywords. Simple linear regression; true resistivity; apparent resistivity; processing time; memory space.

1. Introduction

Resistivity survey plays significant role in many large-scale sites characterization (Dahlin *et al.* 2007; Bery and Saad 2012), groundwater exploration (Saad *et al.* 2012a) and mineral prospecting

(Saad *et al.* 2012b; Song *et al.* 2017). Unfortunately, the measured apparent resistivity data from such surveys cannot be used directly to describe the subsurface (De Donno and Cardarelli 2017). Inversion is therefore required to develop resistivity models that can portray the true subsurface

condition based on the apparent resistivity data. The necessary requirements for the inversion of any geophysical data include fast forward algorithm for calculating theoretical data for initial model parameters, and a technique for calculating derivatives of the data with respect to the model parameters (Narayan *et al.* 1999).

Resistivity inversion has received enormous attention from various researchers over the years. Investigations are still going on to further mitigate the problems associated with its speed and efficacy. Several researches have been conducted to develop and control resistivity inversion mechanisms. The least-squares optimization based techniques are the most commonly and widely explored compared to other approaches (Olayinka and Yaramanci 2000; Loke and Dahlin 2002; Loke and Lane 2004). They have been applied to solve many practical inversion problems of simple and complex structures with recorded remarkable success (Pain *et al.* 2002). They are, however, not without problems. Some of them include non-uniqueness of solutions, great consumption of time and memory space for the iterative computations to converge at minimum data misfit (Gyulai *et al.* 2010; Vachiratietchai and Siripunvaraporn 2013), inability to produce accurate models when the starting initial model is far from the actual solution, emergence of false anomalies at intermediate and final iterative computations of model resistivity (Liu *et al.* 2012) and difficulties in dealing with nonlinear relationships among the modelling (subsurface) parameters (Singh *et al.* 2010; Maiti *et al.* 2012; Srinivas *et al.* 2012; Stanley Raj *et al.* 2014).

Joint inversion technique attempts to address the problems of multiple solutions and false anomalies arising from resistivity inversion. It has performed well with additional data from seismic and ground penetrating radar (GPR) surveys (Zhou *et al.* 2014). Research algorithms that have this functionality are available nowadays, but possibilities to combine several geophysical methods in one inversion are limited. Survey time to acquire extra GPR or seismic data for large sites and picking the first arrival time to provide the needed datasets for the joint inversion are also other factor to consider (Bergmann *et al.* 2014). Finally, even though electrical resistivity and seismic velocity tomograms correlate with one another, very important anomalies as lateral changes in resistivity may not be detected by the seismic method, and can therefore be phased out as false anomalies.

Regression coefficients obtained from linear least squares method can provide alternative means for rapidly estimating the true subsurface resistivity whenever apparent resistivity site measurements are conducted. Nonlinearity in the nature underground resistivity distributions can be taken care of, by using logarithmic transform of datasets, rather than using the actual datasets directly in the regression process. The objective of this research is to develop and assess mathematical models for fast estimation of true subsurface resistivity from apparent resistivity measurements. This is expected to significantly reduce the time and memory capacity needed to process the apparent resistivity data using conventional algorithms. While acknowledging the applicability of linear regression method to solving many scientific problems, the originality of this research lies in its ability to first develop simple models for estimating true subsurface resistivity from apparent resistivity data by regression. This will provide the user with alternative means for rapidly estimating true resistivity without recourse to complex conventional inversion algorithms, thereby saving significant processing time and computer memory space.

2. Study area

This research was carried out at Universiti Sains Malaysia main campus, Pulau Pinang, Malaysia. Pulau Pinang (Penang Island) comprises an area of about 285 km², connected to the mainland by two long bridges. The temperature typically, ranges from 29°–32°C, with the highest temperature occurring between April and June, every year. The average relative humidity ranges from 65–95%, with lowest values recorded between the months of June and September. Average monthly rainfall may reach up to 240 mm, and is not usually lower than 58.6 mm (Pradhan and Lee 2010). Measured resistivity in the area varied from 0 to 6000 Ω m, depending on the nature of materials and degree of saturation (Abdulrahman *et al.* 2016).

Dominant geology is basically granite; classified according to the proportions of alkali feldspars to total feldspars (Ahmad *et al.* 2006). The northern part of the Island is abundantly characterized with alkali feldspars that contain orthoclase to intermediate microcline, they do not show any distinct cross hatched twinning. In the southern part, however, they show well developed cross hatched twinning and are characterized to be microcline

Figure 1. Geology map of Penang Island (modified from Pradhan and Lee 2010).

(Ong 1993). Penang Island is composed of a few soil types; clay materials of Kuala-Kedah, the steep land soil series, which is composed of weathered granitic rocks and is obtained at the central part of the island, fine to medium coarse sandy clay of Rengam series developed on igneous and high grade metamorphic rocks, and Selangor Kangkong association, which is primarily comprised of medium grained clay derived from acid sulphate soils. Alluvium and colluvium series, which constitute fine to medium grained loamy materials derived locally, also exist (Pradhan and Lee 2010). The geologic map of Penang Island showing the study area is given in figure 1.

3. Methodology

3.1 Resistivity data

A typical electrical resistivity method measures underground potential contrast (V) upon injecting specific amount of current (I) into the ground

(figure 2). Resistance (R) is then automatically computed from Ohm's law (equation 1) and subsequently the apparent resistivity, when array geometrical factor is known.

The data utilized for the regression modelling was acquired from ground resistivity survey, using multi-electrode resistivity meter (ABEM Terrameter SAS4000), conducted along two profiles each for the Wenner–Schlumberger and dipole–dipole arrays; one for model calibration (line 1) and the other for model validation (line 2). For the Wenner array however, data along three profiles were acquired for model calibration (lines 1–3) and one profile for validation (line 4). This was necessary to ensure sufficient data for the modelling, as surveys with Wenner array produce less number of data points compared to the remaining two. Each line consisted of 41 stainless steel electrodes at 1 m minimum spacing connected to two smart cables, each of 21 take-outs, using jumper. The current was set between 1 and 100 mA. A computer controlled system Terrameter SAS4000 was used to automatically select the four active electrodes

Figure 2. Four-point electrode configuration with current and potential distribution.

Figure 3. Data points arrangement for a typical resistivity measurement.

for each voltage measurement and to consequently compute the apparent resistivity (equation 2) using the array geometrical factor (k) for each array. Apparent resistivity data points were arranged systematically such that each datum point is specified with its horizontal and vertical coordinates (figure 3)

$$V = IR, \quad (1)$$

$$\rho_a = k \frac{\Delta V}{I}. \quad (2)$$

All the measured apparent resistivity data were subjected to inversion using standard constraint

least-squares (SCLS) inversion algorithm available in Res2Dinv software package, to obtain the true resistivity models and datasets (ρ_t) at same coordinates as the apparent resistivity (ρ_a). The datasets (ρ_a and ρ_t) were then arranged in columns in preparation for the linear regression modelling.

3.2 Linear regression

Generally, least squares based linear regression model with n independent variables (x) in the observations and a dependent variable (y) can be written as:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \varepsilon \quad (3)$$

where β_0 is the intercept, $\beta_1, \beta_2, \dots, \beta_n$, are the regression coefficients and ε is the error term. In this approach, the model converges when the squared sum of the differences between the estimated and observed values is minimum. The values of dependent variable can then be estimated for given values of independent variable(s), if the errors are assumed to be normally distributed with zero mean and constant variance. The distribution of the y_i is thus symmetrical and bell-shaped, with a constant standard deviation at x levels (Balarabe *et al.* 2016).

In this study, true resistivity (ρ_t) is considered as the dependent variable, while apparent resistivity (ρ_a) is the independent variable. Unfortunately, nonlinear nature of the subsurface resistivity restricted the use of datasets directly for the regression modelling. To overcome this, the datasets were transformed into logarithmic scale to satisfy the regression pre-modelling diagnostics such as; tests of linearity, normality of the dependent variable, normality and probability of standardized residuals, heteroscedasticity, axes balance and outliers. Logarithmic transforms of ρ_a and ρ_t , denoted as ρ_A and ρ_T , respectively, are therefore adopted as the new model parameters. Outliers were removed based on Mahalanobis, Cook's and Centred Leverage distances. A modified form of equation (3) can thus be written as equation (4) to fit the new model parameters. First dataset for the three arrays each was imported into SPSS to run the regression (model calibration). Regression coefficients obtained from model calibration were applied to on the second datasets to estimated ρ_T values for validation. Thereafter, ρ_T were transformed back to ρ_t and plotted using Golden Surfer 11 software, at the same colour scale and blanking as the observed true resistivity models, for visual assessment.

The accuracy of each model was assessed using coefficient of determination (R^2), F -test, standard error (SE) and weighted mean absolute percentage error ($wMAPE$), given in equations (5–8), respectively for model calibration, and SE and $wMAPE$ for validation

$$\rho_T = \beta_0 + \beta_1 \rho_A \quad (4)$$

$$R^2 = \frac{\sum (\hat{\rho}_{Ti} - \bar{\rho})^2}{\sum (\rho_{Ti} - \bar{\rho})^2} \quad (5)$$

$$F_{val} = \frac{R^2 / (K - 1)}{(1 - R^2) / (N - K)} \quad (6)$$

$$SE = \left[\frac{1}{N} \sum (\rho_{Ti} - \hat{\rho}_{Ti})^2 \right]^{1/2} \quad (7)$$

$$wMAPE = \frac{1}{N} \sum \left| \frac{(\rho_{Ti} - \hat{\rho}_{Ti})}{\rho_{Ti}} \right| \times 100 \quad (8)$$

where ρ_{Ti} , $\hat{\rho}_{Ti}$ and $\bar{\rho}_{Ti}$ are observed, estimated and average logarithms of true resistivity values respectively, k is the number of independent variables and N is the number of data points.

The performance of the new SLR models with respect to processing time and computer memory space consumed in data processing has been recorded and compared with other established algorithms to further validate the efficacy of the models. SCLS inversion was used as bench-mark, while the Incomplete Gauss–Newton (IGN) algorithm with data compression option, specifically designed to reduce processing time and computer memory space required for the inversion using SCLS technique especially when dealing with many data points, was then compared with the new SLR models.

4. Results and discussion

Figure 4 presents the analyses carried out to check the basic regression assumptions. Figure 4(a) shows that logarithmic transform of the true resistivity data is approximately symmetrical, suggesting a normally distributed dependent variable as against the asymmetrically distributed actual (untransformed) true resistivity data. The histograms depicted by figure 4b show that the standardized residuals obtained from the transformed datasets are also approximately normally distributed, with symmetrical bell-shape and almost zero skewness and kurtosis, as compared to the asymmetrical right skewed standardized residuals of the actual dataset (Cramer 1998; Cramer and Howitt 2004; Doane and Seward 2011). The normal probability curve of expected cumulative probability against observed cumulative probability indicates a steady increase in expected cumulative probability for every increase in the observed cumulative probability, until a nearly straight-line curve is achieved for the transformed dataset (figure 4c). The untransformed data, on the other hand, did not show such linearity.

Figure 4. Basic regression assumptions checks (a) box plot, (b) histogram, (c) normal probability, (d) variables scatter plot, and (e) standardized residual scatter plot.

Table 1. Models summary table.

Models	Coefficients		R^2	F -value	SE		$wMAPE$ (%)	
	β_0	β_1			Cal	Val	Cal	Val
Wenner	1.22	-0.60	0.75	2286	0.12	0.13	3.27	3.49
Wenner-Schlum	1.14	-0.36	0.63	1097	0.16	0.25	3.88	5.72
Dipole-dipole	1.01	0.07	0.47	446	0.21	0.24	5.35	6.07

Note. Cal = Calibration, Val = Validation, Wenner-Schlum = Wenner-Schlumberger.

Figure 4(d) is the scatter plot of the dependent *vs.* independent variables of both actual and transformed datasets for linearity check. Visual assessment of both plots indicates that the transformed data points are approximately linearly distributed as compared to the nonlinear distribution of actual resistivity datasets. Similarly, the scatter plots of the standardized residuals suggest that all the residual data points of the transformed datasets are uniformly distributed between -3 and $+3$ on both axes with almost a zero mean, and without any clear pattern (figure 4e). On the other hand, residuals of the actual dataset are between -2 and $+4$ on horizontal axis, and between -4 and $+6$ on the vertical axis, with non-uniformity everywhere and a noticeable distribution pattern. All the pre-modelling assumption examined in this study have favoured the use of logarithmic transforms of datasets for the regression over the actual datasets.

Table 1 presents summary of the three regression models developed in this study. The generated coefficients β_0 and β_1 , when substituted in equation 4, will give the complete model required for estimating the true resistivity for each array type. The R^2 showcases how much variability in the dependent variable is explained by the independent variable. The R^2 is found to be 0.75, 0.63 and 0.47 for Wenner, Wenner-Schlumberger and dipole-dipole models, respectively. This signifies that 75%, 63% and 47% of variance in the true subsurface resistivity is attributed to the apparent resistivity derived from the three models, respectively. The remaining variance (%), lumped as the model residuals, may be explained by the array sensitivity pattern and geological complexities. The highest value of R^2 obtained with Wenner array further indicates that the array is less affected by sensitivity pattern or other geological constraints (thereby depending mostly on the apparent resistivity measurement) to estimate the true resistivity. All the R^2 values obtained fall within strong to very strong range, according to the absolute strength classification of R^2 established by Evans (1996) and reported

by Beldjazia and Alatou (2016) as: < 0.04 (very weak), $0.04-0.15$ (weak), $0.16-0.35$ (moderate), $0.36-0.63$ (strong) and $0.64-1.00$ (very strong).

The F -test also compares the variance accounted for by each regression model (table 1). The null hypothesis of the test is defined in such a way that the regression models' coefficients are zero. The alternative hypothesis is that at least one of the coefficients is not zero, to allow for the response to be estimated. For the three models developed in this study, the F -values were found to be $F(1, 750) = 2286$, $p < 0.001$; $F(1, 637) = 1097$, $p < 0.001$; and $F(1, 512) = 446$, $p < 0.001$ for the Wenner, Wenner-Schlumberger and dipole-dipole array models, respectively. The two numbers in parenthesis (degrees of freedom) indicate the number of independent variables and total number of data points, respectively. The p -values of less than 0.001 ($< 0.1\%$) associated with each F -value further demonstrates the statistical significance of all the three models; as p -value of 0.05 (5%) or less is required to reject the null hypothesis.

To further assess the models' accuracy to estimating the dependent variables, SE for calibration and validation are respectively computed as 0.12 and 0.13, 0.16 and 0.25, and 0.21 and 0.24 for Wenner, Wenner-Schlumberger and dipole-dipole array models, respectively. The calibration and validation $wMAPE$ are also estimated as 3.27 and 3.49%, 3.88 and 5.72%, and 5.35 and 6.07% for the three models, respectively (table 1). It is important to note that both SE and $wMAPE$ for validation are slightly higher than the corresponding calibration errors. This is because outliers were not removed from the validation datasets as was the case for the calibration datasets. This was deliberately allowed for, since apparent resistivity field data are less frequently filtered for outliers before they are processed to obtain the true subsurface resistivity. Of the three models developed and assessed, the Wenner based model was found to be the most accurate, having the highest R^2 and F -value, with least SE and $wMAPE$.

Table 2. Comparison of the regression models' processing time, memory space consumed and error occurred with established algorithms.

Models	Processing time (s)			Memory (MB)			$wMAPE$ (%)	
	SCLS	IGN	SLR	SCLS	IGN	SLR	IGN	SLR
Wenner	16.2	15.4	1.8	11.4	12.5	0.4	2.7	3.5
Wenner–Schlum	24.2	25.8	2.2	12.3	12.8	0.6	4.1	5.7
Dipole–dipole	27.0	27.5	2.3	12.5	13.2	0.7	4.7	6.1

Note. SCLS = standard constraint least-squares inversion, IGN = incomplete Gauss–Newton with data compression option, SLR = simple linear regression.

Figure 5. Comparison between estimated and observed resistivity contour plots (a) Wenner, (b) Wenner–Schlumberger, and (c) Dipole–dipole array models for visual assessment.

Table 2 provides summary of the regression models' processing time, memory space consumed and errors occurred in comparison with other established algorithms, namely SCLS and IGN. It took about 16.2, 15.4 and 1.8 s, respectively for SCLS, IGN and SLR to process the validation data acquired using the Wenner array. The memory spaces consumed by the SCLS, IGN and SLR in processing the same data are 11.4, 12.5 and 0.4 MB, respectively. This shows that the SLR (Wenner) model is able to cut down up to 80% of the processing time and 96.5% of the computer memory space required by the SCLS algorithm to carry out the inversion. In comparison, the IGN algorithm brought about 5% reduction in processing time and 10% increase in memory space consumed by the SCLS inversion algorithm. Comparable results were also obtained with the Wenner–Schlumberger array model. The SLR (Wenner–Schlumberger) model could save respectively 90 and 95% of the processing time and memory space consumed by the SCLS inversion algorithm, as compared to the IGN algorithm, which brought about respectively 6.6 and 4% increase in time and memory space.

Similarly, the dipole–dipole based SLR model has cut down respectively 91.5 and 94.4% of the processing time and memory space consumed by the SCLS algorithm. This is in contrast with the IGN algorithm with 1.9 and 5.6% increase in time and memory space, respectively. However, the SLR models recorded slightly higher $wMAPE$ s compared to the IGN algorithm. The differences in $wMAPE$ s are 0.8, 2.6 and 2.4% for the Wenner, Wenner–Schlumberger and dipole–dipole arrays, respectively. This may be a compromise between slightly more accurate true resistivity estimates and enormous reduction in processing time and memory space needed especially when dealing too many data points.

Two-dimensional contour plots of the estimated true resistivity derived from the validation datasets were constructed and compared to the corresponding observed true resistivity plots for visual assessment of the models (figure 5). For the Wenner (figure 5a) and dipole–dipole (figure 5c) arrays, the resistivity contour plots depict high resistivity zone (above 1200 Ωm) above 3 m depth and low resistivity zone (below 1200 Ωm) below the depth of about

3 m. High resistivity region at the top left part of the tomogram produced using the observed data with Wenner–Schlumberger array is also noticeable in the estimated data plot (figure 5b). Both the estimated and observed data plots for the three arrays did not differ much from one another in terms of subsurface resistivity distribution, indicating that the models can be relied upon for satisfactory estimations. The Wenner based model has also been proven the best by the visual assessment.

5. Conclusion

Three simple linear regression models for rapid estimation of true subsurface resistivity from apparent resistivity data have been developed and validated in this study. Numerical accuracy assessment carried out suggests that all the R^2 values obtained fall within strong to very strong range, with Wenner model has the highest R^2 (0.75) and F -value (2286) with lowest SE (0.12) and $wMAPE$ (3.27%). Visual assessment using contour plots has also placed the Wenner model at the top of the list, having resistivity distribution closest to its corresponding observed plot. When compared with Standard constraint least-squares (SCLS) inversion and Incomplete Gauss-Newton (IGN) algorithms, the SLR models were found to curtail 80–96.5% of the processing time and memory space required to carry out the inversion with SCLS algorithm. It is therefore concluded that all the three models can rapidly estimate true resistivity whenever the corresponding arrays are used to acquire apparent resistivity data, with the Wenner model being the most accurate.

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