Mathematical expression of discharge capacity of compound open channels using MARS technique

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In this paper, analytical methods, artificial neural network (ANN) and multivariate adaptive regression splines (MARS) techniques were utilised to estimate the discharge capacity of compound open channels (COC). To this end, related datasets were collected from literature. The results showed that the divided channel method with a coefficient of determination (R^2) value of 0.76 and root mean square error (RMSE) value of 0.162 has the best performance, among the various analytical methods tested. The performance of applied soft computing models with $R^2 = 0.97$ and RMSE = 0.03 was found to be more accurate than analytical approaches. Comparison of MARS with the ANN model, in terms of developed discrepancy ratio (DDR) index, showed that the accuracy of MARS model was better than that of MLP model. Reviewing the structure of the derived MARS model showed that the longitudinal slope of the channel (S), relative flow depth (H_r) and relative area (A_r) have a high impact on modelling and forecasting the discharge capacity of COCs.

1. Introduction

Discharge of flow in natural streams is a basic parameter for developing water engineering projects including dam construction, water harvesting, river intakes, etc. (Dehdar-behbahani and Parsaie 2016). Government bodies, therefore, construct hydro-metric stations to directly measure discharge of the flow along important rivers (Lenton and Muller 2012). Another method of estimation of flow discharge is using indirect approaches that apply mathematical operations. Indirect approaches can be divided into analytical methods and modern computations. Calculation of discharge of flow in rivers using analytical approaches is based on classical empirical formulas such as Manning and Chezy formulas. The use of these formulas lead to obvious errors in the river-engineering projects (Myers 1987; Chatila and Townsend 1996; Reddy *et al.* 2014; Fernandes *et al.* 2015; Parsaie 2016;

Parsaie *et al.* 2016b). Researchers have, therefore, attempted to modify and improve accuracy of these formulas in order to use them in river-engineering projects. One powerful approach that has been recently proposed and widely used in river-engineering problems is the compound open channel (COC) idea (Chow 2009; Chow *et al.* 2013). Classical formulas consider the cross-section of flow as en-bloc and one value is considered for roughness coefficients across all parts of the cross-section; whereas in the COC idea, the flow cross-section is separated into main channel and floodplains (Naot *et al.* 1993; Chaudhry 2007). The main advantage of COC compared to classical ones is related to this point. Figure 1 shows the sketch of a COC.

In this figure, n_{mc} and n_{fp} define the roughness coefficients of main channel and floodplains, respectively. H introduces the depth of flow in the main channel, and h is the main channel depth. Using the COC idea, researchers have proposed several

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Figure 1. A schematic of a compound open channel cross-section.

analytical approaches to estimate discharge capacity of natural streams (Subramanya 2009; Mohanty and Khatua 2014) such as the single channel method (SCM), the divided channel method (DCM), and the coherence method (COH). Researchers have attempted to utilise these methods to estimate flow discharge and they have also evaluated the performance of each one in laboratory and field studies (Sahu 2012). In this regard, Seckin (2004) applied SCM, COH and DCM methods to evaluate the discharge capacity of COCs. Results of his study showed that the COH method was more accurate when compared to DCM and SCM. Seckin *et al.* (2009) applied 2D formulas for forecasting discharge capacity of COCs. To this purpose, they assessed and compared 2D formulas and the traditional 1D formulas including SCM and DCM. The results of these studies showed that 2D formulas had better performance compared to traditional formulas. Khatua *et al.* (2012) proposed a modified divided channel method (MDCM) to estimate the discharge capacity of COC. They stated that results of their proposed method could be utilised for practical problems. Recently, by developing the soft computing techniques in water resource research, such as in assessment of water quality (Noori *et al.* 2012, 2015, 2016), river pollution modelling (Najafzadeh and Lim 2015; Parsaie and Haghiabi 2015; Azamathulla *et al.* 2016; Najafzadeh and Tafarjnoruz 2016), scouring downstream of hydraulic structures (Najafzadeh *et al.* 2013, 2016; Noori and Hooshyaripor 2014; Najafzadeh 2016), investigators have applied them to accurately estimate the discharge capacity of rivers based COC concept. In this regard, using artificial neural network (ANN), group method of data handling (GMDH) (Najafzadeh and Zahiri 2015), support vector machine (SVM), genetic expression method (GEM), and M5 model trees (Unal *et al.* 2010; Sahu *et al.* 2011; Azamathulla and Zahiri 2012; Zahiri and Azamathulla 2014) have been employed. Literature review shows that there is no report of evaluation of the performance of multivariate adaptive regression splines (MARS)

models for estimating the discharge of flow in natural streams, based on concept of COC. Therefore, in this study, MARS has been utilised to estimate discharge capacity of COCs. In the following sections of this article, a comparison is made between the results of MARS, analytical approaches and multilayer perceptron (MLP) neural network.

MARS model was introduced by Friedman (1991) and was proposed as a novel approach for modelling the hydrology phenomena. This model has high ability for mapping the relation between the independent and dependent variables in any complex system which is based on dataset. This technique has been successfully used in some areas of hydrology engineering such as estimation of the scour depth, downstream of hydraulic structures, river mixing, river discharge forecasting, rainfall-runoff modelling, etc. (Sharda *et al.* 2008; Zhang and Goh 2014; Samadi *et al.* 2015; Haghiabi 2016a, b; Parsaie *et al.* 2016a). MARS technique is categorised as a smart function fitting methods. This means that MARS technique can define the most effective parameters in mathematical expressions of relation between input(s) and output(s), in complex systems. This point of the MARS model leads to assign more weights to most importance parameters that consequently lead to obtain derived model that is more reliable.

2. Materials and method

Discharge capacity of COCs is proportional to hydraulic and geometric characteristics. Parameters affecting flow discharge are given in equation (1).

$$Q = f(f_{fp}, f_{mc}, A_{fp}, A_{mc}, R_{fp}, R_{mc}, S, h, H - h), \quad (1)$$

where f_{fp} and f_{mc} are the roughnesses of main channel and floodplains; A_{fp} and A_{mc} are the areas of the main channel and floodplains; R_{fp} and R_{mc} are the hydraulic radii of the main channel and floodplains and S is the longitudinal slope of COC. Sahu *et al.* (2011) arranged these variables as

dimensionless parameters (equation 2) to estimate the discharge capacity of COCs and used them as input variables for developing a neural network model. Most of the analytical approaches and some soft computing research are based on dimension parameters.

$$Q = \psi(f_r, A_r, R_r, S, H_r), \quad (2)$$

where f_r is the relative roughness and is defined as f_{mc}/f_{fp} ; A_r is the relative area and is defined as A_{mc}/A_{fp} ; R_r is the hydraulic radius and is defined as R_{mc}/R_{fp} , and H_r is the relative depth and defined as $(H-h)/H$. To evaluate the performance of the

analytical approaches and develop ANN and MARS models, 396 datasets related to flow discharge presented in a peer-reviewed journal were collected. Table 1 presents the summary of collected dataset from references. The histograms of collected dataset are given in figure 2. Datasets were collected from published articles including Knight *et al.* (1984), Wormleaton and Hadjipanios (1985), Wormleaton and Merrett (1990) (UK Flood Channel Facility), Tang *et al.* (1999), Seckin (2004), Atabay and Knight (2006), Khatua *et al.* (2012), Lenton and Muller (2012), Mohanty and Khatua (2014).

Table 1. Summary of collected data related to discharge of flow in the compound channel.

Author	Range	H (m)	h (m)	$H-h$ (m)	B (m)	b (m)	n_{fp}	n_{mc}	S	Q (m ³ /s)
Knight and Demetriou (1984)	Min	0.085	0.08	0.009	0.15	0.08	0.0100	0.0100	0.0010	0.0049
	Max	0.154	0.08	0.078	0.31	0.08	0.0100	0.0100	0.0010	0.0294
	AVG	0.112	0.08	0.036	0.23	0.08	0.0100	0.0100	0.0010	0.0116
	STDEV	0.022	0.00	0.022	0.06	0.00	0.0000	0.0000	0.0000	0.0068
Wormleaton and Hadjipanios (1985)	Min	0.135	0.12	0.015	0.75	0.29	0.0110	0.0099	0.0004	0.0009
	Max	0.210	0.12	0.090	0.75	0.29	0.0210	0.0099	0.0018	0.4800
	AVG	0.167	0.12	0.047	0.75	0.29	0.0160	0.0099	0.0006	0.0372
	STDEV	0.023	0.00	0.023	0.00	0.00	0.0042	0.0000	0.0003	0.0735
Wormleaton and Merrett (1990) (FCF: UK Flood Channel Facility)	Min	0.058	0.05	0.006	0.61	0.20	0.0091	0.0091	0.0001	0.0100
	Max	0.302	0.15	0.152	5.00	0.75	0.0910	0.0210	0.0020	1.1142
	AVG	0.169	0.12	0.050	2.40	0.58	0.0220	0.0113	0.0007	0.3230
	STDEV	0.066	0.05	0.040	1.47	0.26	0.0239	0.0030	0.0009	0.2946
Tang <i>et al.</i> (1999)	Min	0.056	0.05	0.006	0.61	0.20	0.0056	0.0079	0.0020	0.0130
	Max	0.220	0.05	0.170	0.61	0.20	0.0957	0.0390	0.0020	0.2180
	AVG	0.092	0.05	0.042	0.61	0.20	0.0346	0.0170	0.0020	0.0499
	STDEV	0.042	0.00	0.042	0.00	0.00	0.0228	0.0093	0.0000	0.0546
Seckin (2004)	Min	0.060	0.05	0.010	0.61	0.20	0.0090	0.0090	0.0020	0.0148
	Max	0.168	0.05	0.118	0.61	0.20	0.0490	0.0090	0.0020	0.0553
	AVG	0.090	0.05	0.040	0.61	0.20	0.0288	0.0090	0.0020	0.0299
	STDEV	0.027	0.00	0.027	0.00	0.00	0.0177	0.0000	0.0000	0.0117
Atabay and Knight (2006)	Min	0.061	0.05	0.011	0.61	0.20	0.0063	0.0091	0.0020	0.0180
	Max	0.120	0.05	0.070	0.61	0.20	0.0112	0.0115	0.0020	0.1830
	AVG	0.072	0.05	0.022	0.61	0.20	0.0081	0.0098	0.0020	0.0474
	STDEV	0.014	0.00	0.014	0.00	0.00	0.0013	0.0006	0.0000	0.0391
Khatua <i>et al.</i> (2012)	Min	0.136	0.12	0.016	0.22	0.06	0.0100	0.0100	0.0019	0.0087
	Max	0.223	0.12	0.103	0.22	0.06	0.0100	0.0100	0.0019	0.0391
	AVG	0.174	0.12	0.054	0.22	0.06	0.0100	0.0100	0.0019	0.0212
	STDEV	0.031	0.00	0.031	0.00	0.00	0.0000	0.0000	0.0000	0.0111
Ikeda and McEwan (2009)	Min	0.207	0.20	0.007	0.81	0.28	0.0100	0.0100	0.0001	0.0260
	Max	0.278	0.20	0.078	0.81	0.28	0.0100	0.0100	0.0003	0.0730
	AVG	0.242	0.20	0.042	0.81	0.28	0.0100	0.0100	0.0002	0.0486
	STDEV	0.018	0.00	0.018	0.00	0.00	0.0000	0.0000	0.0001	0.0133
Mohanty and Khatua (2014)	Min	0.071	0.07	0.006	1.98	0.17	0.0100	0.0100	0.0011	0.0130
	Max	0.115	0.07	0.050	1.98	0.17	0.0100	0.0100	0.0011	0.1062
	AVG	0.091	0.07	0.026	1.98	0.17	0.0100	0.0100	0.0011	0.0467
	STDEV	0.016	0.00	0.016	0.00	0.00	0.0000	0.0000	0.0000	0.0326

Figure 2. The histogram of collected data involved discharge of flow in compound open channel.

2.1 Analytical approaches

2.1.1 Single channel method (SCM)

The single channel method (SCM) assumes the cross-section of a COC as en-bloc. The central point in SCM is computing equivalent roughness for COC by common methods such as Horton and Einstein formula (equation 3a) and in the following, discharge is calculated by equation (3b).

$$n_e = \frac{\left[\sum_{i=1}^N \left(P_i n_i^{\frac{3}{2}} \right) \right]^{\frac{2}{3}}}{P^{\frac{2}{3}}}, \quad (3a)$$

$$Q = \frac{1}{n_e} A R^{\frac{2}{3}} S^{\frac{1}{2}}. \quad (3b)$$

2.1.2 Divided channel method (DCM)

The divided channel method (DCM) divides the cross section of flow into subareas as main channel and floodplains. DCM assumes uniform velocity in subsections and general discharge is achieved by summation of discharge through subarea. Discharge in the subarea is computed using equation (4). As given in this equation, Manning equation is utilised for evaluating discharge and subscript t is related to the subarea. The segregated border between subsections (main channel and floodplains), as shown in figure 3, could be assumed as vertical, diagonal or horizontal. Modifications have been made on DCM and in this regard, we have DCM with horizontal segregated border, which

are included in the calculation of wetted perimeter ($DCM_{(h-i)}$) DCM with vertical separated lines, which are included in the calculation of wetted perimeter (DCM_{v-i}), and DCM with bisectonal division lines, which are included in the calculation of wetted perimeter (DCM_{b-i}) (Atabay and Knight 2006). A number of commercial packages, such as HEC-RAS, Mike 11 and ISIS, have been developed based on DCM.

$$Q_t = \left(\sum_{i=1}^N \frac{A_i R_i^{\frac{2}{3}}}{n_i} \right) S_0^{\frac{1}{2}}. \quad (4)$$

2.1.3 Coherence method (COH)

Coherence method (COH) is a 1D approach recommended for rectification of transport capacity parameter in a COC. COH is based on the momentum and mass transferring between main channel and floodplains. To this purpose, Ackers (1993) suggested a parameter called coherence factor. As given in equation (5), COH factor is defined as a ratio of the transport capacity parameter of SCM to DCM. If the COH parameter is close to 1, the COC has a hydraulic behaviour similar to a normal channel.

$$COH = \frac{(1 + A_r)^{\frac{3}{2}} / \sqrt{\left(1 + P_r^{\frac{4}{3}} n_r^2 / A_r^{\frac{1}{3}}\right)}}{1 + A_r^{\frac{5}{3}} / (n_r P_r^{\frac{2}{3}})} \quad (5a)$$

$$A_r = \frac{N_{fp} A_{fp}}{A_{mc}}, \quad P_r = \frac{N_{fp} P_{fp}}{P_{mc}}, \quad n_r = \frac{n_{fp}}{n_{mc}}. \quad (5b)$$

In the above equations, N_{fp} denotes the number of floodplains, A denotes the area of cross-section of flow, P denotes the wet perimeter, and n_{mc} and n_{fp} denote the roughness of the main channel and the

floodplains, respectively. The COH factor can get a value between 0 and 1. Whenever COH attains a value of 1, the COC shows a behaviour similar to a normal channel. Ackers (1993) also proposed a coefficient called the discharge adjustment factor (DISADF) to edit discharge capacity. Discharge is corrected using the values derived from figure 4 and equation (6).

$$\text{Region 1: } Q_{COH} = Q_{DCM} - \text{DISDEF}, \quad (6a)$$

$$\text{Regions 2, 3, 4: } Q_{COH} = \text{DISADF} * Q_{DCM}. \quad (6b)$$

2.1.4 Multivariate adaptive regression splines (MARS)

MARS, proposed by Friedman (1991), is a pliable method to map the relationship between the independent and dependent variables in a desired system. MARS method is used to recognize the hidden patterns within dataset in complex systems. Pattern recognition is defined by proposing a number of coefficients and basic functions. The main advantages of MARS include high ability for mapping input parameters and desired outputs, developing simple but robust model and being rational in terms of computational cost. MARS technique is based on simple basic functions defined as given below:

$$|x - t|_+ = \max(0, x - t) = \begin{cases} x - t & x > t \\ 0 & x \leq t \end{cases} \quad (7a)$$

and

$$|t - x|_+ = \max(0, t - x) = \begin{cases} t - x & x < t \\ 0 & x \geq t \end{cases}, \quad (7b)$$

where t denotes the knot. Basic functions are sometimes also called mirrored pair-functions.

Figure 3. Types of separating the boundary between main channel and floodplains. (a–b) vertical, (a–c) diagonal, (a–a) horizontal.

These functions are defined for each input variable such as X_j . Sets of basic functions are defined as

$$C = \{(x_j - t)_+, (t - x_j)_+\}; \{x_{1j}, x_{2j}, \dots, x_{nj}\},$$

$$j = 1, \dots, p. \quad (8)$$

The general form of the function derived from MARS model is written as an adaptive function as follows.

$$y = \beta_0 + \sum_{i=1}^M \beta_i BF_i(X), \quad (9)$$

where β_0 is constant, $BF_i(X)$ is known as basic function and β_i is coefficient of basic functions. The constant and coefficient of derived function

in MARS model are justified using least square error technique. M is the number of basic functions derived from final stage of model development. Developing MARS model consists of two stages. The first is the forward stage where at this stage the number of basic functions is increased to decrease the difference between the results of model and observed data. In the next step of model development to avoid over-parameterisation and over-fitting, pruning some basic functions is considered. At this stage, regarding cross-validation (GCV) criteria as given below, basic functions are pruned.

$$GCV = \frac{SSE}{n \left(1 - \left(\frac{C(B)}{n} \right) \right)^2} \quad (10a)$$

Figure 4. Diagram of adjustment parameter of flow discharge *vs.* relative depth.

Figure 5. Sketch of three-layer ANN architecture.

and

$$C(B) = (B + 1) + dB, \quad (10b)$$

where SSE_i is the sum of square of residuals, n denotes the number of records and $C(B)$ defines a difficulty criteria, which increases by the number

of basic functions. For more information refer Azamathulla *et al.* (2016) and Parsaie *et al.* (2016a).

2.2 Artificial neural networks (ANNs)

ANN is a popular computing technique that was developed to simulate complex problems that are

Figure 6. Correlation between results of analytical approaches *vs.* measured discharge.

based on inputs and outputs. MLP networks are well-known types of ANN that have widely been used for modelling and predicting the engineering phenomena. Preparation of MLP models include a number of steps such as designing the structure of network, choosing of transfer functions, adjusting the values of weights and bias. Different types of transfer functions are proposed for governing the neurons. MLP model could take one or more hidden layers. Figure 5 shows a MLP model including three hidden layers. As shown in figure 5, each input is multiplied with a weight (w_i), following which their summation is added to a bias (b_i) and then passed through the transfer function. After definition of model structure, training algorithm should be considered. Training here implies adjusting the values of weights and biases. This operation is carried out using progressive approaches such as the Levenberg–Marquardt technique. The goal of model training is minimising the errors by comparing the predicted outputs with observed data.

3. Results and discussion

3.1 Results of analytical approach

The analytical approaches MLP and MARS models were assessed using collected datasets, the histograms of which are shown in figure 2. Accuracy of analytical approaches, MLP and MARS models were evaluated by calculation of error indices such as coefficient of determination (R^2) and root mean square error (RMSE). It must be noted that these indices offer an average value for error and do not present any detail about distribution of error

through dataset. Therefore, in addition to computing error indices, it is desirable to plot the results of these methods against the measured data. Figure 6 shows the results of analytical approaches compared to measured data. The values of the error indices for the results of analytical methods are also shown in figure 6. Considering figure 6, DCM_{h-i} , DCM_{h-e} and DCM_{v-i} are more accurate compared to other analytical approaches. The suitable accuracy of these methods can be ascribed to the fact that the cross-section of the COC were divided into a main channel and floodplains and also to the fact that the concept of mass and momentum transferring between subsections of COC were considered. Figure 6 shows the performance of each analytical approach versus measured data. Considering figure 6, it seems that the best analytical approach is DCM_{h-i} , since in addition to having good values for the error indices, it also has a suitable performance for predicting discharge in high value of discharge.

3.2 Results of MLP model

The first step in the development of neural networks is data preparation. This means that part of the dataset should be assigned for model development (calibration stage) and another part should be assigned for model validation (testing stage). The portion of each stage from the collected dataset is defined via a trial-and-error process. However, the experience of the designer and recommendation of investigators who have conducted similar work is useful. Eighty percent of the dataset was considered for calibration, and the remaining 20% for validation. Designing the architecture of ANN

Figure 7. Architecture of developed MLP model.

Figure 8. Performance of MLP model during training stage.

Figure 9. Performance of MLP model during testing stage.

model is also a trial-and-error process, where recommendations of others who utilised other types of soft computing in this regard could be useful. In this research, the recommendation of Sahu *et al.* (2011) was used. Designing the structure of MLP model includes using a number of hidden layer(s), a number of neurons in each layer, assigning suitable transfer function for neurons and training algorithm. Since, the flow in a COC is a fully complex hydraulic phenomena, it was expected that the MLP model was not of a small size. To achieve an optimum structure for the MLP model, the size of model is increased step by step. Different transfer functions including logsig (Log-Sigmoid Transfer Function), tansig (Hyperbolic Tangent Sigmoid Transfer Function), purelin (Linear Transfer Function) were tested. In other words, firstly, a model with one hidden layer involved five neurons (equal to features of inputs) was considered. Then the transfer functions were tested. By choosing the proper transfer function in the next step was to improve the precision of developed MLP model, the number of neurons and hidden layer could be increased. This approach

leads to achieving optimum structure and suitable performance in terms of computation cost. During the development of the MLP model, it was found that the model structure, shown in figure 7, has a suitable performance for predicting flow discharge in COC and increasing the number of hidden layers only increases the computational cost and has no significant effect in improving the performance of model. Figure 7 shows the architecture of the developed MLP model. As can be seen from figure 7, the MLP model consists of two hidden layers that first and second hidden layers include ten and five neurons with the tansig transfer function, respectively. Levenberg–Marquardt technique was used for MLP model learning.

As seen in figure 7 and with respect to equation (2), dimensionless parameters such as f_r , A_r , R_r , S and H_r were considered as input parameters and Q as the output parameter. The performance of MLP model during calibration (training) and validation (testing) stages are given in figures 8 and 9. In these figures, the results of the MLP model along and *vs.* the observed data are shown. Overall, these results of MLP show that the

Table 2. Basic functions and related coefficients of MARS model.

Basic function	Equation	Coefficient
$h_1(x)$	BF1 = $\max(0, 0.00019 - S)$	5504.451
$h_2(x)$	BF2 = $\max(0, S - 0.0001)$	7196.574
$h_3(x)$	BF3 = $\max(0, 0.0001 - S)$	-114469.959
$h_4(x)$	BF4 = $\max(0, f_r - 1.261)$	-0.041
$h_5(x)$	BF5 = $\max(0, A_r - 0.574)$	0.197
$h_6(x)$	BF6 = $\max(0, S - 0.0002) * \max(0, H_r - 0.794)$	4949.876
$h_7(x)$	BF7 = $\max(0, S - 0.0002) * \max(0, 0.749 - H_r)$	-4705.815
$h_8(x)$	BF8 = $\max(0, 1.261 - f_r) * \max(0, 0.342 - A_r)$	-7.416
$h_9(x)$	BF9 = BF2 * $\max(0, H_r - 0.417)$	-4681.751
$h_{10}(x)$	BF10 = BF2 * $\max(0, 0.417 - H_r)$	4427.788
$h_{11}(x)$	BF11 = $\max(0, 0.857 - H_r)$	2.007
$h_{12}(x)$	BF12 = BF11 * $\max(0, S - 0.002)$	-10818.362
$h_{13}(x)$	BF13 = BF11 * $\max(0, 0.002 - S)$	5594.686
$h_{14}(x)$	BF14 = $\max(0, 0.001 - S)$	-183.352
$h_{15}(x)$	BF15 = BF11 * $\max(0, 0.002 - S)$	-7865.379
$h_{16}(x)$	BF16 = $\max(0, S - 0.0011) * \max(0, A_r - 0.456)$	-627.501
$h_{17}(x)$	BF17 = $\max(0, S - 0.0011) * \max(0, 0.456 - A_r)$	5687.774
$h_{18}(x)$	BF18 = BF14 * $\max(0, 0.425 - A_r)$	1309.383
$h_{19}(x)$	BF19 = BF14 * $\max(0, H_r - 0.567)$	-793.312
$h_{20}(x)$	BF20 = BF14 * $\max(0, 0.567 - H_r)$	2103.869
$h_{21}(x)$	BF21 = $\max(0, 0.002 - S)$	6305.094
$h_{22}(x)$	BF22 = $\max(0, S - 0.0019) * \max(0, A_r - 0.456)$	5068.522
$h_{23}(x)$	BF23 = $\max(0, S - 0.002) * \max(0, 0.456 - A_r)$	-38488.855
$h_{24}(x)$	BF24 = $\max(0, 1.261 - f_r) * \max(0, 2.087 - R_r)$	0.715
$h_{25}(x)$	BF25 = $\max(0, A_r - 0.311)$	-0.200
$h_{26}(x)$	BF26 = $\max(0, 0.311 - A_r)$	2.375

performance of MLP is appropriate for predicting flow discharge in a COC.

The extended form of MARS model is given in table 2.

$$Q = -10.436 + \sum_{M=1}^{26} \beta_m h_m(x). \quad (11)$$

4. Results of MARS model

Developing a MARS model similar to MLP model is based on the dataset. To this end, the collected dataset, which was used for preparation of MLP model (training: 80% and testing: 20%), was also used for developing MARS technique. During MARS model development, in the first step, thirty basic functions were considered and in the second (pruning) step, five basic functions were pruned. In the end, the optimal MARS model was derived with 26 basic functions. The general form of obtained MARS model is given in equation (11).

Equation (11) can be used for estimating discharge capacity in a COC. As seen in table 2, S , H_r and A_r have appeared in almost all basic functions. It means that these three parameters are more effective compared to others. This obtained result of the MARS model upholds the results of the study that was conducted by Sahu *et al.* (2011). They declared that the sensitivity analysis of the developed ANN model indicated that H_r and A_r were the most effective parameters for predicting the discharge of flow in COC. Moreover, the performances

Figure 10. Results of MARS model along observed data during training (a) and testing (b) stages.

Figure 11. Results of MARS model *vs.* observed data in training (a) and testing (b) stages.

of the MARS model during preparation stages (training and testing stages) are given in figures 10 and 11. Figures 10 and 11 show the plotted measured data and results of MARS model. Additionally, the results of error indices calculation are also indicated in these figures. From figures 10 and 11, it can be that the performance of MARS model for prediction of flow discharge is very appropriate.

4.1 Comparison of the performances

A comparison of the results of MLP, MARS and DCM_H shows that in the preparation stages, including training and testing stages, MARS model is a bit more accurate than MLP. This assessment was obtained considering the calculation of standard error indices such as coefficient of determination and mean square of errors. Figure 12 shows the outcome of MARS, MLP and DCM_H for testing dataset. As shown in this figure, the best performance of models is related to MARS and MLP. Although the DCM_H has performance in all datasets, but for testing dataset, its precision is reduced.

Standard error indices provide an average value for errors, whereas they do not present detailed information on distribution of errors. Therefore, it is necessary to assess the performance of the model and apply another evaluation criterion. In this paper, the developed discrepancy ratio (DDR) introduced by Noori *et al.* (2010) was applied. This index has also been used in several studies namely

Figure 12. Performance of MARS, MLP and DCM (H) for testing data.

Noori and Hooshyaripor (2014) as well as Dehghani *et al.* (2014). DDR is calculated as follows:

$$\text{DDR} = \left(\frac{\text{Predicted value}}{\text{Observed value}} \right) - 1. \quad (12)$$

Results of DDR calculation for both models in training (a) and testing (b) stages are given in figures 13 and 14. Comparing the performances of

Figure 13. DDR values for MARS (a) MLP and (b) models in training stages.

Figure 14. DDR values for MARS (a) MLP and (b) models in testing stages.

MARS and MLP in terms of DDR index shows that MARS model is more accurate.

5. Conclusion

Predicting flow discharge in rivers is important when developing hydro system projects. COC is a novel concept and an accurate approach proposed for calculating flow discharge in rivers. Several proposed analytical approaches are based on this concept. Recently, by advances in soft computing techniques in most areas, especially in water engineering studies, these models have been applied for predicting flow discharge in COCs. Today, new soft computing models namely MARS have been

proposed for modelling complex systems, which are based on inputs and output dataset. The main advantage of MARS model is its ability to yield clear information about internal processes, which is carried out in the model development process. Results of this study showed that MARS model is very appropriate for predicting flow discharge in a COC. Another advantage of MARS model is related to programing its results for other purposes.

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