Spatial hydrological drought characteristics in Karkheh River basin, southwest Iran using copulas

ESMAEEL DODANGEH^{1,*}, KAKA SHAHEDI¹, JENQ-TZONG SHIAU² and
MARYAM MIRAKBARI³

¹Department of Watershed Management, Sari Agricultural Sciences and Natural Resources University, Sari, Iran.

²Department of Hydraulic and Ocean Engineering, National Cheng Kung University, Tainan 701, Taiwan, Republic of China.

³Department of Rehabilitation of Arid and Mountainous Regions, University of Tehran, Tehran, Iran.

*Corresponding author. e-mail: smaeel.dodangeh@gmail.com

MS received 27 September 2016; revised 16 February 2017; accepted 3 April 2017;
published online 31 August 2017

Investigation on drought characteristics such as severity, duration, and frequency is crucial for water resources planning and management in a river basin. While the methodology for multivariate drought frequency analysis is well established by applying the copulas, the estimation on the associated parameters by various parameter estimation methods and the effects on the obtained results have not yet been investigated. This research aims at conducting a comparative analysis between the maximum likelihood parametric and non-parametric method of the Kendall τ estimation method for copulas parameter estimation. The methods were employed to study joint severity–duration probability and recurrence intervals in Karkheh River basin (southwest Iran) which is facing severe water-deficit problems. Daily streamflow data at three hydrological gauging stations (Tang Sazbon, Huleilan and Polchehr) near the Karkheh dam were used to draw flow duration curves (FDC) of these three stations. The Q_{75} index extracted from the FDC were set as threshold level to abstract drought characteristics such as drought duration and severity on the basis of the run theory. Drought duration and severity were separately modeled using the univariate probabilistic distributions and gamma–GEV, LN2–exponential, and LN2–gamma were selected as the best paired drought severity–duration inputs for copulas according to the Akaike Information Criteria (AIC), Kolmogorov–Smirnov and chi-square tests. Archimedean Clayton, Frank, and extreme value Gumbel copulas were employed to construct joint cumulative distribution functions (JCDF) of droughts for each station. Frank copula at Tang Sazbon and Gumbel at Huleilan and Polchehr stations were identified as the best copulas based on the performance evaluation criteria including AIC, BIC, log-likelihood and root mean square error (RMSE) values. Based on the RMSE values, nonparametric Kendall- τ is preferred to the parametric maximum likelihood estimation method. The results showed greater drought return periods by the parametric ML method in comparison to the nonparametric Kendall τ estimation method. The results also showed that stations located in tributaries (Huleilan and Polchehr) have close return periods, while the station along the main river (Tang Sazbon) has the smaller return periods for the drought events with identical drought duration and severity.

Keywords. Drought; copula; Frank; Clayton; Gumbel; Karkheh River basin; Iran.

1. Introduction

Drought is widely recognized as a complex and multidimensional but least understood natural hazard with pivotal impacts on socio-economic and agricultural activities (Modarres 2010). According to socio-economic damages induced by natural hazards, drought is the leading disaster since it ranks higher than floods and other meteorological disasters (Shamaeizadeh 2009). Drought can be defined with different disciplinary perspectives including meteorological, hydrological, agricultural and socio-economic droughts (Hisdal and Tallaksen 2000). Among these categories, hydrological drought, defined as inadequate surface and subsurface water resources for established water uses of a given water resources management system (Mishra and Singh 2010) is an important issue in arid regions such as Iran since agricultural activities heavily depend on surface and subsurface water resources.

Investigation on droughts receives worldwide attention recently not only due to increasing reported disasters, but also intensified severity and increasing frequency induced by climate change. Frequency analysis for various drought characteristics thus becomes more important than ever. A considerable amount of previous studies on drought frequency analysis focusses on univariate drought characteristics such as severity, duration, and magnitude since the well-developed techniques used in hydrology. For example, Lee *et al.* (1986), Lana and Burgueno (1998) and Cancelliere and Salas (2004) conducted univariate drought frequency analysis of multiyear drought durations, annual extreme episodes of dry days, and drought length of various time-scale hydro-climate variables, respectively.

However, such separate univariate analysis cannot reveal significant correlations among the inherent multi-attribute characteristics of droughts. Therefore, single-variable frequency analysis can only provide limited assessment of such events. Many multivariate distributions have been developed as immediate extensions of univariate distributions, examples being the multivariate normal, bivariate pareto, bivariate gamma, etc. Some studies including Shiau and Shen (2001), Bonaccorso *et al.* (2003), González and Valdés (2003), Loáiciga (2005), and Kim *et al.* (2006) constructed bivariate drought models to explore bivariate probabilistic properties of droughts. The drawbacks of these types of distributions are that: (1) the same

family is needed for each marginal distribution, (2) extensions to more than just the bivariate case are not clear, and (3) parameters of the marginal distributions are also used to model the dependence between the random variables (Favre *et al.* 2004). A construction of multivariate distribution that does not suffer from the drawbacks mentioned above is based on the notation of copulas (Sklar 1959). Copula can preserve both dependence structure and different distribution characteristics of the random variables. Shiau (2006) firstly introduced the copula method for investigating bivariate frequency of droughts to alleviate difficulties of different marginal distributions often used to fit various attributes of droughts such as severity and duration. The copula method thus becomes a popular approach for bivariate or multivariate drought studies since it is easy to link different marginal distributions to construct the joint multivariate distribution function. Copula-based drought studies in literature include Shiau and Modarres (2009), Sernaldi *et al.* (2009), Kao and Govindaraju (2010), Edossa *et al.* (2010), Liu *et al.* (2011), He *et al.* (2011), Shiau *et al.* (2012), Reddy and Ganguli (2012), Shiau and Hsiao (2012), Lee *et al.* (2013), De Michele *et al.* (2013), Sharma and Panu (2014), Saghafian and Mehdikhani (2014), Vergni *et al.* (2015), Wu *et al.* (2015), Xu *et al.* (2015a, b) and among others. Few attempts have been done on joint modeling of drought characteristics obtained from streamflow data. Sadri and Burn (2014) used Archimedean copulas namely Clayton, Frank and Gumbel copula for joint modeling of drought duration and severity extracted from monthly stream flow from 36 non-regulated sites in the Canadian Prairies. Recently Tosunoglu and Kisi (2016) used Archimedean copulas for joint modeling of annual maximum severity (AMS) and corresponding duration (CD) in Coruh basin, Turkey. They showed that Gumbel copula is the most suitable model for joint modeling of the AMS and Cd series.

In copula-based bivariate frequency analysis, estimating parameters to fit an appropriate copula function for observed data is an essential and important step in modelling procedure. Although several goodness-of-fit tests have been developed to find a best-fit copula (Reddy and Ganguli 2012; Lee *et al.* 2013; Fu and Butler 2014), most studies have used a single method for copula parameter estimation. Effects of different parameter-estimation methods on the obtained results have not yet been investigated. Located on the dry belt of

Figure 1. Location map of the Karkheh River basin in the southwest Iran.

the world, Iran is suffering drought-induced water shortage problems. Shamaeizadeh (2009) reported that droughts cause economic loss of 2.5 billion dollars per year for Iran. Significant amount of studies have been devoted to investigate droughts (Shiau and Modarres 2009; Modarres 2010; Modarres and Sarhadi 2010; Zarch *et al.* 2011; Shiau and Hsiao 2012) and stream low flows (Modarres 2008; Dodangeh *et al.* 2014). This study thus aims at deriving severity–duration–frequency characteristics of droughts for Karkheh River basin (southwest Iran) to help decision-makers to construct appropriate drought mitigation strategies in this basin. In addition, effects of different parameter-estimation methods on the frequency of droughts in this basin are also explored.

The remainder of this work is laid out as follows. Section 2 gives an overview of the Karkheh River basin located in southwest Iran. Section 3 presents theory of copula and methods for parameter estimation. Results and discussions are given in the section 4. Finally, the main conclusions of this work are summarized in section 5.

2. Study area and dataset

Located in the southwest Iran, Karkheh River lies between the 46° – 49° E and 31° – 39° N. Karkheh River basin has a drainage area of $43,000 \text{ km}^2$ and encompasses five provinces including Hamedan, Kermanshah, Ilam, Lorestan and Khuzitan (figure 1). Elevation in this basin ranges from a few meters in the south to more than 350 m in the north and east. The main river within this basin is the 900 km long Karkheh River which originates from Zagros mountain ranges and wanders southeastward and finally terminates in the Hawr-Al-Azim swamp. In terms of mean discharge, the Karkheh River is the third largest river in Iran and the largest Karkheh Dam, with a reservoir capacity of 5.6 billion m^3 , is built on this river (Jamali *et al.* 2013).

Although various climatic zones are identified in the Karkheh River basin, the most dominant climatic characteristic is semi-aridity with long and hot summers (Kirby *et al.* 2006). The mean annual precipitation in this basin is approximately

Table 1. *Some basic information for the selected stations in the Karkheh River basin.*

Station ID	Station name	Elevation (m)	Latitude	Longitude	River name
21–159	Tang Sazbon	780	33°34'N	46°51'E	Simereh
21–147	Huleilan	980	33°46'N	47°06'E	Chazman
21–127	Polchehr	1270	34°20'N	47°25'E	Gamasiab

400 mm and generally falls from October through May. However, snow mainly falls in late fall and winter (November–March), with almost no rain in the late summer. Snowmelt is an important component of streamflow during the spring. More than 64% of annual streamflow occurs during the January–May period (Kirby *et al.* 2006).

Rangelands, forests, rain-fed agriculture, and irrigated agriculture are the main land uses in this basin (Muthuwatta *et al.* 2010). The population within this basin is nearly four million and 67% of these people are living in rural areas and heavily depend on the water stored in the reservoirs. Water is primarily used for agriculture, domestic, and fish farming. The Karkheh River basin occupies about 9% of the total irrigated area in Iran, which provides 10% of the country's wheat production (Marjanizadeh *et al.* 2009).

As water is the limited natural resource to develop agricultural activities in this basin, understanding spatial variation of drought characteristics within this basin is an essential issue for decision-makers to construct effective water-resources management and adaptive to climatic extremes. Since this study aims to identify the drought-prone regions within this basin, three streamflow gauging stations along the main river and tributaries are selected. Tang Sazbon station (El. 780 m) is located in the main river (Simereh River), while Huleilan (El. 980 m) and Polchehr (El. 1270 m) are located in the tributaries of Chazman River and Gamasiab River respectively. Daily streamflow data of these three gauging stations, recorded by Iran Ministry of Power, for the period of 1970–2012 are used in this study. Some basic information for these three selected stations are provided in table 1 and locations are shown in figure 2.

3. Methodology

3.1 Drought definition and indices

A quantitative threshold level method (Yevjevich 1967) is used in this work to define hydrological

drought events from streamflow series. A hydrological drought event is defined as a continuous period during which the streamflow falls below a threshold level. Each hydrological drought event is characterized by drought severity (S) and drought duration (D). The drought duration is the consecutive days for which the streamflow is below the reference discharge (threshold level) and drought severity is the cumulative deficit volume for that period. The threshold level is the boundary between the normal and low flows (Zhang *et al.* 2013). Several low-flow indices have been introduced in literature to be used as the threshold levels for streamflow series, which include streamflow drought index (SDI) (Madadgar and Moradkhani 2013), 7-day, 10-year low-streamflow ($Q_{7,10}$) (Reilly and Kroll 2003), surface water supply index (SWSI) (Shafer and Dezman 1982), and reclamation drought index (RDI) (Weghorst 1996). Among these low flow indices, percentile from the flow duration curve (FDC) is frequently applied for both perennial and intermittent streams. The FDC is a relationship between any given discharge value and the percentage of time that this discharge is equalled or exceeded, or in other words, the relationship between magnitude and frequency of streamflow discharges (Smakhtin 2001).

For perennial streams, threshold level between Q_{70} and Q_{95} form the FDC are frequently applied, which are the flows that are exceeded 70% and 95% of the time (Fleig *et al.* 2006). FDCs constructed on the basis of daily flow time series provide the most detailed way of examining duration characteristics of a river (Smakhtin 2001). In this study, the 75-percentile flow (Q_{75}) is used as the threshold level to extract hydrological drought events. The Q_{75} is the flow that exceeded 75% of the time. The reader is referred to Zhang *et al.* (2013) for more details.

3.2 Copula theory

Sklar (1959) introduced the copula theory to model stochastic nature of multivariate process, in which a multivariate distribution function is

Figure 2. Location map of the selected stations within the Karkheh River basin.

constructed using several univariate distribution functions. This theory states that if $F_{X,Y}(x,y)$ is a joint cumulative distribution function (JCDF) of random variables of X and Y with respective marginal cumulative distribution functions (CDFs) $F_X(x)$ and $F_Y(y)$, then a copula function C exists such that

$$F_{X,Y}(x,y) = C(F_X(x), F_Y(y)). \quad (1)$$

Under the assumption of continuous probability density functions $f_X(x)$ and $f_Y(y)$, the joint probability density function thus become

$$f_{X,Y}(x,y) = c(F_X(x), F_Y(y))f_X(x)f_Y(y) \quad (2)$$

where c is the density function of C and is defined as:

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v} \quad (3)$$

Table 2. Archimedean copula used in this study.

Copula	Parameter space	Generator function $\Phi(t)$	JCDF/JDF
Clayton	$\theta \in (-1, \infty) \setminus \{0\}$	$\frac{t^{-\theta}-1}{\theta}$	$C(u, v; \theta) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}, \theta > 0$ $c(u, v) = (\theta + 1)(u^{-\theta} + v^{-\theta} - 1)^{\frac{1}{\theta}-2}(uv)^{-\theta-1}$
Gumbel	$\theta \in (1, +\infty)$	$(-\ln t)^\theta$	$C(u, v; \theta) = \exp \left\{ - \left[-(Lnu)^\theta + (-Lnv)^\theta \right]^{\frac{1}{\theta}} \right\}, \theta \geq 1$ $c(u, v) = C(u, v) \frac{[(-Lnu)(-Lnv)]^{\theta-1}}{uv} \left[(-Lnu)^\theta + (-Lnv)^\theta \right]^{\frac{2}{\theta}-2}$ $\times \left\{ (\theta - 1) [(-Lnu)^\theta + (-Lnv)^\theta]^{-1/\theta} + 1 \right\}$
Frank	$\theta \in (-\infty, \infty) \setminus \{0\}$	$-\ln \frac{e^{-\theta t}-1}{e^{-\theta}-1}$	$C(u, v; \theta) = \frac{1}{\theta} \ln \left[1 + \frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1} \right], \theta \neq 0$ $c(u, v) = \frac{\theta e^{-\theta(u+v)}(e^{-\theta}-1)}{[e^{-\theta(u+v)}-e^{-\theta u}-e^{-\theta v}+e^{-\theta}]^2}$

Note: u and v represent dependent CDFs of univariate distributions of random variables. θ denotes the copula parameter. t denotes the critical probability level.

where u and v represent dependent CDFs of $F_X(x)$ and $F_Y(y)$ and range between 0 and 1.

For bivariate drought frequency analysis, the JCDF of drought severity and drought duration can be constructed by CDFs of drought severity and drought duration and a copula function (Shiau and Modarres 2009). That is,

$$F_{D,S}(d, s) = C(F_D(d), F_S(s)) \quad (4)$$

where $F_{D,S}(d, s)$ is the JCDF of drought severity and drought duration; $F_D(d)$ and $F_S(s)$ represent CDFs of drought duration (D) and drought severity (S), respectively.

In this study, Archimedean class of copulas including Clayton, Gumbel, and Frank copulas, reported in table 2, are used for bivariate drought severity–duration frequency analysis due to the fact that these copulas are easy to construct and they can capture wide range of dependence (Shiau et al. 2007; Reddy and Ganguli 2012; Lee et al. 2013; Chang et al. 2016).

3.2.1 Copula parameter estimation

Identifying the best copula fitting to the observed data requires an efficient parameter-estimation approach and a goodness-of-fit test (Acciolya and Chiyoshi 2004). A considerable amount of parameter-estimation methods has been developed in literature. In this study, the maximum likelihood classified as the parametric method and a non-parametric method of the Kendall τ estimation (Genest and Rivest 1993) are employed to estimate parameters of copula for constructing JCDF of droughts.

The maximum likelihood estimation method needs to establish the logarithmic maximum likelihood function, which is expressed as:

$$L(\theta) = \sum_{i=1}^n \log[c_\theta \{F_X(x_i), F_Y(y_i)\}] \quad (5)$$

where c is the copula density function; F represents the marginal distribution; X and Y are two dependent random variables and x_i and y_i denote the corresponding observed data. The parameter of θ is obtained by maximizing the logarithmic maximum likelihood function. In the likelihood estimator, the marginal distributions $F_X(x_i)$ and $F_Y(y_i)$ are replaced by their empirical representation. The empirical cumulative distribution function (CDF) for a continuous random variable x can be calculated as $F_n(x) = \frac{1}{n+1} \sum_{i=1}^n 1(X < x_i)$, where n is the total number of observed data.

The non-parametric method adopted in this study to estimate the parameter θ is the concordance measurement Kendall's τ which is a rank correlation coefficient and is often used to evaluate the association between two measured quantities. The relationships between Kendall's τ and copula parameter for different Archimedean copulas are given in Genest and Rivest (1993) and Acciolya and Chiyoshi (2004). The following presents the methodology to estimate the value of Kendall's τ .

Let $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$ denote n paired observations of correlated random variables of X and Y . Any pair of observations (x_i, y_i) and (x_j, y_j) are said to be concordant if both $x_i > x_j$ and $y_i > y_j$ or if both $x_i < x_j$ and $y_i < y_j$. Discordant represents the condition that if $x_i > x_j$ and $y_i < y_j$ or if $x_i < x_j$ and $y_i > y_j$.

The Kedall's τ is thus defined as the difference between probabilities of concordant and discordant (Mirakbari *et al.* 2010).

3.2.2 Best fit of copula

The limitation of the copula approach is that there is no specific way to check whether the dependency structure of a dataset is appropriately modeled by the chosen copula (Acciolya and Chiyoshi 2004). Similar to the univariate probabilistic modeling procedure, a goodness-of-fit test is necessary to identify the best-fit copula in modeling observed bivariate drought data. Currently there is no consensus about a statistic criterion used to select the copula that provides the best fit to the data. Breymann *et al.* (2003) and Palaro and Hotta (2006) used the Akaike Information criteria (AIC) to select the best-fit copula. However, various simulation studies show that the Schwarz Information Criterion (SIC or BIC) performs better in large samples whereas the AIC tends to be superior in small samples (Shumway and Stoffer 2011). In this research, both criteria are implemented and the best-fit copula is the one with the lowest values of these criteria. The AIC and the BIC can be defined as follows:

$$\text{AIC} = -2L(\theta) + 2k \quad (6)$$

$$\text{BIC} = -2L(\theta) + k \ln(n) \quad (7)$$

where k is the number of parameters of the copula model and n is the number of observations.

However, information criteria such as AIC are not able to provide any understanding about the power of the decision rule (Genest and Rivest 1993). Goodness-of-fit (GOF) approaches, on the other hand, are able to reject or fail to reject a parametric copula and are thus preferred. Several copula GOF approaches have been proposed in literature. For example, Genest and Rivest (1993) have developed an empirical method to identify the best-fit copula for the Archimedean copulas. Fermanian (2005) proposes a test based on proximity between the smoothed copula density and the estimated parametric density. Genest *et al.* (2006) utilize the non-truncated version of Kendall's process for checking whether the dependence structure of bivariate data is appropriately modeled by the Archimedean copula.

The procedure proposed by Genest and Rivest (1993) to select the most appropriate Archimedean copula is briefly described below.

1. The Archimedean copula has the dependence function of the form

$$C_\phi(x, y) = \phi^{-1}[\phi(x) + \phi(y)] \quad \text{for only } 0 < x, y < 1 \quad (8)$$

where ϕ is the known generator $\phi : 1 \rightarrow [0, \infty] =$ continuous, convex, strictly decreasing function such that $\phi(1) = 0$, $\phi'(x) < 0$, $\phi''(x) > 0$ for all $0 \leq x < 1$.

2. The parametric distribution function of $\Pr\{C(U, V) \leq p\}$, denoted as $K_\phi(p)$, can be expressed as:

$$\begin{aligned} K_\phi(p) &= \Pr\{C(U, V) \leq p\} \\ &= p - \frac{\phi(p)}{\phi'(p)}, \quad 1 \geq p \geq 0 \end{aligned} \quad (9)$$

where p = a constant value (Nelsen 2006).

3. A nonparametric estimate of the $K_\phi(p)$ function, denoted by $K_n(p)$, for each set of observation values (x_i, y_i) , $i = 1, \dots, n$ is obtained as:

$$K_n(p) = \sum_i^n \delta[p - C(x_i, y_i)]/n \quad (10)$$

where $K_n(p)$ = nonparametric estimate of the $K_\phi(p)$; $\delta(\cdot)$ stands for the distribution function of point mass at the origin; n = number of observation values; and $C(x_i, y_i)$ is determined using n set of observation values (x_i, y_i) , $i = 1, \dots, n$ as:

$$C(x_i, y_i) = \text{sum}(T_i) \quad (11)$$

where T_i is calculated as:

$$T_i = \begin{cases} 1 & \forall (x_j, y_j) \text{ if } x_j < x_i \text{ \& } y_j < y_i, j=1, \dots, n \\ 0 & \text{else} \end{cases} \quad (12)$$

After obtaining both estimates of $K_\phi(p)$ and $K_n(p)$, it is possible to calculate performance evaluation criteria such as root mean square error (RMSE), i.e.,

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^n [X_c(i) - X_o(i)]^2} \quad (13)$$

where n is the number of observations; X_c is the theoretical probability of copula and X_o is the empirical probabilities of observations.

Figure 3. Flow duration curves and the corresponding Q_{75} for three stations in the Karkheh River basin.

3.2.3 Joint return period of drought characteristics

The joint return period of drought severity and duration are constructed based on the derived joint cumulative distribution function of droughts in terms of the best-fit copula. The joint return period of droughts are used to estimate the recurrence interval of drought events with certain severity and duration, which is calculated as (Shiau 2006):

$$T_{DS} = T(S \geq s, D \geq d) = \frac{E(L)}{P(S \geq s, D \geq d)} = \frac{E(L)}{1 - F_D(d) - F_S(s) + C[F_S(s), F_D(d)]} \quad (14)$$

where T_{DS} denotes the joint return period for $D \geq d$ and $S \geq s$, here d and s represent specific values of drought duration and severity, respectively; $E(L)$ is the expected drought inter-arrival time which can be estimated from observed droughts; $F_D(d)$ and $F_S(s)$ are cumulative distribution functions of drought duration and severity, respectively; C is the copula function.

4. Results and discussion

4.1 Determination of drought events based on threshold level method

Value of Q_{75} is used as the threshold level in this study to define hydrological droughts. Figure 3 shows the flow duration curves, constructed by

the recorded daily streamflow of 20-year period, for the three selected stations within the Karkheh River basin. Corresponding Q_{75} of 3.47, 11.12 and 16.0 m³/s are obtained for the Polchehr, Huleilan, and Tang Sazbon, respectively. These values indicate that Q_{75} increases with increasing drainage areas within the Karkheh River basin.

Table 3 reports the number of drought events for these three stations, which is equal to 38, 36 and 78 for Tang Sazbon, Huleilan and Polchehr, respectively. Since a large number of minor droughts with short duration or less severity are included in the drought-events series, the approach proposed by Madsen and Rosbjerg (1995) is adopted in this study. This approach uses a predefined percentage of mean drought severity ($r_s E(S)$) and duration ($r_d E(D)$) to exclude minor droughts when drought severity or duration is less than these predefined values. Tallaksen *et al.* (1997) recommend 30% (i.e., $r_d = r_s = 0.3$) is appropriate ratio to remove minor droughts and this criterion is adopted in this study. The $E(S)$ and $E(D)$ statistics respectively indicate the mean drought severity (m³/s.days) and duration(days). The $E(S) = 28$, 15 and 10 days were used to specify meaningful duration of persistently low flows for the event to be called drought in Tang Sazbon, Huleilan and Polchehr stations. The number of drought events after removing minor droughts becomes 21, 23, and 51 for Tang Sazbon, Huleilan and Polchehr, respectively, which are also reported in table 3.

Table 3. Number of droughts before and after pooling minor droughts.

Station	$r_d E(D)$	$r_s E(S)$	Before exclusion	After exclusion
Tang Sazbon	28	98	38	21
Huleilan	15	70	36	23
Polchehr	10	19	78	51

Note. $r_d = r_s = 0.3$; $E(D)$ = mean drought duration and $E(S)$ = mean drought severity.

Table 4. Results of goodness-of-fit tests for marginal distributions CDFs for drought severity.

Station	Distribution type	Distribution parameter	K-S		χ^2		AIC
			Statistic	p-value	Statistic	p-value	
Tang Sazbon	Gamma	$\sigma = 623.28$ $\beta = 0.85$	0.10	0.97	2.24	0.94	309
Huleilan	LN2	$\mu = 5.41$ $\sigma = 1.12$	0.15	0.33	9.86	0.19	323
Polchehr	LN2	$\mu = 3.92$ $\sigma = 1.53$	0.15	0.49	7.69	0.36	338

Table 5. Results of goodness-of-fit tests for marginal distributions CDFs for drought duration.

Station	Distribution type	Distribution parameter	K-S		χ^2		AIC
			Statistic	p-value	Statistic	p-value	
Tang Sazbon	GEV	$u = 74.32$ $\alpha = 53.95$ $k = 0.34$	0.16	0.60	2.24	0.94	232
Huleilan	Exponential	$\xi = 18.18$ $\alpha = 64.63$	0.16	0.26	7.26	0.40	243
Polchehr	Gamma	$\sigma = 1.35$ $\beta = 48.72$	0.14	0.61	4.93	0.66	303

4.2 Fitting univariate drought severity and duration distributions

Observed drought severity and duration are abstracted from these drought events and used to fit a probabilistic distribution separately. There are 13 candidate univariate marginal distribution functions, including generalized extreme value (GEV), 3-parameter log-normal (LN3), generalized Pareto (GPAR), logistic, normal, Weibul, 2-parameter log-normal (LN2), Pearson type 3 (P3), log Pearson type 3 (LP3), generalized logistic (GLOG), extreme value type I (EVI), gamma, exponential distributions (Rao and Hamed 2000), are considered in this study for univariate frequency analysis. Parameter estimation of these distributions is based on the maximum likelihood method. Goodness-of-fit tests are employed to select the best-fit marginal distributions for drought severity and duration time series. Two goodness-of-fit

tests, Kolmogorov–Smirnov (K–S) and chi-square (χ^2) tests are implemented to evaluate whether the data follow the candidate distributions or not. The hypothesis regarding a specific distribution is rejected at the 5% significance level if the p -value for that distribution is less than the 5% significance level, i.e., p -value < 0.05 .

The statistics of K–S (K_n) and chi-square (χ_n^2) tests and the corresponding p -values for drought severity and duration are presented in tables 4 and 5, respectively. The higher statistics values, the less evidence about the appropriateness of the candidate distribution and consequently the higher p -values, the stronger evidence about the null hypothesis and appropriateness of the distributions. In this study, gamma, LN2, and LN2 distributions are identified as the best-fit distribution of drought severity for Tang Sazbon, Huleilan, and Polchehr, respectively, according to the goodness-of-fit tests statistics. For drought duration, GEV,

exponential, and gamma are respectively identified as the best-fit distribution for Tang Sazbon, Huleilan and Polchehr stations. One of the core findings of this study is that the marginal distributions for the Tang Sazbon station in the main river is substantially different from those of Huleilan and Polchehr stations in the upstream. As table 4 indicates, the drought severities for both the Huleilan and Polchehr stations are best modeled by LN2 distribution. However, the gamma distribution is selected as the best model in Tang Sazbon station. On the other hand, the best-fit drought duration model in the Tang Sazbon station is the 3-parameter GEV distribution, while the exponential and gamma distributions are selected as the best models for the Huleilan and Polchehr in the upstream reaches.

4.3 Determining the best-fit copula for bivariate drought severity and duration distribution

Dependence structure between drought severity and drought duration is assessed by the Pearson correlation coefficient and shown in figure 4. The results indicate that high correlation coefficients (>0.8) between the drought severity and duration are observed for all three stations. The greatest correlation of 0.89 exists in Polchehr and the lowest value of 0.84 is observed in the Tang Sazbon.

Constructing the joint severity–duration CDFs by a copula function is investigated in this study. Copula parameter is estimated by the maximum likelihood and Kendall τ estimation methods. Values of copula parameter estimated by these two methods and the corresponding differences for various stations are summarized in table 6. It can be seen that the estimations by parametric method are in good agreement with those from the non-parametric method for Gumbel copula, especially in Huleilan and Polchehr. For Clayton copula, however, there are larger differences between the parametric and nonparametric estimations. Values estimated by Kendall- τ are higher than those obtained by the maximum likelihood method for all stations. This phenomenon is attributed to the tail dependence structure of the CDFs of the observed data which is not consistent with tail behaviour of Clayton copula (Fu and Butler 2014). In this study, tail dependence structure of the observed data, especially for Tang Sazbon and Huleilan stations (left panel in figure 4), is more similar to the Clayton copula in comparison to the Gumbel and Frank copula functions (figure 5).

Relationships between the observed drought severity and duration and CDFs of observed drought severity and duration for each station are shown in left panel and right panel of figure 4, respectively. Figure 5 demonstrates the tail behaviour of various copula functions by generating 1000 pairs of CDFs of drought severity and duration from each copula function by the algorithm of Monte Carlo simulation (Acciolya and Chiyoshi 2004). At a glance, it seems that Clayton copula is an appropriate function according to the comparison between the observed data (left panel in figure 4) and tail behaviour of copula functions (figure 5). It is observed that the association between the observed drought duration and severity becomes weak as the values of both duration and severity increase. For Clayton copula, the degree of association in the upper tail is also less than that in the lower tail. Lee *et al.* (2013) pointed out that the copula function must be determined by considering the association in the CDFs of the observed data (right panel in figure 4) rather than that in the observed data (left panel in figure 4). A different behaviour can be observed when CDFs of drought duration and severity are plotted. By visual inspection, it is revealed that Clayton copula is not appropriate because of the greater dependence in the upper tail of the observed CDF values (right panel in figure 4), which is not consistent with the less dependence of Clayton copula shown in figure 5.

The Frank copula is a symmetric Archimedean copula with uniform dependence in the lower and upper tails. Figure 4(a) shows the observed and CDF values of drought severity and durations for Tang Sazbon station. As it can be seen from this figure, the CDF of the observed data shows a symmetric pattern with an equal dependence in the lower and upper tails. Thus Frank copula seems to be an appropriate copula to fit the bivariate drought data in this station. For two other stations, the association between the drought duration and severity increases with increasing CDF values of the severity and duration. Since Gumbel copula is an asymmetric Archimedean copula with greater dependence in the upper tail than in the lower (Fu and Butler 2014), so it seems to better fit the CDFs of observed data at Huleilan and Polchehr stations in comparison to the Frank copula. Nevertheless, these copula functions are identified as appropriate copulas for describing the dependence structure between the drought characteristics.

The root mean square error (RMSE), AIC, BIC and log-likelihood values, summarized in table 7,

Figure 4. Scatter plots of observed data (left panel) and CDFs (right panel) of drought severity and duration at (a) Tang Sazbon, (b) Huleilan, and (c) Polchehr.

are also provided to identify the best-fit copula for drought severity and duration. Optimal copula (in terms of log-likelihood) is one with the lowest negative likelihood (Patton 2006). The AIC and BIC also have the same ranking as the log-likelihood values. Results show that Frank copula with the lowest values of AIC (−30.18), BIC (−30.13) and

log-likelihood (−15.13) is selected as the best-fit copula for Tang Sazbon station. The Gumbel copula is identified as the best candidate copula to fit drought characteristics at Huleilan and Polchehr stations, with the lowest values of AIC, BIC and log-likelihood respectively equal to −25.75, −25.70, −12.92 for Huleilan and −60.72, −60.67, and −30.39

Table 6. Parameter estimated by Kendall- τ and maximum likelihood methods for various copulas.

Station	Copula	Kendall- τ	Maximum likelihood	(Kendall τ – maximum likelihood)
Tang Sazbon	Clayton	9.47	2.00	7.47
	Frank	21.17	10.68	10.49
	Gumbel	5.73	2.58	3.15
Huleilan	Clayton	6.74	1.10	5.64
	Frank	15.65	6.84	8.81
	Gumbel	4.37	2.60	1.77
Polchehr	Clayton	8.69	3.54	5.15
	Frank	19.59	13.41	6.18
	Gumbel	5.34	4.15	1.21

for Polchehr. The selected best-fit copulas also possess the lowest RMSE values.

Identifying different copula functions for the main river (Frank copula at Tang Sazbon station) and its tributaries (Gumbel copula at Huleilan and Polchehr stations) is attributed to land-use scheme, snowmelt runoff, and groundwater recharge, which induce different streamflow discharge in upstream and downstream reaches. Dependence structure between hydrological drought characteristics in tributaries is dominated by the snow-pack availability. Since streamflow of the tributaries noticeably depends on the snowmelt runoff, snow-pack availability and air-temperature variation may affect the non-linear relationship between drought severity and duration. During cold periods, less snowmelt runoff generated from snow packs leads to longer drought duration when comparing with the drought event with identical drought severity during warm periods. This phenomenon induces a weak association between the drought characteristics in the lower tail. However, drought severity and duration of the extreme drought events, caused by less snow packs, are directly connected to each other and there is strong association between them in the upper tail. Gumbel copula with respective weak and strong association in the lower and upper tails thus fits best to the drought characteristics in tributaries. On the other hand, uniform dependency between the drought severity and duration in the main river, which is not affected by the snow packs, lead to the best-fit copula of Frank in the main river.

A comparison between the parametric and non-parametric methods is also provided in table 7 and figure 6. The parametric estimation method has the higher RMSE values with respect to the

nonparametric estimation for the Tang Sazbon and Polchehr stations. However, for the Huleilan station located in the middle reach, the parametric estimation method performed better than non-parametric Kendall method. So the selected copula functions with the nonparametric estimation procedure were employed to construct the joint cumulative distribution functions for each station.

4.4 Bivariate frequency analysis of drought severity and duration

Figure 7 shows the joint cumulative distribution functions and joint return periods of drought characteristics by the best-fit copula functions. Figure 7(a) shows the JCDF of drought characteristics constructed by Frank copula and figure 7(b and c) demonstrates JCDF of drought severity–duration in Huleilan and Polchehr stations using Gumbel copula. Figure 8(a, b and c) shows the contour plots of the joint probability (left panel) and joint return periods (right panel) for Tang Sazbon, Huleilan, and Polchehr, respectively. These figures reveal that the Huleilan and Polchehr which are located in the tributaries have similar return periods, while the Tang Sazbon located in the main river has the smaller return periods for the same events. For example, a drought event with severity of $300 \text{ (m}^3/\text{s} \cdot \text{days)}$ and duration of 90 days occurs with a return period of 10 yrs at Tang Sazbon station in the main river. However, the identical drought severity and duration event result in longer return periods of >20 and >40 years at Huleilan and Polchehr stations, respectively. On the other hand, a 20-year drought event at Tang Sazbon station has severity greater than $600 \text{ m}^3/\text{s} \cdot \text{days}$

Figure 5. Generated data of drought severity and duration by various copulas at (a) Tang Sazbon, (b) Huleilan, and (c) Polchehr.

and duration longer than 140 days. The identical 20-year drought events are characterized with less severities and durations at Huleilan station ($S < 500 \text{ m}^3/\text{s} \cdot \text{days}$ and $D < 100$ days) and Polchehr station ($S < 200 \text{ m}^3/\text{s} \cdot \text{days}$ and $D < 100$ days) located in tributaries. The results indicated that the main river (Tang Sazbon station) is more susceptible to the hydrological droughts and it can be categorized as the drought-prone area in this basin. Figure 8(a–c) shows the increases in drought

duration with decreasing elevation, that is more sensible in comparison to the drought severity for a given return period. This may be attributed to the groundwater extraction around the main river which utilized for cultivated agricultural lands and lead to the river discharge being below the threshold value for longer periods. Besides, snowmelt runoff recharging, the tributaries are responsible for the decreased drought severity–duration at the Huleilan and Polchehr stations.

Table 7. Copulas goodness-of-fit tests based on the different evaluation criteria and root mean square error (RMSE).

Station	Copula	AIC	BIC	Log-likelihood	RMSE	
					Kendall- τ	Maximum likelihood
Tang Sazbon	Clayton	-22.74	-22.9	-11.42	0.095	0.12
	Frank	-30.18	-30.13	-15.13	0.095	0.10
	Gumbel	-23.40	-23.35	-11.74	0.095	0.10
Huleilan	Clayton	-7.18	-7.13	-3.64	0.066	0.063
	Frank	-18.27	-18.23	-9.18	0.066	0.055
	Gumbel	-25.75	-25.70	-12.92	0.064	0.055
Polchehr	Clayton	-45.67	-45.63	-22.87	0.056	0.064
	Frank	-50.96	-50.91	-25.51	0.058	0.061
	Gumbel	-60.72	-60.67	-30.39	0.057	0.059

Note. Bold numbers represent the best performance.

Figure 6. Comparison of root mean square error (RMSE) between Kendall- τ and maximum likelihood (ML) estimation methods at (a) Tang Sazbon, (b) Huleilan, and (c) Polchehr.

Figure 7. Joint cumulative distribution function (left) and joint return periods (right) of drought severity and duration for (a) Tang Sazbon, (b) Huleilan, and (c) Polchehr.

Comparison was also made for the effect of parameter estimation methods on joint return period of drought severity–duration in this study. Figure 9 demonstrates the joint return period of drought severity–duration for non-parametric Kendall τ (left panel) and parametric ML method (right panel). The results indicate that the ML method estimates drought return periods slightly higher than the ones obtained by Kendall τ method. For instance, the drought event with severity of $600 \text{ m}^3/\text{s} \cdot \text{days}$ and

duration of 120 days has a return period less than 20 years by the Kendall τ method at Tang Sazbon station, which is observed in left panel of figure 9(a). Right panel of figure 9(a) shows that the same event, however, has a return period of 20–40 years by the ML method. Similarly, a drought event with severity of $1000 \text{ m}^3/\text{s} \cdot \text{days}$ and duration of 140 days has a return period of 20–40 years by the Kendall τ and 40–60 years by the ML method. Similar results are also noted at Huleilan and Polchehr stations.

Figure 8. Contours of joint probability (left) and return periods (right) of drought severity and duration for (a) Tang Sazbon, (b) Huleilan, and (c) Polchehr.

5. Conclusions

A bivariate copula-based joint distribution was constructed to explore drought severity and duration in three hydrological stations within the

Karkheh River basin in southwest Iran. Hydrological droughts are defined as the continuous periods with streamflow below Q_{75} , which is estimated from flow duration curves (FDC). Various univariate marginal distributions are used to fit

Figure 9. Contour plots of joint return period *vs* drought severity and duration obtained by Kendall τ (left) and ML methods (right) for (a) Tang Sazbon, (b) Huleilan, and (c) Polchehr.

the drought severity and duration attributes separately and the best-fit model is selected based on the AIC, Kolmogorov–Smirnov and chi-square tests. Archimedean copulas are then employed to

construct the joint cumulative distribution functions by linking the fitted marginal distributions. The nonparametric and parametric parameter-estimation methods are implemented for estimating

copula parameters of observed drought data and various performance evaluation criteria such as AIC, BIC, log-likelihood, and RMSE are used to select best-fit copula. The main findings of this research can be outlined as:

1. The best-fit univariate drought severity and duration distributions are not identical in the main river (Tang Sazbon) and tributaries (Huleilan and Polchehr). Gamma distribution is fitted to the drought severity in Tang Sazbon station, while the LN2 distribution is fitted to the drought severity in Huleilan and Polchehr. Drought duration in Tang Sazbon follows a 3-parameter GEV distribution; however, 2-parameter distributions including exponential and gamma are the best-fit model for Huleilan and Polchehr, respectively.
2. Similar to the univariate models, the best-fit copulas are also different in the main river and tributaries. Frank copula with a uniform tail dependence structure is the best-fit model in the Tang Sazbon, while the Gumbel copula with greater dependence structure in the upper tail is the optimal model in Huleilan and Polchehr. This is possibly due to different land uses and groundwater recharge which differently influence the streamflow between upstream and downstream reaches.
3. The results reveal that the station along the main river (Tang Sazbon) has shorter return periods in comparison to the stations located in tributaries (Huleilan and Polchehr) for the same drought event. Accordingly, for the same return periods, drought severity and durations increases with decreasing elevation in this basin. The main-river region where Tang Sazbon located is the drought-prone area within this basin.
4. Results of bivariate drought frequency analysis indicate that parameter-estimation methods of copulas affect estimation of joint return period and joint probability. Greater drought return periods are obtained by the parametric ML method in comparison to the non-parametric Kendall τ estimation method in the Karkheh River basin.

Acknowledgements

The authors would like to thank Iran Ministry of Power for their kind help and technical support for providing the data for this study. The authors also

appreciate the comments provided by two anonymous reviewers.

References

- Acciolya R D E and Chiyoshi F Y 2004 Modeling dependence with copulas: A useful tool for field development decision process; *J. Petrol. Sci. Eng.* **44**(1–2) 83–91.
- Bonaccorso B, Cancelliere A and Rossi G 2003 An analytical formulation of return period of drought severity; *Stoch. Environ. Res. Risk A.* **17**(3) 157–174.
- Breymann W, Dias A and Embrechts P 2003 Dependence structure for multivariate high-frequency data in finance; *Quant. Struct. Act. Rel.* **3**(1) 1–14.
- Cancelliere A and Salas J D 2004 Drought length properties for periodic-stochastic hydrologic data; *Water Resour. Res.* **40** W02503, doi: [10.1029/2002WR001750](https://doi.org/10.1029/2002WR001750).
- Chang J, Li Y, Wang Y and Yuan M 2016 Copula-based drought risk assessment combined with an integrated index in the Wei River Basin, China; *J. Hydrol.* **540** 824–834.
- De Michele C, Salvadori G, Vezzoli R and Pecora S 2013 Multivariate assessment of droughts: Frequency analysis and dynamic return period; *Water Resour. Res.* **49**(10) 6985–6994.
- Dodangeh E, Soltani S, Sarhadi A and Shiau J T 2014 Application of L-moments and Bayesian inference for low-flow regionalization in Sefidroud basin, Iran; *Hydrol. Process.* **28**(4) 1663–1676.
- Edossa D C, Babel M S and Gupta A D 2010 Drought analysis in the Awash River basin, Ethiopia; *Water Resour. Manag.* **24**(7) 1441–1460.
- Favre A C, Adlouni S E, Perreault L, Thiémonge N and Bobbe B 2004 Multivariate hydrological frequency analysis using copulas; *Water Resour. Res.* **40** 1–12.
- Fermanian J D 2005 Goodness-of-fit tests for copulas; *J. Multivariate Anal.* **95**(1) 119–152.
- Fleig A K, Tallaksen L M, Hisdal H and Demuth S 2006 A global evaluation of streamflow drought characteristics; *Hydrol. Earth Syst. Sci.* **10**(4) 535–552.
- Fu G and Butler D 2014 Copula-based frequency analysis of overflow and flooding in urban drainage systems; *J. Hydrol.* **510** 49–58.
- Genest C and Rivest L P 1993 Statistical-inference procedures for bivariate Archimedean copulas; *JASA* **88**(423) 1034–1043.
- Genest C, Quessy J F and Rémillard 2006 Goodness-of-fit procedures for copula models based on the probability integral transformation; *Scand. J. Stat.* **33**(2) 337–366.
- González J and Valdés J B 2003 Bivariate drought recurrence analysis using tree ring reconstructions; *J. Hydrol. Eng.* **8**(5) 247–258.
- He B, Lu A F, Wu J J, Zhao L and Liu M 2011 Drought hazard assessment and spatial characteristics analysis in China; *J. Geogr. Sci.* **21**(2) 235–249.
- Hisdal H and Tallaksen L M 2000 Drought event definition. Technical report to the *ARIDE Project* No. 6: Supplement to Work Package 2 Hydro-meteorological Drought Activity 2.1 Event Definition, 41p.
- Jamali S, Abrishamchi A, Marino A M and Abbasnia A 2013 Climate change impact assessment on hydrology of

- Karkheh Basin, Iran; *Proceedings of the Institution of Civil Engineers - Water Management* **166**(2) 93–104.
- Kao S C and Govindaraju R S 2010 A copula-based joint deficit index for droughts; *J. Hydrol.* **380** 121–134.
- Kim T W, Valdes J B and Yoo C 2006 Nonparametric approach for bivariate drought characterization using Palmer drought index; *J. Hydrol. Eng.* **11**(2) 134–143.
- Kirby M, Mainuddin M, Ahmad M, Marchand P and Zhang L 2006 Water use account spreadsheets with examples of some major river basins; In: *9th Int. River Symposium*, Brisbane, Australia.
- Lana X and Burgueno A 1998 Spatial and temporal characterization of annual extreme droughts in Catalonia (northeast Spain); *J. Climatol.* **18**(1) 93–110.
- Lee K S, Sadeghipour J and Dracup J A 1986 An approach for frequency analysis of multiyear drought durations; *Water Resour. Res.* **22**(5) 655–662.
- Lee T, Modarres R and Ouarda T B M J 2013 Data-based analysis of bivariate copula tail dependence for drought duration and severity; *Hydrol. Process.* **27**(10) 1454–1463.
- Liu C L, Zhang Q, Singh V P and Cui Y 2011 Copula-based evaluations of drought variations in Guangdong, South China; *Nat. Hazards* **59**(3) 1533–1546.
- Loáiciga H A 2005 On the probability of droughts: The compound renewal model; *Water Resour. Res.* **41**(1) W01009, doi: [10.1029/2004WR003075](https://doi.org/10.1029/2004WR003075).
- Madadgar S and Moradkhani H 2013 Drought analysis under climate change using copula; *J. Hydrol. Eng.* **18**(7) 746–759.
- Madsen H and Rosbjerg D 1995 On the modelling of extreme droughts; In: *Proceedings of Modelling and Management of Sustainable Basin-Scale Water Resource Systems*, IAHS Publications, Oxfordshire, **231** 377–385.
- Marjanizadeh S, Qureshi A S, Turrall H and Talebzadeh P 2009 From Mesopotamia to the third millennium: The historical trajectory of water development and use in the Karkheh River Basin, Iran; Colombo, Sri Lanka, International Water Management Institute, 51p. (IWMI Working Paper 135), doi: [10.3910/2010.206](https://doi.org/10.3910/2010.206).
- Mirakbari M, Ganji A and Fallah S R 2010 Regional bivariate frequency analysis of meteorological droughts; *J. Hydrol. Eng.* **15**(12) 985–1000.
- Mishra A K and Singh V P 2010 A review of drought concepts; *J. Hydrol.* **391** 202–216.
- Modarres R 2008 Regional frequency distribution type of low flow in north of Iran by L-moments; *Water Resour. Manag.* **22**(7) 823–841.
- Modarres R 2010 Regional dry spells frequency analysis by L-moment and multivariate analysis; *Water Resour. Manag.* **24**(10) 2365–2380.
- Modarres R and Sarhadi A 2010 Frequency distribution of extreme hydrologic drought of southeastern semiarid region, Iran; *J. Hydrol. Eng.* **15**(4) 255–264.
- Muthuwatta L P, Ahmad M U D, Bos M G and Rientjes T H M 2010 Assessment of water availability and consumption in the Karkheh River basin, Iran – Using remote sensing and geo-statistics; *Water Resour. Manag.* **24**(3) 459–484.
- Nelsen R B 2006 *An Introduction to Copulas*; Springer, New York.
- Palaro H and Hotta L K 2006 Using conditional copulas to estimate value at risk; *J. Data Sci.* **4**(1) 93–115.
- Patton A J 2006 Estimation of multivariate models for time series of possibly different lengths; *J. Appl. Econometr.* **21**(2) 147–173.
- Rao A R and Hamed K H 2000 *Flood Frequency Analysis*; CRC, Boca Raton, FL.
- Reddy M J and Ganguli P 2012 Application of copulas for derivation of drought severity–duration–frequency curves; *Hydrol. Process.* **26**(11) 1672–1685.
- Reilly C F and Kroll C N 2003 Estimation of 7-day, 10-year low-streamflow statistics using baseflow correlation; *Water Resour. Res.* **39**(9) 1236, doi: [10.1029/2002WR001740](https://doi.org/10.1029/2002WR001740).
- Sadri S and Burn D H 2014 Copula-based pooled frequency analysis of droughts in the Canadian Prairies; *J. Hydrol. Eng.* **19**(2) 277–289.
- Saghafian B and Mehdikhani H 2014 Drought characterization using a new copula-based trivariate approach; *Nat. Hazards* **72**(3) 1391–1407.
- Serinaldi F, Bonaccorso B, Cancelliere A and Grimaldi S 2009 Probabilistic characterization of drought properties through copulas; *Phys. Chem. Earth* **34**(10–12) 596–605.
- Shafer B A and Dezman L E 1982 Development of a surface water supply index (SWSI) to assess the severity of drought conditions in snowpack runoff areas; In: *Western Snow Conference*, Olorado State University, Reno, NV, pp. 164–175.
- Sharma T C and Panu U S 2014 A simplified model for predicting drought magnitudes: A case of streamflow droughts in Canadian Prairies; *Water Resour. Manag.* **28**(6) 1597–1611.
- Shamaeizadeh M 2009 *Low flow regionalization in northern Karoon*; Isfahan University of Technology (in Persian).
- Shiau J T 2006 Fitting drought duration and severity with two-dimensional copulas; *Water Resour. Manag.* **20**(5) 795–815.
- Shiau J T, Feng S and Nadaraiah S 2007 Assessment of hydrological droughts for the Yellow River, China, using copulas; *Hydrol. Process.* **21**(16) 2157–2163.
- Shiau J T and Hsiao Y Y 2012 Water-deficit-based drought risk assessments in Taiwan; *Nat. Hazards* **64**(1) 237–257.
- Shiau J T and Modarres R 2009 Copula based drought severity–duration–frequency analysis in Iran; *Meteorol. Appl.* **16**(4) 481–489.
- Shiau J T, Modarres R and Nadarajah S 2012 Assessing multi-site drought connections in Iran using empirical copulas; *Environ. Model Assess.* **17**(5) 469–482.
- Shiau J T and Shen H W 2001 Recurrence analysis of hydrologic droughts of differing severity; *J. Water Resour. Plan. Manag.* **127**(1) 30–40.
- Shumway R H and Stoffer D S 2011 *Time Series Analysis and its Applications*; Springer, New York.
- Sklar M 1959 Fonctions de r'epartition 'a n dimensions et leurs marges; *Publ. Inst. Statist. Univ. Paris* **8** 229–231.
- Smakhtin V U 2001 Low flow hydrology: A review; *J. Hydrol.* **240** 147–186.
- Tallaksen L M, Madsen H and Clausen B 1997 On the definition and modelling of streamflow drought duration and deficit volume; *Hydrol. Sci. J.* **42**(1) 15–33.

- Tosunoglu F and Kisi O 2016 Joint modeling of annual maximum drought severity and corresponding duration; *J. Hydrol.* **543** 406–422, doi: [10.1016/j.jhydrol.2016.10.018](https://doi.org/10.1016/j.jhydrol.2016.10.018).
- Vergni L, Todisco F and Mannocchi F 2015 Analysis of agricultural drought characteristics through a two-dimensional copula; *Water Resour. Manag.* **29**(8) 2819–2835.
- Weghorst K M 1996 *The Reclamation Drought Index: Guidelines and Practical Applications*; Bureau of Reclamation, Denver, D-8530.
- Wu Z, Lin Q, Lu G, He H and Qu J J 2015 Analysis of hydrological drought frequency for the Xijiang River basin in south China using observed streamflow data; *Nat. Hazards* **77**(3) 1655–1677.
- Xu K, Yang D, Yang H, Li Z, Qin Y and Shen Y 2015 Spatio-temporal variation of drought in China during 1961–2012: A climatic perspective; *J. Hydrol.* **526** 253–264.
- Xu K, Yang D, Xu X and Lei H 2015 Copula based drought frequency analysis considering the spatio-temporal variability in southwest China; *J. Hydrol.* **527** 630–640.
- Yevjevich V 1967 Objective approach to definitions and investigations of continental hydrologic droughts; Hydrology Paper 23, Colorado State University, Fort Collins.
- Zarch M A A, Malekinezhad H, Mobin M H, Dastorani M T and Kousari M R 2011 Drought monitoring by reconnaissance drought index (RDI) in Iran; *Water Resour. Manag.* **25**(13) 3485–3504.
- Zhang Q, Xiao M, Singh V P and Chen X 2013 Copula-based risk evaluation of hydrological droughts in the East River basin, China; *Stoch. Environ. Res. Risk A.* **27**(6) 1397–1406.

Corresponding editor: PRASHANT K SRIVASTAVA