Random fractal characters and length uncertainty of the continental coastline of China

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A coastline is a random fractal object in a geographical system whose length is uncertain. To determine the coastline length of a country or a region, the scaling region and fractal dimension of the coastline is first calculated, and then, the length of the coastline is measured using the scale at the lower limit or near the limit of the scaling region. For this study, the scaling region of the continental coastline of China is determined. The box-counting dimension was calculated with ArcGIS software using 33 scales and a map scale of 1:500,000, and the divider dimension calculated by a C language program. Moreover, the reliability of the Chinese coastline length value, which is widely used currently, is discussed in this paper. The results show that the scaling region of the continental coastline of China is from 0.1 to 400 km. In the scaling region, the box-counting dimension and the divider dimension of the coastline are 1.2004 and 1.0929, respectively. According to fractal theory, the divider dimension more accurately represents the irregularity of a coastline. The length of the continental coastline of China is approximately 21,900 km when the measurement scale is 0.1 km; however, the length is 18,214 km when the scale is 0.25 km, and this value approaches the continental length of China (18,400 km) in popular use today. Although the coastline length is shorter than 21,900 km, the length is acceptable because the measurement scale (0.25 km) is close to the lower limit of the scaling region.

1. Introduction

‘Records of Chinese geography’ is one of the earliest published documents that recorded China’s continental coastline (hereafter referred to as coastline), and the reported length was approximately 8000 km (Kong *et al.* 1914). In 1935, issue 1 of the *Bulletin of Chinese Government Amphibious Map Review Committee* reported that the coastline of China was 10,537 km, and this value has been cited by many documents (He *et al.* 1946; He 1947; Ren 1947). During the initial post-liberation period of the People’s Republic of China in the 1950s, the Chinese authorities performed piecewise

measurements of old charts and updated the coastline length to 14,000 km, a value that was used until 1975 (Li and Jia 1994). In 1972, compass and curve meter methods were used to measure the coastline by the Navy of the People’s Liberation Army of China. In 1975, a length of 18,400.5 km was approved, and this value has been widely used until today (Yong 1985; Wang 1996).

A coastline is a typical geographical fractal object whose length varies according to the size of the measurement scale (Mandelbrot 1967). Hence, the length of a coastline is unscientific and meaningless without certain constraints. Mandelbrot’s research showed that the fractal dimension (D) of

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a coastline is constant, whereas the length varies in accordance with different measurement scales (Mandelbrot 1967, 1975, 1977). The fractal dimension is a characteristic parameter used to describe the irregular extents of coastlines (Goodchild and Mark 1987), and it has a threshold value between 1 and 2. Moreover, coastlines with higher D values are more complex. Methods are available to estimate the fractal dimension for a linear object, such as the divider dimension method (Mandelbrot 1977; Longley and Batty 1989; Carr and Benzer 1991; Singh and Gupta 2013), the box-counting dimension method (Liebovitch and Toth 1989; Longley and Batty 1989; Grassberger 1993; Singh and Gupta 2013) and the stochastic noise dimension method (Mandelbrot 1975; Carr and Benzer 1991). In recent decades, these methods have been used by a number of scholars to calculate the fractal dimensions of coastlines in different regions (table 1).

Fractal types consist of rule fractals and random fractals. Rule fractals have a strict self-similarity on all scales, such as the Koch curve, the Cantor set and the Sierpinski carpet, whereas random fractals show self-similarity only in specific scaling regions. Fractals in geosystems are random fractals and include coastlines, basin boundaries and urban structures (Yang *et al.* 2008). However, few studies have discussed the scaling regions of coastlines. Scaling region would help to decide the correct value of the measurement scale to be used, and hence more accurate length (Srivastava *et al.* 2007). The fractal dimension is inaccurate if the data outside the scaling regions are involved in the calculations (Dimri 1998). Thus, further studies of

the measurement scale of the current coastline's length are required to verify whether it is within the scaling region.

In this paper, both the divider dimension and box-counting dimension methods were applied to calculate the fractal dimension of the coastline of China. This research will explore the uncertainty of the Chinese coastline length and the measurement scale of the current coastline's length, and the coastline length will be examined to determine whether it is within the scaling region by increasing the number of measurement scales and identifying the scaling region. This study is significant because it reveals the fractal geometry characteristics of the coastline of China, defines the constraint conditions of the current coastline's length and verifies the reliability of the length currently in use.

2. Materials and methods

Selecting an appropriate scale map is an essential factor in determining the fractal dimension of the coastline. In this study, the coastline fractal dimension at a map scale of 1:500,000 was measured according to the characteristics of China's coastline (islands, such as Hainan Island and Taiwan Island, were not measured in this study).

2.1 Box-counting method

A total of 10 maps from the Yalu River in the north to the Beilun River in the south of China were successively acquired from Google Earth, using the Google Map V1.1 interceptor software.

Table 1. *Fractal dimension (D) values for various coastlines.*

Coastline	Fractal dimension		References
	Divider	Box-counting	
West Coast, Great Britain	1.25	–	Mandelbrot (1967)
Australia	1.13	–	Mandelbrot (1967)
South Africa	1.02	–	Mandelbrot (1967)
Delaware Bay, New Jersey	1.46	–	Phillips (1986)
East Shore, Gulf of California	1.02	–	Carr and Benzer (1991)
West Shore, Gulf of California	1.03	–	Carr and Benzer (1991)
Pacific Coast, United States	1.00 ~ 1.27	–	Jiang and Plotnick (1998)
Atlantic Coast, United States	1.00 ~ 1.70	–	Jiang and Plotnick (1998)
Maine, United States	–	1.11 ~ 1.35	Tanner <i>et al.</i> (2006)
South Coast, Norway	–	1.52	Feder (1988)
Greenland	1.23	1.26	Singh and Gupta (2013)
Titan lakes, North Pole	1.27	1.32	Sharma and Byrne (2010)
Cres island, Croatia	–	1.118	Paar <i>et al.</i> (1997)
Shangdong and Tianjin, China	–	1.1383	Xu <i>et al.</i> (2014)
China	1.16	–	Zhu <i>et al.</i> (2004)
China	1.195	–	Su <i>et al.</i> (2011)

Note: '–' means no data.

Figure 1. Map showing the technological flow of the box-counting method used for the fractal analysis.

Subsequently, the georeferencing and vectorisation functions in ArcGIS 10.0 were used at a scale of 1:500,000 in the obtained maps.

The smallest scale of the box size used to calculate the coastline fractal dimension is based on the resolution of the map used, and the resolution varies from 0.3 to 0.5 mm map units. On the map with a scale of 1:500,000, the smallest actual distance varies from 150 to 250 m based on the resolution, whereas, the largest scale was not strictly defined and may be equidistant, exponential or random without influencing the fractal dimension calculations (Zhu *et al.* 2000; Sharma and Byrne 2010; Singh and Gupta 2013). In this study, the measurement scales were piecewise equidistant, and as many scales as possible were used to find the scaling regions of China's coastline. The following 33 measurement scales were used: 0.01, 0.05, 0.1, 0.2, 0.5, 1, 1.5, 2, 2.5, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100, 150, 200, 250, 300, 400, 500, 600, 700 and 800 km.

ArcGIS 10.0 was used to obtain the number of boxes (N), and these data were then analysed using Excel 2007 software. The technological flow for the fractal analysis of the coastline is shown in figure 1.

We then used Excel 2007 to draw the $\ln N - \ln r$ curve based on the corresponding number of boxes for each scale. After the data outside the straight-away in the curve segment (i.e., scaling region) were removed, a fitting equation was automatically generated:

$$\ln N = C - D \ln r, \quad (1)$$

where C is a constant, and D is the fractal dimension. In practice, the slope of $\ln N - \ln r$ is the fractal dimension, and C is the vertical intercept.

The coastline of China is roughly divided into two sections by Hangzhou Bay (figure 2). The plain coast is dominant and less complex on the northern coast section from the Yalu River to Hangzhou Bay, whereas bedrock and biological coasts are dominant and more complex on the south coast section from Hangzhou Bay to the Beilun River (Yong 1985; Wang 1996). To investigate these coastline differences, we calculated the fractal dimensions for the north coast section, the south coast section and the entire coastline of China.

2.2 Divider method

The divider method is suitable to use on vector linear features represented by coordinate pairs (Jiang and Plotnick 1998). The calculation of fractal

Figure 2. Sketch map of the continental coastline of China.

dimension for individual data lines was acquired by a C language program (Hayward *et al.* 1989; Longley and Batty 1989). First, we determined the step lengths (r). We used up to 20 step lengths (2.5, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100, 150, 200, 250 and 300 km) because these step lengths are all within the scaling regions of the Chinese coastline for the obtained box-counting dimension. Then, we measured the coastline from one side to the other with different divider step lengths and maintained a record of each number of steps taken (figure 3). Finally, the fractal dimension of the coastline was determined by the box-counting method calculation formula presented above.

3. Results

3.1 Scaling regions for the coastline of China

Because the number of scales of the box-counting method is much larger than that of the divider

method, we determined the coastline sections and the scaling regions for the different sections by drawing the $\ln N - \ln r$ scatter using the obtained data (figure 4). As illustrated in figure 4, we can conclude that the scaling regions presented certain differences in the various sections because of differences in the degree of irregularity between the different sections of coastline, with the more irregular coastlines presenting wider scaling regions. However, in the south coast section, the coastline is circuitous, and the scaling regions range from 0.1 to 600 km, whereas in the north coast section, the scaling regions range from 0.05 to 300 km. In terms of the entire coastline of China, the scaling regions range from 0.1 to 400 km.

3.2 Fractal dimensions of China's coastlines

Data inside the scaling regions that were derived from the box-counting method and the divider method were used to draw the $\ln N - \ln r$ curve, which was used to determine the fractal dimensions of the coastlines (figure 4).

As illustrated in figure 4, the divider dimension is larger than the box-counting dimension for the same section of coastline. For the entire coastline of China, the north coast section and the south coast section, the box-counting dimensions are 1.0929, 1.0655 and 1.1117, respectively, whereas the divider dimensions are 1.2004, 1.1204 and 1.2565, respectively. The box-counting dimension of China's coastline in our study is smaller than that calculated by Zhu *et al.* (2004), and the divider dimension is larger than that calculated by Su *et al.* (2011) as shown in table 1. This difference, however, may have been caused by the different map scales and different measurement scales used by different researchers as well as by measuring errors. We can also conclude from figure 4 that the fractal dimension of the coastline in the south coast section is greater than that in the north coast section, regardless of the box-counting dimension or divider dimension, and this result is consistent with the degree of irregularity between the different sections of coastline.

As shown in figure 4, the divider dimensions are larger than the box-counting dimensions, and this result is inconsistent with the results of others reports (Sharma and Byrne 2010; Singh and Gupta 2013). As presented in figure 4, the box numbers from the box-counting method are larger than the step numbers from the divider method for the same measurement scale, and this difference is a result of the box being metered as a grid, regardless of how the grid covers the length of the coastline, whereas the step length of the line segments are equal for the divider method. However, the D -value rate between the box numbers and step numbers (D -value rate = $[(N_{\text{box}} - N_{\text{step}})/N_{\text{box}}] \times 100\%$) shows a power function decrease with reductions of measurement scale (r) (figure 5), and this decrease will inevitably lead to variations in the slope of the $\ln N - \ln r$ curve, as calculated by the box-counting method and the divider method. When the measurement scale is large, the $\ln N - \ln r$ curves for the box-counting and divider methods are separated and the curve of the box-counting method is above the curve of the divider method. As the measurement scales decrease, the two curves gradually approach. However, the $\ln N - \ln r$ slope of the divider method is larger than that of the box-counting method, which means that the fractal dimension of the divider method is larger than that of the box-counting method.

4. Discussion

According to fractal theory, selected pattern elements should be compatible with the shapes of geometric objects, when covering methods are used to measure fractal dimensions, which means that linear random fractals should be covered by steps of different lengths (i.e., the divider method), planar fractals should be covered by grids of different lengths (i.e., the box-counting method) and three-dimensional fractals should be covered by different cubes. Hence, the box-counting dimension cannot accurately describe the degree of irregularity of coastlines, whereas the divider dimension provides

Figure 3. Algorithm for calculation of number of steps.

Figure 4. Scaling regions and fractal dimensions of the continental coastline of China calculated by the divider method and box-counting method.

Figure 5. Variations of the D-value rates between the number of dividers and boxes along with different measurement scales.

the characteristic parameter that represents the degree of irregularity of coastlines. Therefore, the true fractal dimension of China's coastline should be 1.2004, at a map scale of 1:500,000.

A series of measurement scales of N within the scaling regions was calculated by the fitting equation $\ln N = 9.5333 - 1.2004 \ln r$, and the length of the coastline was calculated at different measurement scales using $L = N \times r$ (figure 6).

As illustrated in figure 6, the length of the coastline shows an exponential increase as the measurement scale value decreases within the scaling regions. For example, $r = 400$ km, $L = 4156$ km; $r = 200$ km, $L = 4776$ km; $r = 100$ km, $L = 4586$ km; $r = 50$ km, $L = 6304$ km; $r = 10$ km, $L = 7801$ km; $r = 5$ km, $L = 9996$ km; $r = 1$ km, $L = 13,798$ km; $r = 0.5$ km, $L = 15,853$ km; $r = 0.25$ km, $L = 18,214$ km; $r = 0.2$ km, $L = 19,047$ km; and $r = 0.1$ km, $L = 21,883$ km. Therefore, in the scaling regions, the measuring scales and fractal dimensions must be considered with regard to the length of the coastline, or the length measurement will be meaningless. In practice, the smallest measurement scales possible within the scaling regions should be selected to calculate

Figure 6. Different lengths of the continental coastline of China as calculated by the different measurement scales using the fractal dimension of 1.2004.

the length of the coastline after the scaling regions and fractal dimensions are determined. The lower limit of China's coastline scaling regions is 0.1 km; thus, the length of the coastline should be approximately 21,900 km.

When the measuring scale is 0.25 km, the length of the coastline (18,214 km) is close to the value currently in wide use (18,400 km). This measurement scale is within the scaling region and near the lower limit of the scaling region; consequently, the current coastline length of China is accurate and acceptable. Therefore, the fractal dimension (1.2004) and the measurement scale (0.25 km) should be included, along with the length of China's coastline, because the length of China's coastline is only accurate under specific constraint conditions.

5. Conclusions

A coastline is a type of random fractal in geographical systems. The scaling region of China's coastline ranges from 0.1 to 400 km at a map scale of 1:500,000. The divider dimension is larger than the box-counting dimension for linear random fractal objects, and it also more accurately represents the degree of irregularity of the coastline more accurately relative to the

box-counting dimension. The divider dimension and the box-counting dimensions of China's coastline are 1.2004 and 1.0929, respectively. The length of the coastline increases as the measurement scale decreases within the scaling region. When the measurement scale is 0.25 km, which is within the scaling region and near the lower limit of the scaling region, the length of the coastline (18,214 km) is close to the value currently in wide use (18,400 km). In addition, the fractal dimension (1.2004) and measurement scale (0.25 km) must be noted along with the length of the coastline of China, because even when the fractal dimension and coastline length are determined using the same method, their values will differ if the maps are at different scales. This paper discusses a case study of the Chinese continental coastline at the map scale of 1:500,000. Moreover, this review mainly measures the fractal dimensions of two sections of the coastline divided by Hangzhou Bay. In fact, differences in the degree of irregularity occurred in each section of the coastline, especially north of Hangzhou Bay. In addition, more objective measurements are obtained if the coastline is subdivided when calculating the fractal dimension and coastline length.

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