Estimation of earthquake source parameters in the Kachchh seismic zone, Gujarat, India, using three component S-wave spectra

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Earthquake source parameters and crustal Q_0 values for the 138 selected local events of (M_w :2.5–4.4) the 2001 Bhuj earthquake sequence have been computed through inversion modelling of S-waves from three-component broadband seismometer data. SEISAN software has been used to locate the identified local earthquakes, which were recorded at least three or more stations of the Kachchh seismological network. Three component spectra of S-wave are being inverted by using the Levenberg–Marquardt non-linear inversion technique, wherein the inversion scheme is formulated based on ω^2 source model. SAC Software (seismic analysis code) is being utilized for calculating three-component displacement and velocity spectra of S-wave. The displacement spectra are used for estimating corner frequency (in Hz) and long period spectral level (in nm-s). These two parameters play a key role in estimating earthquake source parameters. The crustal Q_0 values have been computed simultaneously for each component of three-component broadband seismograph. The estimated seismic moment (M_0) and source radius (r) using S-wave spectra range from $7.03E+12$ to $5.36E+15$ N-m and 178.56 to 565.21 m, respectively. The corner frequencies for S-wave vary from 3.025 to 7.425 Hz. We also estimated the radiated energy (E_S) using velocity spectra, which is varying from $2.76E+06$ to $4.07E+11$ Joules. The estimated apparent stress drop and static stress drop values range from 0.01 to 2.56 and 0.53 to 36.79 MPa, respectively. Our study also reveals that estimated Q_0 values vary from 119.0 to 7229.5, with an average Q_0 value of 701. Another important parameter, by which the earthquake rupture process can be recognized, is Zuniga parameter. It suggests that most of the Kachchh events follow the frictional overshoot model. Our estimated static stress drop values are higher than the apparent stress drop values. And the stress drop values are quite larger for intraplate earthquakes than the interplate earthquakes.

Keywords. Source parameters; Zuniga parameter; apparent stress; 3-component S-wave spectra; Levenberg–Marquardt non-linear inversion technique; crustal Q_0 values.

1. Introduction

Earthquakes have been occurring in the Kachchh region of Gujarat (Rajendran and Rajendran 2001).

Two catastrophic intraplate earthquakes have occurred in the Kachchh region, within a time span of 182 yrs. One of them was great Allah Bund earthquake of 1819 ($M_w = 7.8$), occurred

about 100 km NW of the 2001 Bhuj earthquake epicentre (Rajendran and Rajendran 2001). However, lack of availability of instrumental data, the estimation of source parameters is not well defined exactly. On 26th January 2001, the deadliest intraplate earthquake ever to strike in India occurred on a south dipping reverse fault in the Kachchh region of Gujarat state, which is situated in the northwestern part of India. Moment magnitude of this earthquake was reported to be $M_w = 7.7$ and the focal depth in the Kachchh rift zone was reported to be 23 km (Mandal *et al.* 2004a, b; USGS 2001). Around 20,000 people were killed by this earthquake. The intensity of this earthquake was reported to be X+ on MM scale (Rastogi *et al.* 2001). In 1956, another moderate earthquake ($M_w = 6.0$) had occurred in a region near Anjar, which is located south of the epicentre of the 2001 Bhuj earthquake (Chung and Gao 1995). The main motivation of the estimation of source parameters in the Kachchh region is the aftershock sequence of the 2001 Bhuj main shock, which is continuing until today with the occurrences of $M_w \geq 4.0$ earthquakes (Mandal 2007). The Kachchh in India and New Madrid in USA, only two intraplate regions have experienced large earthquakes of moment magnitude $M_w \geq 7.7$. But the estimated stress drop value ranges from 100 to 140 bars for (1811–1812) New Madrid region (Johnston and Schweig 1996), which have less stress drop values compared to 2001 Bhuj earthquake stress drop value, which is around 200 bars.

Some people have estimated stress drop values using aftershock sequence of the 2001 Bhuj main shock. Negishi *et al.* (2001) obtained a static stress drop ranging from 12.6 to 24.6 MPa, for an equivalent source radius ranging from 20 to 25 km. Singh *et al.* (2004) suggest that a frictional sliding model with an average dynamic stress drop of about 12 MPa and the ratio of rupture to shear wave velocity of 0.7 could satisfactorily explain the source complexity observed for the 2001 Bhuj main shock. While Bodin and Horton (2004) estimated a static stress drop of 16.0 ± 2 MPa and the total rupture area was about 1300 km². Saha *et al.* (2012) obtained stress drop and seismic moments ranging from 0.01 to 20 MPa and 1.12×10^{12} to 4×10^{16} N-m, respectively. Kumar *et al.* (2001) estimated stress drop varying from 0.1 to 14.4 MPa, for an equivalent source radius ranging from 139.1–933.9 m. Our estimated stress drop values vary from 0.53 to 36.79 MPa. Interplate earthquakes tend to have lower static stress drops than intraplate earthquakes

(Kanamori and Anderson 1975), but this stress drop is high even among intraplate events. The high-stress drop indicates that the near-field strong ground motions from this earthquake may have been particularly strong. According to Mandal and Johnston (2006), the stress drop values for higher magnitude events (above $10^{14.5}$ N-m for Bhuj aftershocks) are scattering in nature whereas they exhibit more systematic nature ($M_0^3 \propto \Delta\sigma$) for smaller seismic moment (smaller aftershocks) values. Systematic nature ($M_0^3 \propto \Delta\sigma$) for smaller seismic moment values exhibit the size dependency of stress drop. Despite the source parameter estimates for other stable continental region, earthquakes globally do not show any size dependency. The large stress drops are estimated to be confined to the main rupture zone of the 2001 Bhuj earthquake and are possibly related to the large crustal stress/strain associated with a brittle, competent intrusive body of mafic nature.

2. Geology and tectonics of the Kachchh region

The oldest rift zone in India is the Kachchh rift zone; the Bhuj earthquake occurred in the Kachchh rift, an intraplate region of rifted Precambrian craton in northwestern India. During the break-up of Gondwana in the Jurassic, this area was affected by rifting in E–W direction. The main characterization of the Kachchh region is Quaternary/Tertiary sediments, Deccan volcanic rocks and Jurassic sandstones resting on Archean basement (Gupta *et al.* 2001). The structural style of this basin is unique, and this type of structural style is not found anywhere in India. Major tectonic settings in Kachchh are a series of E–W trending uplifts along master faults with uplands surrounded by lowlands, as shown in figure 1. The rift zone is bounded by the Nagar Parker fault in the north and the Kathiawar fault in the south (Biswas 1987). In this region, the major faults oriented in the E–W trending Allah Bund Fault (ABF), Kachchh Mainland Fault (KMF), Island Belt Fault (IBF), and Katrol Hill Fault (KHF). Besides, several NE–SW trending small faults/lineaments are observed (Biswas 1987).

Geophysical studies such as seismic, gravity and magnetotelluric surveys reveal that an undulating basement with 2–5 km deep sediments, a Moho depth at 35–43 km, and a lithospheric depth at 62–77 km are present in the Kachchh region (Gupta *et al.* 2001; Reddy *et al.* 2001; Mandal 2011;

Figure 1. This figure shows location of 11 broadband seismograph stations marked by blue colour triangles and events (solid red circles). The seismological network of this region is deployed by the CSIR-NGRI. The stations are abbreviated as VJP: Vajepar, TPR: Tappar, MTP: Motapaya, GDD: Gadhada, BCH: Bhachau, NGR: Nagor, BHU: Bhuj, TPM: Tappar (Mundra), MOU: Mouvana, SIV: Shivlaka, JUM: Jumkinariya. Fault system, KMF: Kachchh Mainland Fault, KHF: Katrol hill fault, ABF: Allah bund fault, IBF: Island belt fault, NPF: Nagar Parkar fault, NWF: North Wagad fault, GF: Gedi fault.

Mandal and Pandey 2011). Overall, the tectonic setting in the region is very complex, as it consists of faults of multiple orientations and different natures (Biswas 1987).

3. Seismic network and aftershock data

Twelve digital broadband seismometer stations were installed by the CSIR-NGRI, Hyderabad, in the Kachchh seismically active zone. The data were recorded continuously on 24 bit REFTEK digital recorder with 100 samples per second sampling rate and a three component broadband seismograph with a natural period of 30–120 s for recording local, regional and teleseismic events. In our study, we selected only local events (138) of moment magnitude $M_w \geq 2.5$. Here, we used data from January

2013 to March 2014 (within 100 km distance from the epicentre).

4. Methodology used for estimating source parameters

In this work, a two-stage methodology has been used for estimating the source parameters, which is as follows:

1. Spectral analysis of S-wave using SAC (seismic analysis code) from three component broadband seismograph data.
2. Levenberg–Marquardt non-linear inversion technique has been used to invert the displacement spectra (Fletcher 1995), wherein a correction for frequency-dependent Q ($Q(f)=102f^{0.98}$) (Mandal *et al.* 2004a, b) is applied.

Table 1. V_p and V_s models obtained from the 1-D travel time inversion (Mandal 2007).

Depth to the top layer (km)	V_p (km/s)	V_s (km/s)
0	2.92	0.90
2	5.99	3.49
5	5.99	3.77
10	6.18	3.60
16	6.07	3.60
22	6.59	3.73
29	7.20	3.99
34	6.78	3.44
40	8.20	4.70

Local earthquakes occurring in the Kachchh region have been located using SEISAN software. First, the raw binary earthquake data files have been converted into Seisan (.sei) files by using different batch files consisting of various commands of archive utility of REFTEK Inc., USA. Next, these .sei files were registered in the SEISAN software. Then, the arrival times of the clearly visible P and S waves are picked from the vertical and horizontal components of broadband seismograms respectively, using HYPO71PC program which is integrated with SEISAN software (Lee and Valdes 1985). One-dimensional crustal velocity model is used consisting of six layers varying in velocity with depth. The layers at different depths are 0.0, 2, 10, 16, 29, 40 km, with the corresponding P-wave velocities of 2.92, 5.93, 6.18, 6.40, 6.97 and 8.20 km/s (Mandal 2007) shown in table 1. At least three to four stations are needed to locate an event. The location parameters are latitude, longitude, epicentre distance, focal depth, origin time and magnitude. An azimuthal gap of less than 180° of the studied seismic network provides rms value of 0.04s for average location. The estimated errors of hypocentral parameters are relatively accurate, latitude and longitude error of an event is <1.2 km, and focal depth estimates <2.5 km. After successfully locating the local events (location parameters are shown in supplementary table S1), the displacement and velocity spectra of S-wave from the three-component digital recordings are calculated using a SAC script. Here, displacement spectra (window length of 2.56 s) are used for computing source parameters and velocity spectra (window length of 5.12 s) are used for computing radiated seismic energy.

The Levenberg–Marquardt method is a generalized inversion technique used to solve the

non-linear least square problems. The Levenberg–Marquardt algorithm is a combination of two minimization techniques, the Gauss–Newton method and gradient decent method. It is relatively good method as it facilitates the faster convergence of the solution and minimizes the error between the inverted and observed spectra. The inversion procedure is as follows.

4.1 Inversion procedure

Taking the attenuation factor into consideration, the spectral amplitudes at a hypocentral distance R from the source can be written as:

$$A(f, R) = (A_0/R^{-\gamma}) \exp(\pi Rf/Q_0V_s). \quad (1)$$

Here, Q_0 is the frequency independent S-wave quality factor; A_0 is the amplitude at the source, f is the frequency, V_s is the velocity of the S-wave and γ is the geometrical spreading.

The exponential term can be written as:

$$\pi Rf/Q_0V_s = 1/2 * 2(\pi Rf/Q_0V_s) = \omega t^*/2 \quad (2)$$

where $t^* = R/Q_0V_s$.

Equation (1) can be linearized by taking natural logarithm on both sides of the equation and assuming $\gamma = 1$ (Fletcher 1995).

$$\ln A(f, R) = \ln A_0 - \ln R - \omega t^*/2 \quad (3)$$

A_0 is the source term, and it can be written as:

$$A_0 = \Pi_0/B_4^{0.5} = \Pi_0/[1 + (f/f_c)^4]^{0.5}. \quad (4)$$

Substituting equation (2) in equation (1) and taking $t^* = 0$

$$\ln[A(f, R)] = \ln\{\Pi_0/[1 + (f/f_c)^4]^{0.5}\} - \ln(R). \quad (5)$$

The B_4 term is expanded in a Taylor series as a Newton–Raphson’s method for finding roots of non-linear equations (Press 1992) and yields the final equation:

$$\ln A(f) = \Pi_0 - 0.5 \ln B_4 + 2B_4^{-1}(f/f_c)^4(\Delta f_c/f_c) \quad (6)$$

‘ R ’ term is dropped because of the fact that the formula for moment is corrected from the geometrical spreading effect. Using least-squares algorithm with several iterations solves the model parameters.

The error is minimized in least squares sense by solving

$$D = GM. \quad (7)$$

Here G is the sensitivity matrix, M is the model parameter matrix and D is the data matrix. Therefore,

$$M = [\ln \Pi_0, \Delta f_c]^T, \\ G = [1, 2B_4^{-1}(f/f_c)^4/f_c],$$

and

$$D = [A(f) + 0.5 \ln B_4]. \quad (8)$$

Using Newton's method for quasi-linear equation,

$$\Delta M = (G^T G)^{-1} G^T \Delta D \quad (9)$$

where ΔM is the change in model parameters and ΔD is the difference between observed and predicted data. The solution is found iteratively $M_k = M_0 + \Delta M_{k-1}$, where M_0 is the initial model. Because the sensitivity matrix is near to singular here.

$$\Delta M = (G^T G + \lambda I)^{-1} G^T \Delta D. \quad (10)$$

In the above equation (10), λ is the Levenberg–Marquardt adjustable damping parameter and ' I ' is the identity matrix.

In this connection, the inversion starts with initially guessed values such as corner frequency, long period spectral level, Q_0 value (for Kachchh region, $Q_0 = 102$), epicentral distance for a particular station and damping parameter (λ). The λ value varies from 0.01 to 500. The input of corner frequency and spectral level is visually identified for respective spectrum and λ value which starts with 0.01, 1, 10, 20, 50, 80, 100, 120, 150, ..., 500. The above procedure is iterated until the best fit spectra are obtained and is shown in figure 2 which consists of different stations with different magnitudes. At the best fit spectra, the value of Q_0 and model parameters is simultaneously attained. The model parameters are corner frequency (Hz) and long period spectral level (nm-s). Figure 3 shows the misfit, and it consists of several lines incorporated by using different parameters.

The corner frequency (Hz) and long period spectral level (nm-s) are the foremost input parameters for estimating source parameters. Here computed corner frequencies are most robust because

the average value of S-wave spectra from three-component broadband seismograms has been taken into consideration. The entire source parameters estimation is mainly dependent on the corner frequency which is a transition zone between low and high frequencies. After obtaining the two model parameters, we have computed other source parameters like earthquake moment magnitude, seismic moment, stress drop, apparent stress, radiated energy, and source radius, using their respective empirical equations. The formulas are mentioned below:

$$\text{Seismic moment } (M_0) = 4\pi\rho V_s^3 \Pi_0 / FR_{\theta\phi} \quad (\text{Keilis-Borok 1959}) \quad (11)$$

$$\text{Source radius } (r) = (2.34 * V_s) / (2\pi f_c) \quad (\text{Brune 1970}) \quad (12)$$

$$\text{Stress drop } (\Delta\sigma) = M_0 * (f_c / 0.498)^3 \quad (\text{Madariaga 1976}) \quad (13)$$

where ' V_s ' and ' ρ ' are the S-wave velocity in m/s and rock density in kg/m³ at the source respectively. ' Π_0 ' and ' f_c ' represent the long-period spectral level (in nm-s) and corner frequency (in Hz) respectively. ' R ' is the hypocentral distance in m, while ' F ' represents the free surface amplification factor, which is assumed to be 2 for the Kachchh region. The radiation factor ($R_{\theta\phi}$) is considered to be 0.55 for the Kachchh region (Singh *et al.* 2004; Mandal and Johnston 2006). The estimated M_0 , r , and $\Delta\sigma$ are in N-m, m, MPa, respectively using the above empirical formulae. The moment magnitude is estimated using equation (14).

$$M_w = (2/3) \log_{10}(M_0) - 6.06 \quad (\text{Hanks and Kanamori 1979}) \quad (14)$$

4.2 Radiated seismic energy

S-wave energy (E_S) associated with each earthquake in the frequency range of 0.2–25 Hz has been computed using equation (15) (Singh and Ordaz 1994). Here 1-D P-velocity and S-velocity structure of the Kachchh region has been used for computing radiated seismic energy (Mandal 2007). 1-D velocity model is shown in table 1.

Radiated seismic energy

$$E_S = \frac{4\pi R^2 \left[\frac{G^2(R)}{R^2} \right] \beta}{F_s^2} \times \left[2 \int_0^\infty [V_N^2(f) + V_E^2(f) + V_Z^2(f)] e^{\frac{2\pi f R}{\beta Q(f)}} df \right] \quad (15)$$

Figure 2. Some of best fit spectra for different stations with different magnitudes. The figures are plotted between frequency in Hz on x-axis and log spectral amplitude in nm-s on y-axis. The estimated S-wave spectra (black colour lines) and green colour line indicate the best fit curve. The date of occurrence, origin time, corner frequency and moment magnitude are shown in their respective plots.

Figure 3. The figure consists of misfit spectra with several lines. These several lines incorporated by using different parameters such as input value of corner frequency, damping factor and long period spectral level.

Here ' β ' is the shear-wave velocity which has been taken according to table 1, ' ρ ' is the density, ' f ' is the corner frequency, R is the hypocentral distance, $Q(f)$ is the quality factor for Kachchh region which is considered as 102 (Mandal *et al.* 2004a, b), V_N , V_E , V_Z are the velocity spectra of three components (N-S, E-W and vertical), ' F_s ' represents the free surface amplification factor which is assumed to be 2 for the Kachchh region and $G(R)$ is the geometrical spreading term, which can be written as:

$$G(R) = \{R \text{ for } R \leq R_0\}$$

$$G(R) = \{\sqrt{RR_0} \text{ for } R > R_0\}$$

where R_0 is 100 km. S-wave velocity spectra have been used for computing radiated seismic energy, the range of frequency spectra is between 0.2 and 25 Hz.

4.3 Apparent stress

Apparent stress is defined as the multiplication of average stress ' σ ' released during the faulting and unknown seismic efficiency factor ' η ', given by (Aki 1966):

$$\text{Apparent stress } (\sigma_a) = \frac{\mu E_S}{M_0}. \quad (16)$$

Here M_0 and E_S are seismic moment (N-m) and radiated seismic energy (Joules), respectively, and rigidity modulus $\mu = 3.3 \times 10^{10} \text{ N/m}^2$.

4.4 Zuniga parameter

Earthquake rupture process can be identified by Zuniga parameter assuming three idealized cases Orowan's model, overshoot model and partial stress drop model which are discussed here.

Let σ_1 , σ_2 be the initial and final average stresses, and σ_f be the frictional stress releases during faulting, M_0 is the seismic moment, μ is the rigidity, and σ_a is the apparent stress.

The radiated seismic energy can be written as (Savage and Wood 1971):

$$E_s = [(\sigma_1 - \sigma_2)/2 + (\sigma_2 - \sigma_f)](M_0/\mu)$$

$$= \sigma_a(M_0/\mu). \quad (17)$$

In the above equation, the average stress difference between initial σ_1 and final stress σ_2 gives the static stress drop ($\Delta\sigma$).

Case 1: If $\sigma_2 = \sigma_f$ Orowan's model (Orowan 1960), then equation (17) becomes

$$\left[\frac{\mu E_s}{M_0} \right] = \frac{\Delta\sigma}{2} = \sigma_a. \quad (18)$$

Case 2: If $\sigma_2 < \sigma_f$ overshoot model (Savage and Wood 1971), then equation (17) becomes

$$\left[\frac{\mu E_s}{M_0} \right] < \frac{\Delta\sigma}{2} \quad \text{and} \quad \sigma_a < \frac{\Delta\sigma}{2}. \quad (19)$$

Case 3: If $\sigma_2 > \sigma_f$ Partial stress drop-model (Brune 1970), then equation (17) becomes

$$\left[\frac{\mu E_s}{M_0} \right] > \frac{\Delta\sigma}{2} \quad \text{and} \quad \sigma_a > \frac{\Delta\sigma}{2}. \quad (20)$$

Zuniga parameter: Zuniga proposed a model based on the above three cases in 1993 called ε_Z , by which earthquake rupture process can be identified. ε_Z can be calculated by static stress drop and apparent stress using equation (21), which is mentioned below:

$$\varepsilon_Z = \frac{\Delta\sigma}{\sigma_a + \frac{\Delta\sigma}{2}}. \quad (21)$$

According to Zuniga, if $\varepsilon_Z > 1.0$ satisfies the overshoot model and if $\varepsilon_Z < 1.0$ satisfies the partial stress drop model.

5. Error analysis

Error in source parameters has been calculated by estimating standard deviation and mean of the moment (M_0), source radius (r) and stress drop ($\Delta\sigma$). The average seismic moment and source radius are determined by following equations (Archuleta *et al.* 1982):

$$\langle M_0 \rangle = \text{antilog}[(1/NS)\Sigma \log M_{0i}] \quad (22)$$

and

$$\langle r \rangle = [(1/NS)\Sigma r_i]. \quad (23)$$

The standard deviations (sd) of the log moment are computed using the equation given below:

$$sd(\log \langle M_0 \rangle) = \{(1/NS - 1) \times \Sigma [\log M_{0i} - \log \langle M_{0i} \rangle]^2\}^{0.5}. \quad (24)$$

Multiplicative factor is also calculated using below equation:

$$E_{mo} = \text{antilog}\{sd(\log \langle M_0 \rangle)\}. \quad (25)$$

Similarly, we calculated the standard deviation of source radius and finally the standard deviation of stress drops are estimated using the equation as given below (Fletcher 1995):

$$\sigma_{std} = \Delta\sigma \{(\sigma_m/M_0)^2 + 9(\sigma_r/r)^2\}^{0.5}. \quad (26)$$

From the above equations, the standard deviation of the moment, stress drop, source radius, corner frequency and E_{mo} have been computed for the selected 138 Bhuj aftershock sequence data. Here we used three components of (vertical and two horizontal components) broadband seismograph of Kachchh seismological network of CSIR-NGRI which are recorded on three or more stations. The maximum and minimum values of standard deviation in corner frequency, source radius and stress drop, ranges from 0.004 to 0.378 Hz, 0.46 to 37.77 m, 0.008 to 5.41 MP, respectively are shown in supplementary table S2.

6. Results and discussion

The main aim of this paper is a robust and accurate estimation of source parameters and crustal Q_0 values. Our estimated results reveal that the seismic moment (M_0), source radius (r) and stress drop values of 138 selected local events (M_w : 2.5–4.4) are varying from $7.03\text{E}+12$ to $5.36\text{E}+15$ N-m, 178.56–565.21 m, and 0.53–36.79 MPa, respectively. There are different understandings in earthquake source scaling where one group of seismologists believes in constantly scaled energy (E_R/M_0) (Choy and Boatwright 1995; McGarr 1999; Ide and Beroza 2001; Perez-Campos and Beroza 2001; Ide *et al.* 2003; Prieto *et al.* 2004), while the other group advocates that radiated energy increases with M_0 (Kanamori *et al.* 1993; Abercrombie 1995; Mayeda and Walter 1996; Prejean and Ellsworth 2001; Richardson and Jordan 2002; Mori *et al.* 2003; Stork and Ito 2004; Mayeda *et al.* 2005). An attempt has been made in this paper to obtain stable and reliable scaling relationships of earthquake source parameter estimates for small to moderate earthquakes occurring in Kachchh, Gujarat.

Are the earthquakes self-similar over their entire size range (Abercrombie 1995; Ide and Beroza 2001)? Different nature of normalized radiated seismic energy related to large and small earthquakes (Mayeda *et al.* 2005) state that earthquakes are not self-similar. The scaling relationships of earthquakes with $M_w \leq 3$ have been the subject of considerable controversy over the last two decades (Morasca *et al.* 2005). Studies of such events have shown that a constant source dimension (of about 100 m) apparently characterize these smaller magnitude earthquakes (Abercrombie 1995). Seismic moment is proportional to the source dimension cubed, corresponding to the stress drop being

Figure 4. The seismic moment (N-m) plotted against source radius (m) on the logarithmic scale with constant stress drop lines.

Figure 5. Plot between $\log(M_0)$ in N-m and $\log(\text{std})$ in MPa.

independent of magnitude (Abercrombie and Rice 2005). It is inevitable to access the seismic hazard in a particular region with the knowledge of source scaling.

Figure 4 shows the seismic moment plotted against the source radius on a logarithmic scale with the constant stress drop lines. The logarithmic of the estimated seismic moment plotted against the logarithmic of static stress drop values clearly shows a break in scaling at $M_0 = 10^{14.91}$, which is shown in figure 5. From figure 4, we notice that similar break in linear scaling also identified at $M_0 = 8.128 \times 10^{14}$, where larger earthquakes are showing deviation from self-similar scaling and also noticed a break in source scaling between larger and smaller earthquakes at $r = 251.57$ m. Corner frequency is plotted against seismic moment (M_0) (figure 6), and it is observed that the seismic moment and corner frequencies follow non-self similarity. The cluster of lower magnitudes ($M_w < 3$) and higher magnitudes ($M_w > 3.5$) are separated

Figure 6. Shows a plot drawn between corner frequency (Hz) and seismic moment (N-m).

Figure 7. Showing the plot between the energy E_S (joules) vs. seismic moment M_0 (N-m).

into two regions. The calculated corner frequencies are ranging from 3.025 to 7.425 Hz. The maximum error in corner frequency is 0.378 Hz (table 3).

E_S (in joules) and M_0 (in N-m) values are listed in table 3 and is plotted in figure 7; the black coloured lines are of constant values of 5×10^{-4} , 5×10^{-5} and 5×10^{-6} . Smaller earthquakes are scattered and also show the systematic increase in scaled energy with increasing seismic moment. Most of the events lie between 5×10^{-5} and 5×10^{-6} and larger events are lying between 5×10^{-4} and 5×10^{-5} . The ratio of E_S and M_0 values are less for smaller earthquakes, which agrees with the global observation (Jost *et al.* 1998; Richardson and Jordan 2002). The average value of E_S/M_0 is 1.128×10^{-5} . Figure 8 shows a variation of estimated seismic energy-to-moment ratio (E_S/M_0) with moment magnitude with lines of different ϵ values. It has been noticed that the energy may vary from a given seismic moment (figures 7 and 8). The ratio of energy to moment E_S/M_0 with

Figure 8. The plot is taking $\varepsilon = [E_s/M_0]$ on y-axis, seismic moment magnitude on x-axis.

Figure 9. The graph drawn between apparent stress vs. static stress drop.

moment magnitude looks to be scattered at $\varepsilon > 1$, smaller earthquakes have more scattered nature at $M_w < 3.0$. Therefore, it can be revealed that smaller earthquakes radiate less radiated energy per unit fault slip than large events. From the above discussions, it can be concluded that the rupture dynamics of small and large earthquake events are different (Abercrombie and Rice 2005). Apparent stress ' σ_a ', Brune model with complete stress drop ($\Delta\sigma_b$) estimates and the relation $\sigma_a = 0.43\Delta\sigma_b$ are also shown in figure 9 (Singh and Ordaz 1994). Finally, we noticed that the Brune's model with complete stress drop values is higher than the apparent stress drop values and also our results satisfy the frictional overshoot model, i.e., ($\sigma_a < \Delta\sigma_b$) suggested by Savage and Wood (1971).

The estimated apparent stress drop and static stress drop values range from 0.01 to 2.56 and 0.53 to 36.79 MPa, respectively. The variation of stress drop values with focal depth is shown in figure 10, and the values are shown in table 3. Our estimated stress drops are larger than the other

Figure 10. Represents the plot between focal depths with static stress drop values.

Figure 11. Represents about Zuniga parameter, the graph plotted between moment magnitude and Zuniga parameter.

earthquakes occurred in different areas. The larger stress drops obtained at which the hypocentral distance ranges from 30 to 60 km. The reasons for obtaining higher stress drop values are the presence of crustal mafic intrusive bodies, aqueous fluids in the lower crust, and it was suggested by the earlier tomographic study of the region (Saha *et al.* 2012). According to Zuniga parameter ε_Z , it is observed that the studied Kachchh events follow the frictional overshoot model. All events satisfying the condition, i.e., $\varepsilon_Z > 1$, are shown in figure 11. The Zuniga parameter is the best identification of earthquake rupture parameter. The limits of ' ε_Z ' are $0 < \varepsilon_Z < 2$.

An estimation of crustal Q_0 values in a seismically active region plays an important role in risk studies. In the above connection, all available frequency-dependent Q for Kachchh region are discussed here. Singh *et al.* (2004) determined Q value of Indian shield by using the spectral analysis attenuation of Lg waves and the estimated Q value was $Q(f) = 800f^{0.42}$ with the dataset of four

earthquakes recorded in the distance ranges from 240 to 2400 km. Later Mandal *et al.* (2004a, b) computed a frequency dependent Q (Q_C at 1 Hz) value of 102 for Kachchh region through the study of Q_C . They computed Q values from Gupta *et al.* (2006), $Q_C = 106f^{1.11}$, $Q_C = 148f^{1.04}$ by Sharma *et al.* (2008). The low Q values obtained for the Kachchh region suggest that the region is, relatively, more attenuative and heterogeneous by Mandal *et al.* (2004a, b), Gupta *et al.* (2006), and Sharma *et al.* (2008). Whereas according to Singh *et al.* (2004) and Bodin *et al.* (2004), the waves penetrate the deeper parts of the crust when propagating longer distances and therefore, attenuation is expected to be dependent on distance and thus obtained large Q values in the deeper portion of the crust.

In the Kachchh region, the attenuation properties are deviating with the other tectonically active regions of California, Turkey and Italy. The Q values in these active regions range from 100 to 300. Bodin *et al.* (2004) obtained the Q value of 790 for Kachchh region and concluded that the seismic wave amplitudes decay much slower with distance in Kachchh region than in interplate regions (plate-boundary regions). The location with ground motion attenuation most similar to Kachchh is the New Madrid seismic zone in the central United States, with a Q of 900. Now our study shows that the estimated value of frequency-independent Q_0 varies from 119.0 to 7229.5 with an average value of 701. Saha *et al.* (2012) inferred that the frequency independent average crustal Q_0 value is 840. As the stations (shown in figure 1) BCH, TPR and VJP are located in the Kachchh basin (composed mostly with alluvium and soft sediment), the obtained values of Q are low due to the thick top Quaternary sediments in the basin. The remaining stations (i.e., MTP, BHU, NAGO and TPM) which are situated in the region associated with Mesozoic uplifts, show higher Q values. As we go away from the main rift zone, an increase in Q values is observed (at MOU, JMN, MTP and TPM sites), which can be explained in terms of decrease in material heterogeneity in the less deformed surrounding regions (Biswas *et al.* 2016). Low Q values at lower frequencies can be explained in terms of fluid movement in the causative fault zone and/or fluid-filled cracks in the Kachchh seismic zone (Mandal *et al.* 2004a, b; Padhy 2009), while high Q zones (high velocity zones) could be attributed to the presence of rigid brittle less attenuative zones in the MRZ and also in the

Figure 12. Represents the plot between focal depths with crustal Q_0 values.

surrounding regions (Mandal and Chadha 2008). The Q_0 factor value plotted against the focal depth and the graph is shown in figure 12.

Later we carried out a comparative study of source parameters between 2001 Bhuj earthquake and other intraplate earthquakes that occurred in India. The estimated stress drops of 2001 Bhuj (M_w 7.7) and 1993 Latur (M_w 6.2) earthquakes are reported to be 21 and 7 MPa, respectively (Baumbach *et al.* 1994; Antolik and Dreger 2003). The estimated static stress drop was 20 MPa for Jabalpur earthquake of M_w 5.7 (Singh *et al.* 1999). The reservoir triggered Koyna earthquake sequence (1994–1997) suggests that the estimated stress drop ranging from 0.03 to 19 MPa for a moment magnitude ranging from 1.5 to 4.7 (Mandal *et al.* 1998). We reported that our estimated stress drop value 36.79 MPa for M_w 4.4 is larger than the 1993 Latur earthquake of M_w 6.3 (~ 7 MPa, Baumbach *et al.* 1994). This could be attributed to the rift tectonics of the Kachchh region, but the Latur region is mainly characterized by tectonics of stable continental regions.

7. Conclusions

The estimated source parameters are robust because we have used three components of S-wave spectra of broadband seismograph using SAC software. Fletcher's inversion technique (1995) was used to invert the three component S-wave spectra and at best-fit spectra; simultaneously crustal Q_0 -values were calculated for 138 selected aftershocks of 2001 Bhuj earthquake. The estimated apparent stress drop and static stress drop values range

from 0.01 to 2.56 and 0.53 to 36.79 MPa, respectively. The seismic moment (M_0), source radius (r) and corner frequencies range from $7.03\text{E}+12$ to $5.36\text{E}+15$ N-m, 178.56 to 565.21 m, and 3.025 to 7.425 Hz, respectively. We also estimated the radiated energy (E_S) using velocity spectra, which is varying from $2.76\text{E}+06$ to $4.07\text{E}+11$ Joules.

We noticed that the seismic energy might vary from a given seismic moment. The ratio of energy to moment E_S/M_0 with moment magnitude looks to be scattered at $\varepsilon > 1$, smaller earthquakes have more scattered nature at $M_w < 3.0$, which suggests that smaller earthquakes radiate less radiated energy per unit fault slip than large events. The estimated apparent stress drop values are much smaller than the static stress drop values. We calculated Zuniga parameter (ε_Z) (Zuniga 1993) using these two stress drops. We suggest that most of the studied Kachchh events follow the frictional overshoot model, i.e., $\varepsilon_Z > 1$.

The computed crustal Q_0 values ranging from 119.0 to 7229.5 with an average Q_0 value of 701 for Kachchh region are nearly equal to New Madrid region ~ 900 . The estimated stress drop, seismic moment and source radius show a clear deviation from self similarity. We notice that the stress drop values increase with increasing seismic moment. But there is a break in scaling, the larger earthquakes ($M_0 > 10^{14.91}$) are showing one trend and smaller earthquakes ($M_0 < 10^{14.91}$) are showing another trend.

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