Prediction of longitudinal dispersion coefficient using multivariate adaptive regression splines

AMIR HAMZEH HAGHIABI

Water Engineering Department, Lorestan University, Khorramabad 0098-663-4200289, Iran.
e-mail: ahaghiabi@gmail.com

In this paper, multivariate adaptive regression splines (MARS) was developed as a novel soft-computing technique for predicting longitudinal dispersion coefficient (D_L) in rivers. As mentioned in the literature, experimental dataset related to D_L was collected and used for preparing MARS model. Results of MARS model were compared with multi-layer neural network model and empirical formulas. To define the most effective parameters on D_L , the Gamma test was used. Performance of MARS model was assessed by calculation of standard error indices. Error indices showed that MARS model has suitable performance and is more accurate compared to multi-layer neural network model and empirical formulas. Results of the Gamma test and MARS model showed that flow depth (H) and ratio of the mean velocity to shear velocity (u/u^*) were the most effective parameters on the D_L .

1. Introduction

Assessment of river water quality is a major factor for developing environmental projects. Therefore, governments construct sampling stations along the rivers to continuously monitor river water quality. Unfortunately, sometimes, rivers are considered as a place for sewage disposal that causes pollution. Therefore, study of mixing flow to reduce degree of contamination has become one of the main subjects in river water quality projects (Leta *et al.* 2014). When a source of pollution, which has a density close to density of water, is injected into the river, it spreads in the river and is transmitted downstream due to molecular motion and turbulence of flow. Studies on river water quality have focused on the mechanism of mixing flow to reduce the effect of pollution on public environmental health. Dispersion coefficient is the main parameter which is discussed in river water quality problems (Ji 2008; Benedini and Tsakiris 2013). At the initial stage

of injection of pollution, dispersion mechanism is three-dimensional whereas with widespread pollution in the flow section and moving, the pollution along the river dispersion mechanism assumes to be one-dimensional (figure 1). As seen in figure 1, in stage (V), dispersion is almost one-dimensional.

Several experimental studies have been conducted on dispersion coefficient in rivers. Most of these studies are based on routing tracer concentration along the river (Atkinson and Davis 2000; Davis and Atkinson 2000; Davis *et al.* 2000; Disley *et al.* 2015; Parsaie and Haghiabi 2015a, b, c, d). Due to the high cost of experimental studies, investigators have attempted to implement mathematical methods for modelling D_L . In mathematical approaches, the governing equations have been solved by powerful methods such as finite difference, finite volume and finite element methods (Stamou 1992; Wang *et al.* 2015). The advection–dispersion equation (ADE) is the governing equation on one-dimensional pollution transmission in rivers.

Keywords. River water quality; artificial neural network; longitudinal dispersion coefficient; pollution transmission.

Figure 1. Schematic map of dispersion mechanism in rivers (Benedini and Tsakiris 2013).

Equation (1) presents one-dimensional ADE without any sink or source terms (Chao *et al.* 2007).

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D_L \frac{\partial^2 C}{\partial x^2} \quad (1)$$

where C is the concentration of pollution, D_L is the longitudinal dispersion coefficient, u is the mean velocity of river flow. El Kadi Abderrezak *et al.* (2015) and Parsaie and Haghiabi (2015a, b, c, d) investigated the role of D_L in numerical modelling of pollution transmission in rivers. They stated that accurate prediction of D_L has significant effect on numerical simulation; therefore, investigators have used powerful approaches, especially soft-computing techniques, for predicting this parameter. In this regard, the use of artificial neural network (ANN) (Kashefipour and Falconer 2002; Noori *et al.* 2015a), radial basis function neural network (RBF) (Parsaie and Haghiabi 2015a, b, c, d), genetic programming (gp) (Azamathulla and Ghani 2011), support vector machine (SVM) (Azamathulla and Wu 2011), adaptive neuro-fuzzy inference system (ANFIS) (Riahi-Madvar *et al.* 2009; Noori *et al.* 2015a), group method of data handling (GMDH) (Najafzadeh and Sattar 2015), and gene expression programming (GEP) (Sattar and Gharabaghi 2015) have been reported. Based on the reports, accuracy of soft-computing methods is high compared to empirical formulas. Parsaie and Haghiabi (2015a, b, c, d) proposed a novel approach for numerical simulation of pollution transmission in rivers based on using ANN model together with finite volume method (FVM). They used ANN to estimate D_L and FVM for solving ADE. They stated that using soft-computing techniques for predicting internal coefficients of governing differential equations results in an increase in the accuracy of numerical simulation. In this paper, multivariate adaptive regression splines (MARS) is used as a new approach in hydrology for modelling and prediction of D_L . To compare performance of MARS model with other types of soft-computing

techniques, multilayer perceptron neural network (MLP) as a common type of artificial neural network is also developed. To define the most effective parameters on D_L , the Gamma test (GT) was applied. Results of GT are used to evaluate the results of MARS model as well. MARS model was developed by Friedman (1991) and has been used in some fields of water resource management even today. MARS technique has been successfully applied for predicting scour depth at downstream of spillways, river discharge forecasting, rainfall-runoff modelling, etc. (Sharda *et al.* 2008; Zhang and Goh 2014; Samadi *et al.* 2015).

2. Materials and methods

Initially, a review is conducted on the influence of hydraulic and geometric parameters on D_L . Then, using the Buckingham, dimensionless parameters are derived. Later, using the Gamma test, the most effective parameters on D_L are defined. At the end, two soft-computing techniques including ANN and MARS model are developed and compared.

2.1 Relevant parameters and dimensional analysis

Longitudinal dispersion coefficient is proportional to properties of fluid, hydraulic conditions and river geometry (cross-sections and path line). All effective parameters can be written as the following function:

$$D_L = f_1(\rho, \mu, u, u_*, h, w, s_f, s_n) \quad (2)$$

where ρ is fluid density; μ is dynamic viscosity; w is width of cross-section; h is flow-depth; u_* is shear velocity, s_f is longitudinal bed shape and s_n is sinuosity. The Buckingham theory was applied as dimensional analysis approach to derive effective dimensionless parameter on D_L . Derived

Table 1. Empirical equations for estimating longitudinal dispersion coefficient.

Sl. no.	Author	Equation
1	Elder (1959)	$D_L = 5.93hu_*$
2	McQuivey and Keefer (1974)	$D_L = 0.58 \left(\frac{h}{u_*} \right)^2 uw$
3	Fisher (1967)	$D_L = 0.011 \frac{u^2 w^2}{hu_*}$
4	Li <i>et al.</i> (1998)	$D_L = 0.55 \frac{wu_*}{h^2}$
5	Liu (1977)	$D_L = 0.18 \left(\frac{u}{u_*} \right)^{0.5} \left(\frac{w}{h} \right)^2 hu_*$
6	Iwasa and Aya (1991)	$D_L = 2 \left(\frac{w}{h} \right)^{1.5} hu_*$
7	Seo and Cheong (1998)	$D_L = 5.92 \left(\frac{u}{u_*} \right)^{1.43} \left(\frac{w}{h} \right)^{0.62} hu_*$
8	Koussis and Rodriguez-Mirasol (1998)	$D_L = 0.6 \left(\frac{w}{h} \right)^2 hu_*$
9	Li <i>et al.</i> (1998)	$D_L = 5.92 \left(\frac{u}{u_*} \right)^{1.2} \left(\frac{w}{h} \right)^{1.3} hu_*$
10	Kashefipour and Falconer (2002)	$D_L = 2 \left(\frac{u}{u_*} \right)^{0.96} \left(\frac{w}{h} \right)^{1.25} hu_*$
11	Tavakollizadeh and Kashefipour (2007)	$D_L = 7.428 + 1.775 \left(\frac{u}{u_*} \right)^{1.752} \left(\frac{w}{h} \right)^{0.62} hu$
12	Rajeev and Dutta (2009)	$D_L = 10.612 \left(\frac{u}{u_*} \right) hu$

Table 2. Ranges of collected data related to D_L .

Data range	W (m)	H (m)	U (m/s)	U^* (m/s)	D_L (m ² /s)
Min	11.9	0.2	0.0	0.0	1.9
Max	711.2	19.9	1.7	0.6	1486.5
Avg	73.2	1.5	0.5	0.1	115.3
Stdev	106.9	2.3	0.4	0.1	218.7

dimensionless parameters are shown in equation (3) (Seo and Cheong 1998),

$$\frac{D_L}{hu_*} = f_2 \left(\rho \frac{uh}{\mu}, \frac{u}{u_*}, \frac{w}{h}, s_f, s_n \right). \quad (3)$$

Flow-in nature, especially in rivers, is always turbulent. Therefore, the Reynolds number $\rho(uh/\mu)$ can be ignored and bed form and sinusitis path parameters cannot be measured clearly, as well. Therefore, their effects can be considered as flow resistant, which is seen in flow depth. Dimensionless parameters that can be clearly measured are given as below (Seo and Cheong 1998; Seo and Baek 2004):

$$\frac{D_L}{hu_*} = f_2 \left(\frac{u}{u_*}, \frac{w}{h} \right). \quad (4)$$

Developing empirical formulas and soft-computing models are based on these dimensionless parameters. Table 1 presents the most famous empirical formulas, which have been presented for D_L calculation.

Soft-computing models are prepared based on the dataset; therefore, for this purpose, about 116 datasets related to equation (4), were used, the ranges of which are given in table 2 (Zeng and Huai 2014).

2.2 Gamma test (GT)

The Gamma test is used to examine the relationship between inputs and outputs in numerical datasets without the need to construct estimation model. The Gamma test is used for estimating the variance of output before modelling, even though the model is unknown. This error variance estimate presents a target mean squared error that any smooth non-linear function should attain on unseen data. Suppose we have a set of observed data represented by:

$$((x_1, \dots, x_M), y) = (x, y) \quad (5)$$

where vector $X = (x_1, \dots, x_M)$ is the *input*, confined to a closed bounded set $C \in R^M$ and scalar y is the corresponding *output*, without loss of generality. The only assumption made is that relationship of the system is in the following form:

$$y = f(x_1, \dots, x_M) + r \quad (6)$$

where f represents a smooth function and r denotes an indeterminable part, which may be due to real noise lack of functional determination in assumed input/output relationship. The Gamma test is used to return a data-derived estimate for $\text{Var}(r)$ without knowing the underlying function (f) just directly from the data. Estimate of model's output variance called the Gamma statistic represented by Γ , cannot be accounted for by a smooth data model. The Gamma test is derived from the Delta function of input vectors:

$$\delta_M(k) = \frac{1}{M} \sum_{i=1}^M |x_{N[i,k]} - x_i|^2 \quad (7)$$

where $x_{N[i,k]}$ denotes the index of k th nearest neighbour to x_i , and $|\cdot|$ denotes Euclidean distance. Thus, $\delta_M(k)$ is the mean square distance to k th nearest neighbour. The corresponding Gamma function of output values is:

$$\gamma_M(k) = \frac{1}{2M} \sum_{i=1}^M (y_{N[i,k]} - y_i)^2. \quad (8)$$

The Gamma test computes the mean-squared k th nearest neighbour distances $\delta(k)$, ($1 \leq k \leq k_{\text{Max}}$) and the corresponding $\gamma(p)^2$. In order to compute Γ , the best line is constructed for p points ($\delta_M(k)$, $\gamma_M(k)$) and vertical intercept (Γ), is returned as the gamma value. The regression line slope is also returned to show the complexity of model f . V_{ratio} is the standardized results considering $\Gamma/\text{Var}(y)$. It returns a scale invariant noise estimate, which normally lies between zero and one (Noori *et al.* 2009, 2015a, b).

2.3 Artificial Neural Networks (ANNs)

ANN is a nonlinear mathematical model that is able to simulate arbitrarily complex nonlinear processes of any systems that are in terms of inputs and outputs. In many complex mathematical problems that lead to solve complex nonlinear equations, multilayer perceptron networks are common types of ANN widely used in the research. To use MLP model, definition of transferring functions, weights and bias should be considered. Due to the nature of the problem, different activity functions in neurons can be used. An ANN may have one or more hidden layers. Figure 2 demonstrates a three-layer neural network consisting of inputs layer, hidden layer (layers) and outputs layer. As shown in figure 2, w_i is the weight and b_i is the bias for each neuron. Weight and biases' values will be assigned progressively and corrected during training process comparing predicted outputs with

known outputs. Such networks are often trained using back propagation algorithm. In the present study, ANN was trained by Levenberg–Marquardt technique since this technique is more powerful and faster compared to conventional gradient descent technique.

2.4 Multivariate adaptive regression splines (MARS)

MARS technique is a novel approach in soft-computing that applies a series of simple linear regressions. As mentioned in the introduction section, MARS was introduced by the mathematician Friedman (1991). It is a high-precision technique for modelling systems which is based on dataset. This approach separated the computational space into sub-ranges of input variables (predicting parameters) and defined the relationship between input parameters and output variable. In other words, this technique has high ability to characterize the relationship between independent variables and dependent variable in each desired phenomenon. This process was carried out by fitting a simple regression on each input parameter for predicting the output. MARS separates the space of input parameters into various units and then fits a spline function on these units. These elements of regressions are called basic function of MARS methods. One of the main advantages of MARS method is highlighting input parameters which have prominent effect on output parameter. This method could be used for small and big datasets. A basic function that gives information about the relationship between inputs and output parameters is defined as equation (9).

$$h_m(x) = \text{Max}(0, C - x)$$

or

$$h_m(x) = \text{Max}(0, x - C) \quad (9)$$

Figure 2. Sketch of three-layer ANN architecture.

where h is the basic function, x is the input parameter, and C is the threshold value of the independent (input) parameter of x . It is notable that C is introduced in scientific paper via knots and is derived using the least square method. The general form of MARS is introduced as equation (10).

$$Y = f(x) = \beta_0 + \sum_{m=1}^M \beta_m h_m(x) \quad (10)$$

where Y is output parameters, β_0 is constant value, M is the number of functions, $h_m(x)$ is M th basic function and β_m is the corresponding coefficient of $h_m(x)$. Developing MARS includes two steps. In the first step, all basic functions are prepared where overfitting can occur. Therefore, to prevent overfitting in the next step, basic functions, which are less important are pruned with generalized cross-validation (GCV) criteria that is given in equation (11).

$$\text{GCV} = \frac{\frac{1}{n} \sum_{i=1}^n (y_i - f(x_i))^2}{\left(1 - \left(\frac{C(B)}{n}\right)\right)^2} \quad (11)$$

where n denotes the number of observations and $C(B)$ denotes complexity criteria which increase by the number of basic functions (12).

$$C(B) = (B + 1) + dB. \quad (12)$$

2.5 Neural network model development

Developing MLP model as a soft-computing technique is based on the dataset. Therefore, for this purpose, collected dataset was divided into two groups as training and testing. Data selection for ANN model training and testing process was carried out by random approach. The percentage of each group portfolio from total dataset will be determined during model preparation. Designing ANN model structure is more based on designer experience; however, recommendations of investigators who have conducted similar researches were also taken into consideration. Designing ANN model includes a type of neural network model, number of hidden layer(s), number of neurons in each layer, and defining suitable transfer function for neurons of hidden and output layers and learning algorithm.

3. Results and discussion

Performance of empirical formulae was assessed for predicting D_L and is shown in figure 3. For this purpose, three standard error indices including R^2 , root mean square error and mean absolute

error were used. Results of error indices calculation for each empirical formula are given in table 3. As presented in table 3, all empirical formulae have determination of correlation $< 50\%$ and this accuracy is not suitable for practical usage. As seen from table 3, the Tavakollizadeh and Kashefipour formula (2007) is the accurate among empirical formulae.

3.1 Results of Gamma test

In this study, to define the most important effective parameters on D_L , five scenarios were considered. In each scenario, the effect of the input variables was evaluated. In the first scenario, all input variables were used for the GT and in the next scenarios, one of the input variables was removed and again the GT was analyzed. Result of each scenario is given in table 4. The GT parameters including gamma, gradient, standard error and V_{ratio} were chosen as criteria for defining the most important parameters. Scenarios, which had minimum value for the GT parameters showed the most influential input parameters on output parameter. Variation of V_{ratio} is between 0 and 1. The point notable here is that the more this factor is close to zero, the more accurate the related scenario can predict the output regarding related input parameters.

Reviewing table 4, it is realized that scenario number (1) which involves all input variables has minimum value for the GT parameters. Table 4 shows that removing U/u^* causes significant decrease in the Gamma value; therefore, it was found that these parameters are the most important parameters on D_L modelling. Variation of gamma for dataset based on all input variables is given in figure 4. Figure 4 shows that the gamma curves are almost flat after point 100. It means that for modelling discharge coefficient regrading collected dataset, qualification of 100 dataset (85% of all datasets) is sufficient.

3.2 Results of ANN model development

Developing ANN model includes choosing dataset for training and defining portfolio of training and testing groups from total dataset, choosing the number of hidden layer(s), number of neurons in each hidden layer(s), choosing transfer function for neurons and finally choosing learning algorithm. To provide MLP model for predicting D_L , utilities of Matlab software were used. Table 5 presents a summary of trial-and-error process for preparing ANN model. To develop ANN model, 70% of collected dataset was considered for training, 15% for validation and the rest (15%) for testing. Validation of dataset was considered to avoid overlearning. As can be seen from table 5, ANN model, which has 10

Figure 3. Performance of empirical formulas to calculate D_L .

Table 3. Performance of empirical formulae to calculate D_L .

Equation	R^2	RMSE	MAPE
Elder (1959)	0.076	230.05	0.96
McQuivey and Keefer (1974)	0.004	4612990	465028
Fisher (1967)	0.326	1301.03	3.25
Li <i>et al.</i> (1998)	0.056	230.38	0.87
Liu (1977)	0.326	24932.62	60.58
Iwasa and Aya (1991)	0.238	183.83	1.25
Seo and Cheong (1998)	0.383	394.92	4.40
Koussis and Rodriguez-Mirasol (1998)	0.200	312.85	2.81
Li <i>et al.</i> (1998)	0.351	4404.65	24.75
Kashefipour and Falconer (2001)	0.343	525.30	3.51
Tavakollizadeh and Kashefipour (2007)	0.451	5861.86	42.84
Rajeev and Dutta (2009)	0.395	262.20	8.95

Table 4. Results of Gamma test analysis in the absence of one variable.

Row	Inputs	Output	Gammas	Gradient	Standard error	V_{ratio}
1	$h, u^*, U/u^*, W/h$	D_L	0.0935	0.0773	0.0339	0.3740
2	$u^*, U/u^*, W/h$	D_L	0.0929	0.1272	0.0302	0.3714
3	$h, U/u^*, W/h$	D_L	0.0957	0.1477	0.0375	0.3828
4	$h, u^*, W/h$	D_L	0.0028	0.4068	0.0514	0.0113
5	$h, u^*, U/u^*$	D_L	0.0753	0.1716	0.0565	0.3011

Figure 4. Variation of the Gamma test and standard error with unique data points.

neurons with logsig as transfer function in the first layer, is accurate and adding the number of hidden layer(s) and neurons in the layer has no significant effect on increasing model performance.

Performance of ANN model during stages of model preparation (training, validation and testing) is shown in figures 5–7. In figures 5–7, results of ANN model were plotted against measured data. In addition, to understand the performance of ANN model for training and testing data, error distribution was also plotted. Moreover, to define error concentration, error histogram was plotted.

As can be seen from the histogram of error, density of error is aggregated around zero. In overall, the performance of MLP model is more suitable especially when compared to empirical formulae.

3.3 Results of MARS model development

Similar to other types of soft-computing models, developing MARS model is based on the dataset. For this purpose, collected dataset (equation 4) was randomly divided into two groups of training and testing. Eighty five percent of the total data

Table 5. Performance and summary of ANN model during development stage.

Row	N-H-L	F-HL&TF	S-HL&TF	R^2	RMSE	MAE	R^2	RMSE	MAE
1	1	5-tansig	—	0.55	0.286	0.0	0.77	0.04	0.015
2	1	8-tansig	—	0.83	0.084	0.03	0.75	0.03	0.018
3	1	5-logsig	—	0.86	0.04	0.016	0.85	0.05	0.012
4	1	8-logsig	—	0.83	0.046	0.024	0.72	0.052	0.016
5	1	10-logsig	—	0.93	0.044	0.015	0.88	0.003	0.054
6	2	9-logsig	5-tansig	0.93	0.053	0.028	0.86	0.060	0.012
7	2	12-tansig	5-tansig	0.95	0.063	0.035	0.95	0.068	0.015

Figure 5. Performance of MLP model during training stage.

Figure 6. Performance of MLP model during validation stage.

Figure 7. Performance of MLP model during testing stage.

Table 6. The Basic function and related coefficients of MARS model.

Basic function	Equation	Coefficient
$h_1(x)$	$BF1 = \max(0, 0.63651 - u/u^*)$	2.3410
$h_2(x)$	$BF2 = \max(0, h - 0.14655)$	- 9.7300
$h_3(x)$	$BF3 = \max(0, 0.14655 - h)$	16.4736
$h_4(x)$	$BF4 = BF2 * \max(0, u/u^* - 0.7162)$	292.4690
$h_5(x)$	$BF5 = BF2 * \max(0, 0.7162 - u/u^*)$	100.7754
$h_6(x)$	$BF6 = BF1 * \max(0, 0.13793 - h)$	- 102.7943
$h_7(x)$	$BF7 = \max(0, 0.0974 - w)$	2.8615
$h_8(x)$	$BF8 = \max(0, w - 0.24124)$	1.4645
$h_9(x)$	$BF9 = \max(0, 0.24124 - w)$	- 14.3571
$h_{10}(x)$	$BF10 = \max(0, u/u^* - 0.7162) * \max(0, w - 0.24124)$	37.6451
$h_{11}(x)$	$BF11 = \max(0, u/u^* - 0.7162) * \max(0, 0.24124 - w)$	8.2545
$h_{12}(x)$	$BF12 = \max(0, u/u^* - 0.44949)$	- 0.3316
$h_{13}(x)$	$BF13 = BF1 * \max(0, w - 0.065808)$	2.0688
$h_{14}(x)$	$BF14 = BF1 * \max(0, h - 0.1141)$	- 105.5226
$h_{15}(x)$	$BF15 = BF1 * \max(0, 0.1141 - h)$	103.4887
$h_{16}(x)$	$BF16 = BF9 * \max(0, h - 0.03144)$	98.6693
$h_{17}(x)$	$BF17 = BF9 * \max(0, 0.03144 - h)$	- 99.1440
$h_{18}(x)$	$BF18 = BF3 * \max(0, w - 0.067996)$	- 118.6282
$h_{19}(x)$	$BF19 = BF3 * \max(0, 0.067996 - w)$	97.9521

was considered for training and the rest (15%) for testing group. During MARS model preparation, in the first step, 30 basic functions were considered and in the second step (pruning step), 11 basic functions were pruned and finally, optimal MARS model with 19 basic functions was derived. Pruning criteria, which were introduced with GVC parameter in the development of MARS model, were derived equal to 0.0063. As mentioned in the section of a review on MARS model, each basic function has coefficient and constant, which was adjusted in MARS model development process derived using the least square method. The general

form of obtained MARS model is given in equation (13). The extended form of MARS model is given in table 6.

$$D_L = 0.560 + \sum_{m=1}^{19} \beta_m h_m(x). \quad (13)$$

Equation (13) can be used for longitudinal dispersion coefficient in rivers. Considering table 6, u/u^* and h have appeared in almost all basic functions. It means that these two parameters, compared to other parameters, are more effective on D_L . This result from MARS model upholds the

Figure 8. Performance of MARS model during training stage.

Figure 9. Performance of MARS model during testing stage.

results of the GT model. Moreover, by considering the results of assessing performance of empirical formula and MARS model results, it is clear that empirical formula, which has more weight over these formula such as Tavakollizadeh and Kashefipour (2007), is more accurate compared to others. Performance of MARS model during development process (training and testing stages) is given in figures 8 and 9. As can be seen from figures 8 and 9, it is found that performance of MARS model for prediction of D_L is more suitable. Another advantage of developed MARS model is in terms of utility in programming the results of MARS model for developing other river pollution packages.

4. Conclusion

River pollution has become the most important issue in environmental engineering. Dispersion coefficient (D_L) is one of the main parameters in river water quality studies. Recently, by advances in soft-computing techniques in hydrology engineering, investigators attempt to use these models for predicting D_L . Among the soft-computing techniques, multi-layer perceptron neural network

as primary model has been used in most areas of engineering and science, especially in hydrology engineering. Researchers have reported that this model is a black box and does not give any information about internal model process. Recently, by proposing the newest types of soft-computing techniques such as multivariate adaptive regression splines (MARS) in addition to proposing high-performance techniques for predicting engineering phenomena, this model gives full information to designers and readers. In other words, the internal process of MARS model is clear for readers and users. In this paper, developing MARS model for predicting D_L was considered. Results of this research show that MARS model has high performance for predicting D_L and gives full information about internal model process to the readers. Users can easily prepare MARS model for other researches.

References

- Atkinson T and Davis P 2000 Longitudinal dispersion in natural channels: I. Experimental results from the River Severn, UK; *Hydrol. Earth Syst. Sci.* **4**(3) 345–353.

- Azamathulla H and Ghani A 2011 Genetic programming for predicting longitudinal dispersion coefficients in streams; *Water Resour. Manag.* **25**(6) 1537–1544.
- Azamathulla H M and Wu F-C 2011 Support vector machine approach for longitudinal dispersion coefficients in natural streams; *Appl. Soft Comput.* **11**(2) 2902–2905.
- Benedini M and Tsakiris G 2013 Water quality modelling for rivers and streams; Springer Science & Business Media.
- Chao X, Jia Y, Shields F D, Wang S S and Cooper C M 2007 Numerical modeling of water quality and sediment related processes; *Ecol. Modelling* **201**(3) 385–397.
- Davis P and Atkinson T 2000 Longitudinal dispersion in natural channels: 3. An aggregated dead zone model applied to the River Severn, UK; *Hydrol. Earth Syst. Sci.* **4**(3) 373–381.
- Davis P, Atkinson T and Wigley T 2000 Longitudinal dispersion in natural channels: 2. The roles of shear flow dispersion and dead zones in the River Severn, UK; *Hydrol. Earth Syst. Sci.* **4**(3) 355–371.
- Disley T, Gharabaghi B, Mahboubi A A and McBean E A 2015 Predictive equation for longitudinal dispersion coefficient; *Hydrol. Process.* **29**(2) 161–172.
- Elder J W 1959 The dispersion of a marked fluid in turbulent shearflow; *J. Fluid Mech.* **5**(4) 544–560.
- El Kadi Abderrezzak K, Ata R and Zaoui F 2015 One-dimensional numerical modelling of solute transport in streams: The role of longitudinal dispersion coefficient; *J. Hydrol.* **527** 978–989.
- Fisher B H 1967 The mechanics of dispersion in natural streams; *J. Hydraul. Div.* **93**(6) 187–216.
- Friedman J H 1991 Multivariate adaptive regression splines; *The Annals of Statistics* **19** 1–67.
- Iwasa Y and Aya S 1991 Predicting longitudinal dispersion coefficient in open channel flows; In: *Proceedings of International Symposium on Environmental Hydraulics*, Hong Kong, pp. 505–510.
- Ji Z-G 2008 Hydrodynamics and water quality: Modeling rivers, lakes, and estuaries; John Wiley & Sons.
- Kashefipour S M and Falconer R A 2002 Longitudinal dispersion coefficients in natural channels; *Water Res.* **36**(6) 1596–1608, doi: 10.1016/S0043-1354(01)00351-7.
- Koussis A D and Rodriguez-Mirasol J 1998 Hydraulic estimation of dispersion coefficient for streams; *J. Hydraul. Eng.* **124** 317–320.
- Leta O, Shrestha N, de Fraine B, van Griensven A and Bauwens W 2014 Integrated water quality modelling of the river Zenne (Belgium) using open MI; In: *Advances in Hydroinformatics* (eds) P Gourbesville, J Cunge and G Caignaert, Springer Singapore, pp. 259–274.
- Li Z H, Huang J and Li J 1998 Preliminary study on longitudinal dispersion coefficient for the gorges reservoir; In: *Proceedings of the seventh International Symposium Environmental Hydraulics*, 16–18 December, Hong Kong, China.
- Liu H 1977 Predicting dispersion coefficient of stream; *J. Environ. Eng. Div.* **103**(1) 59–69.
- McQuivey R S and Keefer T N 1974 Simple method for predicting dispersion in streams; *J. Environ. Eng. Div.* **100**(4) 997–1011.
- Najafzadeh M and Sattar A A 2015 Neuro-Fuzzy GMDH approach to predict longitudinal dispersion in water networks; *Water Resour. Manag.* **29**(7) 2205–2219.
- Noori R, Karbassi A, Farokhnia A and Dehghani M 2009 Predicting the longitudinal dispersion coefficient using support vector machine and adaptive Neuro-Fuzzy inference system techniques; *Environ. Eng. Sci.* **26**(10) 1503–1510.
- Noori R, Deng Z, Kiaghadi A and Kachooangi F T 2015a How reliable are ANN, ANFIS, and SVM techniques for predicting longitudinal dispersion coefficient in natural rivers? *J. Hydraul. Eng.* 04015039.
- Noori R, Yeh H-D, Abbasi M, Kachooangi F T and Moazami S 2015b Uncertainty analysis of support vector machine for online prediction of five-day biochemical oxygen demand; *J. Hydrol.* **527** 833–843.
- Parsaie A and Haghiabi A 2015a Computational modeling of pollution transmission in rivers; *Appl. Water Sci.*, pp. 1–10, doi: 10.1007/s13201-015-0319-6.
- Parsaie A and Haghiabi A 2015b The effect of predicting discharge coefficient by neural network on increasing the numerical modeling accuracy of flow over side Weir; *Water Resour. Manag.* **29**(4) 973–985.
- Parsaie A and Haghiabi A 2015c Predicting the longitudinal dispersion coefficient by radial basis function neural network; *Modeling Earth Systems and Environment* **1**(4) 1–8.
- Parsaie A and Haghiabi A H 2015d Calculating the longitudinal dispersion coefficient in river. Case study: Severn River, UK; *Int. J. Sci. Res. Environ. Sci.* **3**(5) 199–207.
- Rajeev R S and Dutta S 2009 Prediction of longitudinal dispersion coefficients in natural rivers using genetic algorithm; *Hydrol. Res.* **40**(6) 544–552.
- Riahi-Madvar H, Ayyoubzadeh S A, Khadangi E and Ebadzadeh M M 2009 An expert system for predicting longitudinal dispersion coefficient in natural streams by using ANFIS; *Expert Systems with Applications* **36**(4) 8589–8596.
- Samadi M, Jabbari E, Azamathulla H and Mojallal M 2015 Estimation of scour depth below free overfall spillways using multivariate adaptive regression splines and artificial neural networks; *Engineering Applications of Computational Fluid Mechanics* **9**(1) 291–300.
- Sattar A M A and Gharabaghi B 2015 Gene expression models for prediction of longitudinal dispersion coefficient in streams; *J. Hydrol.* **524** 587–596.
- Seo I and Baek K 2004 Estimation of the longitudinal dispersion coefficient using the velocity profile in natural streams; *J. Hydraul. Eng.* **130**(3) 227–236.
- Seo I and Cheong T 1998 Predicting longitudinal dispersion coefficient in natural streams; *J. Hydraul. Eng.* **124**(1) 25–32.
- Sharda V, Prasher S, Patel R, Ojasvi P and Prakash C 2008 Performance of multivariate adaptive regression splines (MARS) in predicting runoff in mid-Himalayan micro-watersheds with limited data/Performances de régressions par splines multiples et adaptives (MARS) pour la prévision d'écoulement au sein de micro-bassins versants Himalayens d'altitudes intermédiaires avec peu de données; *Hydrol. Sci. J.* **53**(6) 1165–1175.
- Stamou A I 1992 Improving the numerical modeling of river water quality by using high order difference schemes; *Water Res.* **26**(12) 1563–1570.
- Tavakollizadeh A and Kashefipour S M 2007 Effects of dispersion coefficient on quality modeling of surface waters; In: *Proceedings of the sixth International Symposium on River Engineering*, Ahwaz, Iran, 16–18 October 2007, pp. 67–78.
- Wang Z-Y, Lee J W and Melching C 2015 *Water Quality Management. River Dynamics and Integrated River Management*; Springer Berlin-Heidelberg, pp. 555–631.
- Zeng Y and Huai W 2014 Estimation of longitudinal dispersion coefficient in rivers; *J. Hydroenviron. Res.* **8**(1) 2–8.
- Zhang W and Goh A T C 2014 Multivariate adaptive regression splines and neural network models for prediction of pile drivability; *Geosci. Frontiers* **7**(1) 45–52.

MS received 18 December 2015; revised 17 March 2016; accepted 23 March 2016