Finite-difference modelling of anisotropic wave scattering in discrete fracture model

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The presence of fractures in reservoir rocks causes scattering of seismic wave energy. In this paper, we utilize the finite-difference modelling technique to study these scattering effects to gain more insights into the effects and assess the validity of using anisotropic wave scattering energy as a diagnostic tool to characterize fractured hydrocarbon reservoirs. We use a simplified fractured reservoir model with four horizontal layers with a fractured-layer as the third layer. The fractures are represented by grid cells containing equivalent anisotropic medium by the use of the linear slip equivalent model. Our results show that the scattered energy, quantified through estimates of the seismic quality factor (Q) is anisotropic, exhibiting a characteristic elliptical ($\cos 2\theta$) variations relative to the survey azimuth angle θ . The fracture normal is inferred from the minor axis of the Q ellipse. This direction correlates with the direction of maximum wave scattering. Minimum wave scattering occurs in the fracture strike direction inferred from the major axis of the Q ellipse. These results provide more complete insights into anisotropic wave scattering characteristics in fractured media and thus, validate the practical utility of using anisotropic attenuation attribute as an additional diagnostic tool for delineation of fracture properties from seismic data.

Keywords. Finite difference; standard-staggered grid; fractures; attenuation; anisotropy.

1. Introduction

Fractures with preferential alignment cause anisotropy in P-wave attributes such as amplitude, velocity, travel time and AVO gradients. Analysis of the azimuthal variations in these attributes is currently an entrenched method for the delineation of fracture properties from seismic data (Rathore *et al.* 1995; Li 1999; Hall *et al.* 2000; Wang *et al.* 2007). Aligned fractures have also been shown to cause anisotropy in P-wave attenuation which has been of great interest to scientists in recent years, as it could provide additional information

especially on the fracture size (i.e., orientation and scale length). Equivalent medium theories (Tod 2001; Chapman 2003) envisage anisotropic characteristics in seismic wave attenuation for wave propagation in a medium containing aligned vertical fractures. Actually, this characteristic anisotropic behaviour has been seen both in field and laboratory data (Clark *et al.* 2001; Chichinina *et al.* 2006; Luo *et al.* 2006; Maultzsch *et al.* 2007; Clark *et al.* 2009; Ekanem *et al.* 2013) and has been associated with fracture properties. Research has shown that fractures with lengths comparable with the seismic wavelength in reservoir rocks scatter seismic

wave energy (Schultz and Toksöz 1995; Willis *et al.* 2006; Burns *et al.* 2007; Xu *et al.* 2010; Ekanem *et al.* 2014). The scattering effect occurs more in the fracture normal direction when compared to the fracture strike direction. This anisotropic wave scattering could provide helpful clues on the fracture properties (Willis *et al.* 2006 and Burns *et al.* 2007). Quantifying the amount of the scattering energy through an estimate of the seismic quality factor (Q) and hence the attenuation factor could help to provide more insights into the effects and assess the validity of using anisotropic wave scattering energy as an analytical tool to characterize fractured hydrocarbon reservoirs to complement the use of other seismic attributes. Despite the concerted effort in research and development related to seismic characterization of fractured reservoirs using anisotropic wave scattering, pragmatic utilization of this attribute in geophysical exploration is still restricted perhaps as a result of the ambiguity in its quantification and difficulty in its interpretation in terms of rock properties (Jeng *et al.* 1999; MacBeth 1999; Rongrong *et al.* 2006). Thus, the task of using anisotropic wave scattering for fracture prediction in the Earth's crust still needs more understanding of the underlying principle.

Seismic finite-difference modelling to estimate the degree of wave scattering caused by suite of aligned fractures could improve our understanding of the scattering effects and the resultant attenuation characteristics, thus providing a means of validating existing fracture inversion schemes from seismic data. A number of techniques have been put forward for solving numerically the wave equation in order to model the propagation of seismic waves in a given medium. These techniques, sometimes regarded as the direct methods, can explain all kinds of waves and also suitable for complex sub-surface structures (Bansal and Sen 2008). Numerically, the wave equation can be solved by the use of the finite-element method, the pseudo-spectral method or the finite-difference method. These methods have their respective advantages and disadvantages (Graves 1996). For example, the finite-element method can handle irregular grids and boundaries but its computational process is complicated and time-consuming (Bansal and Sen 2008; Hua *et al.* 2009). The finite-difference method has the advantage of being very efficient and having high speed (Igel *et al.* 1995; Hua *et al.* 2009) and has been successfully utilized to model the propagation of the seismic wave in

elastic media. Kelly *et al.* (1976) generated synthetic seismic data in 2-D acoustic media by the use of the finite-difference technique. The standard staggered-grid finite-difference scheme, first introduced by Madariaga (1976) and later by Virieux (1984, 1986) which makes use of the velocity–stress formulation to model seismic wave propagation in 2-D elastic media has since become very popular in modelling seismic wave propagation (Moczo *et al.* 2000). Several attempts have been made to extend the existing 2-D finite-difference algorithms to 3-D elastic media which have been successful especially because of advances in computer technology (Igel *et al.* 1995; Graves 1996; Moczo *et al.* 2000; Willis *et al.* 2006; Bansal and Sen 2008; Xu *et al.* 2010).

In this paper, we used the generic 3-D anisotropic elastic finite-difference scheme to model the propagation of seismic waves through layered anisotropic media. Specifically, we used a simplified fractured reservoir model with four horizontal layers with the fractured layer as the third layer. The finite-difference scheme makes use of a standard staggered-grid with an explicit 8th-order operator in space and 2nd-order operator in time to solve the first-order wave equation expressed in terms of velocity and stress. The wavefield components are discretized in different numerical grids in order to solve the wavefield spatial derivatives at the required grid locations. The fractures are represented by grid cells containing equivalent anisotropic medium using the equivalent medium theory of Coates and Schoenberg (1995). We extracted 2-D section from the 3-D cube in four azimuths relative to the fracture strike direction (0 azimuths) and computed the interval values of Q in the fractured-layer using the classical spectral ratio method. Our results provide significant insights into anisotropic wave scattering characteristics in fractured media and thus, validate the practical utility of using anisotropic wave scattering attribute as an additional diagnostic tool for fracture prediction in the Earth's crust from seismic data.

2. Theoretical foundations

2.1 Fracture representation

A medium with a set of aligned vertical fractures can be thought of as an equivalent azimuthally anisotropic medium for seismic wave propagation provided the scale length of the fractures is

much smaller than the wavelength of the seismic wave. Thus, the equivalent medium theories for materials with aligned fractures can be used to establish a link between seismic anisotropy and fracture properties. The common models developed to simplify the study of fracture-induced seismic anisotropy include the Hudson's model (1980, 1981), the linear slip model (Schoenberg 1980) and the Thomsen model (1995).

Hudson's model (1980 and 1981) considers an isotropic background medium containing a set of thin, penny-shaped ellipsoidal cracks or inclusions. The cracks are assumed to have a random distribution in terms of position and either aligned or randomly orientated. The crack density is low ($\ll 1$) and the mean crack shape is assumed to be circular with a radius smaller than the wavelength of the associated seismic wave. The cracks are isolated which precludes the possibility of fluid flow into or out of the cracks during seismic wave propagation. However, the cracks may either be empty or filled with a solid or fluid material (Hudson 1981). If the cracks have a preferred orientation, the resulting medium is anisotropic and the effective elastic (stiffness) tensor C is given by:

$$C = C^{(0)} + \alpha C^{(1)} + \alpha^2 C^{(2)} \quad (1)$$

$C^{(0)}$ is the stiffness tensor of the isotropic background, $C^{(1)}$ are the 1st order corrections due to the presence of the cracks and $C^{(2)}$ accounts for the interactions between the cracks. α is the crack density defined mathematically as:

$$\alpha = NaV^{-1} \quad (2)$$

where N is the number of cracks, V is the volume of the base material and a is the mean crack radius. The first order correction depends on the elastic parameters of the uncracked solid medium and the cracks' response to normal and shear traction which in turn depend on the crack aspect ratio and the elastic moduli of the material filling the cracks. In seismological applications, Hudson's model is used to derive the crack density from the measured magnitude of the azimuthal anisotropy. However, it can be seen from equation (2) that the same crack density can be obtained from many small cracks as well as from few large cracks within the same volume of material. This indicates the inability of the model to distinguish between the anisotropy caused by micro-cracks and that caused by macro-cracks.

The linear slip model (Schoenberg 1980) models fractures as non-welded interfaces between two elastic media. The stress caused by the propagation of the seismic wave is continuous across these interfaces while displacement is not, which implies that slip occurs (Coates and Schoenberg 1995; Schoenberg and Sayers 1995). Unlike Hudson's (1980, 1981) model, the linear slip assumes nothing about the shape of the fractures. The small displacement Δu and the stress σ are linearly related to the fracture compliance, S according to the equation (Coates and Schoenberg 1995):

$$\Delta u = S\sigma \cdot n \quad (3)$$

where n is a unit vector normal to the fracture. With the linear slip model, the effective stiffness tensor of the resulting medium can be obtained from the inverse of the effective compliance S as follows:

$$C = S^{-1} = [S_b + S_f]^{-1} \quad (4)$$

S_b is the background compliance of the host rock, S_f is the compliance of the fractures and C is the effective stiffness of the resulting medium. The compliance tensor S is made up of two independent elements: the normal fracture compliance S_N and the tangential fracture compliance S_T respectively. The linear slip model is particularly useful in the determination of the elastic constants of media with lower symmetry than the hexagonal, for instance, materials with more than one set of aligned fractures (Schoenberg and Douma 1988; Schoenberg and Sayers 1995; Sayers 2002). Coates and Schoenberg (1995) used the linear slip model to model faults and fractures through finite-difference modelling. They computed the effective compliance of each cell in the finite difference grid and the effective stiffness of the resulting medium using equation (4) and then replaced the properties of every cell intersected by the fault or fractures by the effective medium properties of the background rock and the fracture compliance.

Both the Hudson (1980, 1981) inclusion-based model and the linear slip model of Schoenberg (1980) are frequency-independent equivalent medium theories. In both cases, the fractures or cracks are assumed to be isolated with respect to fluid flow. This is a strong approximation, but these models do describe conditions in the high-frequency limit, where the fluid does not have sufficient time to move due to wave-induced

pressure gradients. In addition, only the fracture or crack porosities are considered by the models. However, in a real geological setting, the host rock may also be porous and fluid exchange between the pores in the fractures and the pores in the host rock could have tremendous effect on the measured anisotropy of the rock (Mukerji and Mavko 1994; Thomsen 1995). Based on these ideas, Thomsen (1995) developed a model which takes into account the porosity of the host rock and assumes that the cracks and pores are hydraulically connected. The exchange of fluid between the pores and the cracks allows for the equilibration in the local fluid pressure in both the pores and the cracks. This model can thus be regarded as the low-frequency limit, in the sense that the frequency is sufficiently low to allow adequate time for the fluid pressure to equilibrate locally between the pores and nearby cracks (Thomsen 1995). The magnitude of the resulting anisotropy is higher because the fluid-filled cracks or fractures are much more compliant than the isolated ones. Thomsen (1995) found that his model matched with the laboratory measurements of Rathore *et al.* (1995) on samples containing aligned cracks in a porous matrix to a greater extent than the isolated crack model of Hudson (1981).

In this work, we used the linear slip model of Schoenberg (1980) to model discrete fractures. Xu *et al.* (2010) used the same approach to model seismic wave propagation in a medium with aligned fractures to investigate the influence of fracture cell spacing on the resulting wavefield and found that the magnitude of the P-wave anisotropy increased systematically with decreasing fracture cell spacing while the degree of scattering in the wave energy weakens with fracture cell spacing. Cui (2015) also successfully used the linear slip model utilizing the finite difference schemes to simulate various fractured media which include horizontally, vertically and orthogonally fractured media respectively.

2.2 Velocity–stress formulation in anisotropic media

The strain–stress relationship for a linear elastic and anisotropic medium can be expressed by the generalized Hooke's law as:

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} \quad (5)$$

where c_{ijkl} is the elastic or the stiffness tensor which has 81 components in the general case, σ_{ij} is the

stress tensor and ε_{kl} is the strain tensor. From the equations of Newton's second law, the elastodynamic wave equation which relates stiffness to displacement can be written in the form (in the presence of forces (f_i)):

$$\rho \frac{\partial^2 u_i(x, t)}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} + f_i, \quad (6)$$

where ρ is density, u_i is the i th component of the displacement and t is time. Substituting for stress from equation (5) and rearranging gives the wave equation as:

$$\rho \frac{\partial^2 u_i(x, t)}{\partial t^2} = \frac{\partial}{\partial x_j} [c_{ijkl}(x) \varepsilon_{kl}(x, t)] + f_i. \quad (7)$$

Combining equation (7) with the definition of the strain tensor (equation 8) gives the general wave equation for a 3D inhomogeneous anisotropic media (equation 9).

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (8)$$

$$\rho \frac{\partial v_i(x, t)}{\partial t} = \frac{\partial}{\partial x_j} \left[c_{ijkl}(x, t) \frac{\partial u_{k(x, t)}}{\partial x_l} \right] + f_i, \quad (9)$$

v_i is particle velocity. Also, by differentiating equation (5) with respect to time and noting the expression for the strain tensor in equation (8), we can write:

$$\frac{\partial \sigma_{ij}}{\partial t} = c_{ijkl} \frac{\partial}{\partial x_l} v_{kl}. \quad (10)$$

Equations (9) and (10) constitute the first-order linear partial differential velocity–stress equations for a general inhomogeneous anisotropic media. These equations can be solved by the use of the finite-difference schemes. Taylor's expansion is used to perform temporal and spatial derivatives of the quantities in the equations and any order of Taylor series expansion can be performed. The 4th order Crank–Nicolson approximation (1947) can be used to approximate the first-order derivatives in the spatial coordinates as follows:

$$\frac{\partial v}{\partial x} \approx \frac{-v \left(\left(i + \frac{3}{2} \right) \Delta x \right) + 27v \left(\left(i + \frac{1}{2} \right) \Delta x \right) - 27v \left(\left(i - \frac{1}{2} \right) \Delta x \right) + v \left(\left(i + \frac{3}{2} \right) \Delta x \right)}{24\Delta x} \quad (11)$$

The first order derivative in time can be approximated by a 2nd order scheme as:

$$\frac{\partial v}{\partial t} \approx \frac{v\left(i + \frac{1}{2}\right) \Delta t - v\left(i - \frac{1}{2}\right) \Delta t}{\Delta t}. \quad (12)$$

The approximations in equations (11 and 12) can be obtained from the linear combination of different Taylor expansions (Fornberg 1988):

$$\begin{aligned} v(x + \Delta x) \approx & v(x) + \frac{\Delta x}{1!} \frac{\partial v}{\partial x} + \frac{\Delta x^2}{2!} \frac{\partial^2 v}{\partial x^2} \\ & + \frac{\Delta x^3}{3!} \frac{\partial^3 v}{\partial x^3} + O\Delta x^4. \end{aligned} \quad (13)$$

As an illustration, a 4th order approximation of a first-order derivative that is used in the implementation of the staggered grid finite difference modelling can be achieved from four Taylor expansions on 4 points centred around $x = 0$ as follows:

$$\begin{aligned} v\left(x + \frac{\Delta x}{2}\right) \approx & v(x) + \frac{\Delta x}{2} \frac{\partial v}{\partial x} + \frac{\Delta x^2}{8} \frac{\partial^2 v}{\partial x^2} \\ & + \frac{\Delta x^3}{24} \frac{\partial^3 v}{\partial x^3} + \frac{\Delta x^4}{96} \frac{\partial^4 v}{\partial x^4} + O\Delta x^5 \end{aligned} \quad (14)$$

$$\begin{aligned} v\left(x - \frac{\Delta x}{2}\right) \approx & v(x) - \frac{\Delta x}{2} \frac{\partial v}{\partial x} + \frac{\Delta x^2}{8} \frac{\partial^2 v}{\partial x^2} \\ & - \frac{\Delta x^3}{24} \frac{\partial^3 v}{\partial x^3} + \frac{\Delta x^4}{96} \frac{\partial^4 v}{\partial x^4} + O\Delta x^5 \end{aligned} \quad (15)$$

$$\begin{aligned} v\left(x + \frac{3\Delta x}{2}\right) \approx & v(x) + \frac{3\Delta x}{2} \frac{\partial v}{\partial x} + \frac{9\Delta x^2}{8} \frac{\partial^2 v}{\partial x^2} \\ & + \frac{27\Delta x^3}{24} \frac{\partial^3 v}{\partial x^3} + \frac{81\Delta x^4}{96} \frac{\partial^4 v}{\partial x^4} + O\Delta x^5 \end{aligned} \quad (16)$$

$$\begin{aligned} v\left(x - \frac{\Delta x}{2}\right) \approx & v(x) - \frac{3\Delta x}{2} \frac{\partial v}{\partial x} + \frac{9\Delta x^2}{8} \frac{\partial^2 v}{\partial x^2} \\ & - \frac{27\Delta x^3}{24} \frac{\partial^3 v}{\partial x^3} + \frac{81\Delta x^4}{96} \frac{\partial^4 v}{\partial x^4} + O\Delta x^5. \end{aligned} \quad (17)$$

Subtracting equation (15) from equation (14) gives:

$$\begin{aligned} C_1 = & v\left(x + \frac{\Delta x}{2}\right) - v\left(x - \frac{\Delta x}{2}\right) \\ \approx & \Delta x \frac{\partial v}{\partial x} + \frac{2\Delta x^3}{24} \frac{\partial^3 v}{\partial x^3} + O\Delta x^5. \end{aligned} \quad (18)$$

Also, subtracting equation (17) from equation (16) gives:

$$\begin{aligned} C_2 = & v\left(x + \frac{3\Delta x}{2}\right) - v\left(x - \frac{3\Delta x}{2}\right) \\ \approx & 3\Delta x \frac{\partial v}{\partial x} + \frac{54\Delta x^3}{24} \frac{\partial^3 v}{\partial x^3} + O\Delta x^5. \end{aligned} \quad (19)$$

The third order term can be eliminated from the linear combination of C_1 and C_2 to give the 4th order approximation as

$$\begin{aligned} \frac{27C_1 - C_2}{24\Delta x} \approx & \frac{\partial v}{\partial x} + O\Delta x^4 \\ & 27\left(v\left(x + \frac{\Delta x}{2}\right)\right) - v\left(x - \frac{\Delta x}{2}\right) \\ \approx & \frac{-v\left(x + \frac{3\Delta x}{2}\right) + v\left(x - \frac{\Delta x}{2}\right)}{24\Delta x} + O\Delta x^4. \end{aligned} \quad (20)$$

In solving equations (9) and (10) using finite-difference modelling, all the quantities must first be discretized. A common discretization method is the use of the standard staggered grid (SSG) scheme (Madariaga 1976; Virieux 1984, 1986) which has the advantage of having the differential operators centred at the same point in space and time, enabling the spatial derivatives to be computed at half the grid size (Graves 1996; Bansal and Sen 2008). The different components of the stress tensor and the velocities must be defined at different locations on the grids. The stiffness tensor and the diagonal elements of the stress tensor are defined at the corner of the cell, while the off-diagonal element of the stress tensor is defined at a different location. This necessitates the interpolation of some of the elements of the stiffness tensor. Similarly, the buoyancy (reciprocal of density) also has to be interpolated since it is not defined at the same location as the components of the particle velocity. In the 3-D case, the stress, stiffness and velocity components have to be defined at seven different locations (Igel *et al.* 1995) and as in the 2-D case, some of the elements of the stiffness tensor and the buoyancy also have to be interpolated to the locations where they are required.

2.3 Boundary conditions

Two boundary conditions are commonly employed in seismic wave modelling; namely the absorbing boundary conditions and the free surface conditions on the top side of the computational domain (Virieux *et al.* 2012). The effects are assumed to be well described by variations of the physical properties of the medium for internal boundaries (Kelly *et al.* 1976; Kummer and Behle 1982).

The absorbing boundary condition is applied at the edges of a numerical model to attenuate undesired edge reflections in the recorded wavefield. There are three types of absorbing boundary conditions that are commonly used. The first type, proposed by Clayton and Engquist (1977) is the wavefield-prediction-based boundary condition. This type of absorbing boundary condition is sufficient for low incident wave angles, but does not work well for large incident wave angles (Liu *et al.* 2017). The second type of absorbing boundary condition is the damping boundary condition. The condition involves the multiplication of the wavefield values with an exponential damping factor in the boundary region (e.g., Cerjan *et al.* 1985). The major obscurity with this boundary condition is the difficulty in the determination of the damping factor (Liu *et al.* 2017). The third type of absorbing boundary condition is the perfectly matched layer (PML) boundary condition proposed by Berenger (1994) for electromagnetic wave simulation and later modified for electrodynamics by Chew and Liu (1996) and Festa and Vilotte (2005). Perfectly matched layers are anisotropic absorbing layers added at the outer edge of the numerical model. This boundary condition is widely utilized in numerical modelling due to its ability to absorb incident waves of any angle of incidence and any frequencies (Liu *et al.* 2017).

The free surface boundary conditions generally require that the stress must be zero at the free surface which corresponds to the top surface of the finite difference grid (Gottschamer and Olsen 2001). On the alternative, the method of the image can be utilized to implement the free surface along a virtual plane located half a grid interval above the top surface of the finite difference grid (Virieux 1986). The stress is made to be zero at the free surface by using a virtual plane located half a grid interval above the free surface where the stress must have opposite values to that located just below the free surface.

3. Hypothetical model and data acquisition

The hypothetical model consists of four isotropic horizontal layers (figure 1). To simulate a fractured reservoir model, a set of aligned vertical fractures was introduced into the third layer by the use of the technique of Coates and Schoenberg (1995). This makes the layer to become HTI. Table 1 gives details of the parameters of the model. The exact

Figure 1. Geometry of the hypothetical model. The third layer constitutes the anisotropic layer with vertical aligned fractures. The fractures produce a horizontally transverse isotropic medium with the fast wave direction parallel to the z-axis. The source (red star) was located on the surface of the model at ($x = 500$ m, $y = 1500$ m and $z = 0$), while the receivers were spread out in the x- and y- directions at a spacing of 10 m in each direction along the black dotted lines.

values of the model parameters or model set-up are not significant for the analysis as long as the model is reasonable and resemble an actual geological situation. The fractures are represented by grid cells each of size $10 \text{ m} \times 10 \text{ m} \times 10 \text{ m}$ and the entire model was divided into $300 \times 300 \times 300$ grids. The thickness of each fracture cell is equivalent to the length of a single grid cell (10 m) and the fracture spacing is regarded as the distance from one vertical fracture cell to the next vertical fracture cell. We made the same assumption as in Vlastos *et al.* (2003) that the normal and tangential fracture compliances are the same and have a value of $5.6 \times 10^{-10} \text{ mPa}^{-1}$ and computed the synthetic data from the model with a fracture cell spacing of 120 m. Lubbe and Worthington (2006) argued that the fracture compliance values used by Vlastos *et al.* (2003) are relatively high and maintained that for numerical modelling to be of practical use, the required values of the fracture compliance need to be obtained from field measurements although this might be a difficult task. Nevertheless, the tangential and normal compliance ratio of unity assumed in the numerical modelling is indicative of the case of dry fractures (e.g., Liu *et al.* 2000).

For the source wavelet, we used a Ricker wavelet with a centre frequency of 20 Hz. The source was placed on the model's surface at $x = 500$ m, $y = 1500$ m and $z = 0$. The P-wave

length in the fractured-layer (labelled HTI in figure 1) is 131 m, making the ratio of the fracture cell spacing to the seismic wavelength approximately 0.9. According to the condition of equivalence of a fractured medium to an anisotropic medium under the long wavelength approximation of the equivalent medium theory (Bakulin *et al.* 2000):

Seismic wavelength > fracture spacing > fracture opening.

In our study, 131 m > 120 m > 10 m: implying that our hypothetical model satisfies the condition of equivalence of a fractured medium to an anisotropic medium. Thus, the simulated fractures are expected to cause scattering of seismic waves which results in the seismic code. The receivers were spread out in the x- and y-directions at a fixed spacing of 10 m in each direction as illustrated in figure 1. The synthetic data were computed from the model and recorded at 1 ms sampling rate and 3 s total time. We extracted a 2-D section from the 3-D cube in four azimuthal angles (0, 26°, 45° and 90°), all with respect to the 0 azimuth which corresponds to the fracture strike direction.

The reflections from the model boundaries are considered as undesirable waves in the final recorded synthetic data and therefore need to be suppressed. To this end, the edges of the model were set to be absorbing boundaries by applying an exponentially decaying function of the form given by Cerjan *et al.* (1985) at the limited boundary layers to attenuate the energy at the boundary of the model. The recorded wavefield for the four azimuths considered is shown in figure 2. The red arrow indicates direct waves travelling on the model surface from the source to the receiver, the blue arrow indicates the second layer top reflection while the green and yellow arrows indicate the top and bottom fractured-layer reflections respectively. The black arrow indicates the top fractured-layer P-S converted wave. Scattering effects caused by the simulated fractures are noticeable in the four

gathers below the top fractured-layer reflection (green arrow, figure 2). These effects, however, are difficult to distinguish in the four gathers due to auto-scaling of software used in generating the plots. The software scales the amplitudes of the traces based on the maximum amplitude of the first trace in the gathers. The receiver spacing changes in figure 2(a–d) depending on the azimuth considered as a 2-D section was extracted from the 3-D cube in four different azimuths with 0° azimuth corresponding to the fracture strike direction. Our aim is to analyze the top and bottom fractured-layer reflected waves to quantify the scattering effects through attenuation estimates in the four azimuthal gathers to observe if these effects could give any useful information about the fracture orientations.

4. Attenuation estimation—spectral ratio method

Attenuation is usually measured from seismic data through Q estimation. In practice, estimates of Q are commonly made by the use of the spectral ratio method partly because the method is easier to use and very stable (Hauge 1981; Pujol and Smithson 1991; Dasgupta and Clark 1998), and also due to the fact that it can easily remove the effects of geometric spreading. The method involves the use of the ratios of the amplitude spectra A_1 and A_2 recorded at two receiver depths which corresponds to travel times t_1 and t_2 respectively according to the equation:

$$\ln \frac{A_2^2}{A_1^2} = \ln \frac{P_2}{P_1} = 2 \ln (RG) - \frac{2\pi f}{Q} (t_2 - t_1) \quad (21)$$

where f = frequency, R = reflectivity term, G = geometrical spreading factor, P_1 and P_2 are the respective power spectra (square of amplitudes), while Q is the seismic quality factor. Q is assumed to be frequency independent within the bandwidth used and is given by the slope of the least-squares regression of the Logarithm of the Power Spectral Ratio (LPSR) and frequency as:

$$\text{slope} = -\frac{2\pi (t_2 - t_1)}{Q}. \quad (22)$$

The intercept term on the vertical axis $\{\ln(RG)\}$ corresponds to a measure of elastic losses and is

Table 1. Hypothetical model parameters for studying the effects of discrete fractures.

Layer	v_p (m/s)	v_s (m/s)	ρ (kg/m ³)	Thickness (m)
1	1500	100	1000	400
2	2314	1100	1150	800
3	2610	1300	1750	800
4	3100	1800	2200	Half-space

Figure 2. 2D gather extracted from the 3D cube at (a) 0° azimuth, (b) 26° azimuth, (c) 45° azimuth, and (d) 90° azimuth. The red arrow indicates direct waves, the blue arrow indicates the second layer top reflection, while the green and yellow arrows indicate the top and bottom fractured-layer reflections respectively. The black arrow indicates top fractured-layer P-S converted wave.

Figure 3. Reference event – top layer (layer 1) reflection at 0 m offset and 0° azimuth (a) time series and (b) amplitude spectrum.

Figure 4. Sample spectral plots for the HTI layer reflection at 800 m offset. The black colour indicates top layer reflection, while the blue colour indicates bottom layer reflection.

a function of energy partitioning and geometrical spreading.

The accuracy of Q measurements from the spectral ratio method depends on trace windowing and regression frequency bandwidth (Sams and Goldberg 1990; Pujol and Smithson 1991). When large window times are used, noise interference may be included in the analysis window and this will result in spectral holes in the Fourier amplitude spectra (Sams and Goldberg 1990) and thus, erroneous Q values when least-squares regression is

applied. The use of a short time window might reduce these interference effects; however, this might lead to undersampling of the data at low frequencies and cause spectral instability (e.g., Sams and Goldberg 1990). A trade-off between these two limits is to use a time window that is long enough to include only the target events and short enough to eliminate the possible inclusion of any noise interference in the analysis window. The linear trend in the LPSR–frequency regression predicted by equation (21) can be distorted by the

Figure 5. Plots of maximum amplitude of bottom fractured layer reflection *versus* azimuth. The wave amplitude decreases to a minimum at 90° azimuth.

Figure 6. Log Power Spectral Ratio (LPSR) against frequency plots for the top HTI layer reflection. The inserted legend indicates the various offsets.

Figure 7. Log Power Spectral Ratio (LPSR) against frequency plots for the bottom HTI layer reflection. The inserted legend indicates the various offsets.

effects of multiple scattering together with other noise interference that might be included in the analysis window (Spencer *et al.* 1982). This will produce strong local variations and notches in the computed spectra and lead to oscillation of the LPSR– frequency plot. Consequently, the accuracy of Q measurements from the spectral ratio method also depends on the frequency bandwidth used for regression (e.g., Spencer *et al.* 1982; Sams and Goldberg 1990; Pujol and Smithson 1991). More stable results of Q values can be obtained with a wider regression bandwidth but this, however, might lead to regression analysis being carried out outside the signal bandwidth. The choice of the frequency bandwidth for linear regression is thus very subjective and a good practice is to use the linear portion of the LPSR– frequency plot where the frequency bandwidth lies approximately within the signal bandwidth.

In this paper, we utilize the spectral ratio method to compute the interval Q values in the fractured-layer for the four azimuthal synthetic gathers. We used the first trace in the 0 azimuth gather of the top layer (layer 1) reflection at 0 offsets (figure 3) as the reference event. The pulse shape of this event is proportional to the time

derivative of the input Ricker wavelet. Thus, its peak frequency is higher than that of the input Ricker wavelet.

The ratios of the power spectra of the target events (top and bottom fractured-layer reflections) and that of the reference event were computed with a constant FFT window of 160 ms according to equation (21). Sample power spectral plots are shown in figure 4 for the fractured-layer reflection at 800 m offset for the four azimuths considered. The black colour indicates the top layer reflection while the blue colour indicates bottom layer reflection. There is a little drop in the peak frequencies of the bottom layer compared to the top layer reflection, indicative of scattering attenuation induced by the simulated fractures. Figure 5 shows the plots of the maximum amplitudes of the bottom fractured layer reflections *versus* azimuths at fixed offsets of 200, 400, 800 and 1000 m, respectively. The wave amplitude decreases to a minimum at 90° azimuth which corresponds to the known fracture normal direction from the hypothetical model. Sample plots of the LPSR against frequency for selected offsets for the reflections from the top and bottom of the target layer are shown in figures 6 and 7 correspondingly. The plots show

Figure 8. Q profiles against offsets. There is a systematic decrease in the Q values with increasing offsets.

approximate linearity within a frequency range of 5–40 Hz. This frequency bandwidth is common to all the reflections analyzed and also lies within the signal bandwidth and was consequently used for the least-squares regression analysis. We then computed the Q value down to the reflector for a given offset from the slope of the least-squares regression which is given by equation (22).

The interval Q values in the fractured layer can be computed using the layer-stripping method (e.g., Dasgupta and Clark 1998; Behura and Tsvankin 2009; Shekar and Tsvankin 2011; Reine *et al.* 2012). Here, the layer-stripping method of Dasgupta and Clark (1998) was used to compute the interval Q_i values in the target layer from the pair of Q values (Q_1 and Q_2) calculated for the top and bottom of the layer respectively from equation (23).

$$Q_i = \frac{[t_2 - t_1]}{t_2/Q_2 - t_1/Q_1}. \quad (23)$$

5. Results and analysis

Figure 8 shows the results of our Q estimation from the azimuthal gathers. The Q values decrease systematically both with increasing offset and angle from the 0 azimuths corresponding to the direction of fast wave propagation (see figure 8). The seismic wave amplitude decays exponentially with distance of propagation, culminating in the decrease in Q values with increasing offset. Thus,

the wave experiences more attenuation as it travels through longer distances in the medium. Maximum wave scattering occurs in the direction which is perpendicular to the fracture strike direction (i.e., 90° azimuth), while minimum wave scattering occurs parallel to the fractures (i.e., 0 azimuth). These directions correlate with the direction of maximum and minimum attenuation, respectively.

Azimuthal variations in the induced attenuation are well fitted with a cosine function of the form given by Maultzsch *et al.* (2007) as:

$$\Delta Q^{-1} = A + B \cos[2(\theta - \theta_o)] \quad (24)$$

where A is an arbitrary constant, B is the amplitude of the attenuation anisotropy and θ_o is the direction perpendicular to the fractures in which maximum attenuation or wave scattering occurs. These fits are shown in figure 9 for the azimuths considered. B increases systematically with increasing offsets or incidence angles (figure 10a). The cosine fits show that maximum wave scattering and hence attenuation occurs in the 90° azimuth which corresponds to the known fracture normal direction from the hypothetical model (figure 10b). These cosine fitting results are consistent with the amplitude results shown in figure 5.

More analysis of our results reveals that the azimuthal variations in the induced attenuation in the HTI layer are well fitted with the ellipse (figure 11). The form and size of the ellipse depend on the magnitude of the semi-minor and semi-major

Figure 9. Cosine fits of the Q results. The anisotropic attenuation amplitude increases with offsets. The induced attenuation has a maximum value in the 90° azimuth.

Figure 10. Inverted parameters from the cosine fits of the Q results. (a) The amplitude of the attenuation anisotropy increases with increasing offsets and (b) the angle of maximum wave scattering (maximum attenuation) – 90° azimuth which corresponds to the fracture normal direction. A minimum offset of 200 m is needed for the anisotropy to occur.

axes. These plots, similar to slowness surfaces constitute a suitable means of representing anisotropic attenuation graphically. At a given azimuthal angle θ from the north direction corresponding to the y-axis, the Q value corresponds to the distance from the centre of the ellipse to the surface. The fracture normal is inferred from the minor axis of the Q ellipse. This direction correlates with the direction of maximum wave scattering

(90° azimuth). Minimum wave scattering occurs in the fracture strike direction (0° azimuth) inferred from the major axis of the Q ellipse. However, a minimum offset–depth ratio of 0.2 to the top of the HTI layer corresponding to a source–receiver offset of 200 m is required for the azimuthal variations in the induced attenuation to occur. These findings show a good consistency with the cosine fitting results.

Figure 11. Ellipse fitting of the Q results. The centre of the ellipse is at (0, 0). At a given azimuthal angle θ from the north direction corresponding to the y-axis, the Q value corresponds to the distance from the centre of the ellipse to the surface (red arrow). The minor and major axes of the ellipse correspond to the fracture normal and fracture strike directions respectively. No variations occur at 0 m offset.

The results of experimental studies with laboratory scale models and numerical simulation (e.g., [Schultz and Toksöz 1995](#); [Willis et al. 2006](#); [Burns et al. 2007](#); [Ekanem et al. 2013, 2014](#)) have shown that fractures with lengths on the order of seismic wavelength cause scattering of seismic waves. The scattering occurs more in the direction normal to the fractures than in the fracture strike direction. Our results are consistent with these findings, indicating that anisotropic wave scattering could provide useful information on fracture properties.

6. Concluding remarks

Loss of seismic wave energy (attenuation) could be the result of two effects. Firstly, it could result from energy conversion to heat and fluid displacement as the wave passes through a fluid-saturated medium; commonly regarded as intrinsic or anelastic attenuation (e.g., [Johnson et al. 1979](#); [Winkler and Nur](#)

[1982](#); [Chapman 2003](#), etc). Secondly, it could be the result of scattering and tuning effects; commonly termed apparent or extrinsic attenuation (e.g., [Schultz and Toksöz 1995](#); [Willis et al. 2006](#); [Burns et al. 2007](#), etc). The resultant effect of both forms of attenuation is called effective attenuation. In this work, we have used the 3-D finite-difference technique to study anisotropic wave scattering in discrete fracture model based on the linear slip model of [Schoenberg \(1980\)](#). The results of our study demonstrate that a population of aligned fractures with length comparable with the seismic wavelength causes scattering which exhibits a characteristic anisotropic behaviour with respect to the fracture properties. We have quantified the amount of wave scattering through attenuation estimates and shown that the induced wave attenuation exhibits a characteristic anisotropic behaviour which could provide useful information about the fracture properties.

The induced attenuation caused by the scattering from the fractures (averaged over the frequency

range used) increases both with increasing offset and angle θ from the 0 azimuth. This anisotropic attenuation exhibits a characteristic elliptical variation which allows the orientations of the fractures to be obtained from the axes of the ellipse. The fracture normal is inferred from the minor axis of the Q , which is the 90° azimuth. This direction correlates with the direction of maximum wave scattering (maximum attenuation) and slow wave velocity. Minimum wave scattering (minimum attenuation) occurs in the fracture strike direction (0 azimuth) inferred from the major axis of the Q ellipse which corresponds to the direction of fast wave propagation. The azimuthally varying induced attenuation is also well fitted with a $\cos 2\theta$ function which reveals that maximum wave scattering occurs in the 90° azimuth, showing consistency with the ellipse fitting results. The anisotropic attenuation amplitude increases with increasing offset and has a maximum value of 0.6% at an offset of 1200 m. Though the analysis of four azimuthal data may not be adequate for the study of anisotropic wave scattering, it is useful to demonstrate that the scattering characteristics can provide a means of obtaining the known fracture orientations in the hypothetical model. Our findings are in excellent agreement with the results of empirical experiments in the laboratory and thus, validate the practical utility of using anisotropic wave scattering as a potential tool to delineate fractures from seismic data to complement the use of other seismic attributes, at least for the relatively simple geometries of subsurface structure investigated here.

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