Effect of irregularity on torsional surface waves in an initially stressed anisotropic porous layer sandwiched between homogeneous and non-homogeneous half-space

ANUP SAHA*, SANTIMOY KUNDU, SHISHIR GUPTA and PRAMOD KUMAR VAISHNAV

Department of Applied Mathematics, Indian School of Mines, Dhanbad 826 004, India.

**Corresponding author. e-mail: sahaanup1989@gmail.com*

The present paper is concerned with the propagation of torsional surface waves in an initially stressed anisotropic porous layer sandwiched between homogeneous and non-homogeneous half-space. We assume the quadratic inhomogeneity in rigidity and density in the lower half-space and irregularity is taken in the form of rectangle at the interface separating the layer from the lower half-space. The dispersion equation for torsional waves has been obtained in a closed form. Velocity equation is also obtained in the absence of irregularity. The study reveals that the presence of irregularity, initial stress, porosity, inhomogeneity and anisotropy factor in the dispersion equation approves the significant effect of these parameters in the propagation of torsional waves in porous medium. It has also been observed that for a uniform media, the velocity equation reduces to the classical result of Love wave.

1. Introduction

The study of surface waves in elastic media is very important due to their devastating damage capabilities during earthquake and for homogeneous, non-homogeneous and layered media, it has been central interest to theoretical seismologists in recent time. Many papers have been published on the subject. Some valuable information about the propagation of seismic waves is available in Ewing *et al.* (1957). The propagation of Love waves in water-saturated soil underlain by a heterogeneous elastic medium has been discussed by Chakraborty and Dey (1982). Dey *et al.* (1996) studied propagation of Love waves in heterogeneous crust over a heterogeneous mantle. Chattopadhyay *et al.* (2011a) investigated the propagation of torsional surface waves in an inhomogeneous layer over an inhomogeneous half-space. Georgiadis *et al.* (2000) showed the existence of torsional surface waves in

a gradient-elastic half space whereas Davini *et al.* (2008) studied the propagation of torsional waves in a thin rectangular domain using asymptotic approach. Gupta *et al.* (2012a) pointed that in a homogeneous layer over a heterogeneous half-space torsional waves do exist. The commendable works by Islam *et al.* (2014), Chattopadhyay *et al.* (2012c, 2013b), Kalyani *et al.* (2008), Pal and Mandal (2014) in the study of seismic waves may be cited.

The Earth is considered as an initially stressed body with a layered structure. These initial stresses exist due to various reasons, viz., temperature, atmospheric pressure, gravity variation, slow process of creep, etc. The presence of initial stress has remarkable effect on the phase velocity of surface wave. Gupta *et al.* (2013b) discussed the propagation of torsional surface waves in an inhomogeneous layer over an initially stressed inhomogeneous half-space. Kepceler (2010) examined and found the existence of torsional wave dispersion relations in a

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pre-stressed bi-material compounded cylinder with an imperfect interface. Ahmed and Abo-Dahab (2010) pointed out the existence of Love waves in an orthotropic granular layer under initial stress overlying a semi-infinite granular medium. References can be made to Kundu *et al.* (2014a, b, c), Manna *et al.* (2013), Sharma and Garg (2006), Ozturk and Akbarov (2009), Vishwakarma and Gupta (2013) for their excellent contributions in investigating seismic waves in various media under various circumstances.

Recently, the phenomenon of wave propagation in fluid-saturated porous media has been attracting the attention of seismologists and physicists. The existence of porosity is a common happening in crustal and reservoir rocks in the Earth. Such existence may play a crucial role on the phase velocity of surface waves. Ghorai *et al.* (2010) discussed the problem of Love waves in a fluid-saturated porous layer under a rigid boundary and lying over an elastic half-space under gravity. Chattaraj *et al.* (2011b) formulated the dispersion equation of the propagation of torsional surface wave in anisotropic poroelastic medium under initial stress. Gupta *et al.* (2013c) delivered the idea of introducing propagation of Love wave in a porous layer under the effect of linearly varying directional rigidities. The works done by Son and Kang (2012), Chattaraj *et al.* (2014a), Akbarov *et al.* (2011), Wang and Zhang (1998), and Arora and Tomar (2007) may also be cited.

In the present paper, we study the propagation of torsional surface waves in an initially stressed anisotropic porous layer sandwiched between homogeneous and non-homogeneous half-space. The irregularity has been taken at the lower interface and the rigidity and density in the lower half-space have been assumed to vary quadratically. We have observed that torsional surface wave exists in such porous media. The study shows that the irregularity as well as inhomogeneity, anisotropy, initial stress and porosity have significant effects in the propagation of torsional waves. For graphical representation, MATLAB software has been used to generate results. It has also been observed that if the upper or lower half-space is neglected, the dispersion equation for homogeneous layer reduces to the classical result of Love wave.

2. Formulation of the problem

For the study of torsional surface waves, we consider a cylindrical coordinate system. The model consists of an anisotropic porous layer of thickness H under compressive initial stresses $P = -\tau_{rr}$ along radial direction lying between a homogeneous and a non-homogeneous half-space with one

rectangular irregularity at the lower interface. The non-homogeneity is assumed both in rigidity and mass density. We consider the irregularity of depth h and length $2m$.

The origin of the cylindrical coordinate system (r, θ, z) is located at the interface separating the layer from the lower half-space and z is directed downwards. The following variations in rigidity and density are assumed into account.

We considered the transformations for the upper half-space,

$$\mu = \mu_0, \quad \rho = \rho_0. \quad (1)$$

For the porous layer,

$$\mu = \mu_1, \quad \rho = \rho_1. \quad (2)$$

For lower half-space,

$$\mu = \mu_2 (1 + az)^2, \quad \rho = \rho_2 (1 + bz)^2, \\ a > 0, \quad b > 0, \quad (3)$$

where μ and ρ are rigidity and density of the media respectively and a, b are constants having dimension that are inverses of length.

We consider the equation of irregularity as:

$$z = \varepsilon F(r), \quad (4)$$

where the function $F(r)$ and irregularity ε are defined in Appendix.

3. Solution for half-spaces

If r and θ be radial and circumferential coordinates respectively and if the wave travels along the radial direction only, the dynamical equations of motion may be written as (Love 1927):

$$\left. \begin{aligned} \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} &= \rho \frac{\partial^2 u}{\partial t^2}, \\ \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2\sigma_{r\theta}}{r} &= \rho \frac{\partial^2 v}{\partial t^2}, \\ \frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} &= \rho \frac{\partial^2 w}{\partial t^2}, \end{aligned} \right\} \quad (5)$$

where $\sigma_{rr}, \sigma_{r\theta}, \sigma_{rz}, \sigma_{\theta\theta}, \sigma_{\theta z}, \sigma_{zz}$ are the respective stress components and u, v, w are the respective displacement components.

Now the stress-strain relations are given by

$$\sigma_{ij} = \lambda \Omega \delta_{ij} + 2\mu e_{ij}, \quad (6)$$

where λ and μ are Lamé's constants, $e_{ii} = \Omega$ and e_{ij} is defined in Appendix.

Now for the propagation of torsional surface wave in radial direction and causing displacement in azimuthal direction only, we have the following displacement components:

$$u = 0, \quad w = 0, \quad v = v(r, z, t). \quad (7)$$

Using equations (6 and 7), we have the only non-vanishing equation of motion without body force as:

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2\sigma_{r\theta}}{r} = \rho \frac{\partial^2 v}{\partial t^2}. \quad (8)$$

For an elastic medium, the stresses are related to the displacement by

$$\sigma_{r\theta} = \mu \left(\frac{\partial v}{\partial r} - \frac{v}{r} \right), \quad \sigma_{z\theta} = \mu \frac{\partial v}{\partial z}. \quad (9)$$

Using equation (9), equation (8) reduces to

$$\begin{aligned} \mu(z) \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} \right) v + \frac{\partial}{\partial z} \left(\mu(z) \frac{\partial v}{\partial z} \right) \\ = \rho(z) \frac{\partial^2 v}{\partial t^2}. \end{aligned} \quad (10)$$

For wave changing harmonically in time and propagating along radial direction, we have:

$$v = V(z) J_1(kr) e^{i\omega t} \quad (11)$$

where k is the wave number, ω is the angular frequency and $J_1(kr)$ is the Bessel's function of the first order and first kind and thus the equation (11) takes the form

$$\frac{d^2 V}{dz^2} + \frac{\mu'(z)}{\mu(z)} \frac{dV}{dz} - k^2 \left(1 - \frac{c^2}{c_s^2} \right) V(z) = 0, \quad (12)$$

where c_s is the shear wave velocity and c is the torsional wave velocity which are defined in Appendix.

3.1 Solution for upper half-space

In the upper half-space, we considered

$$\mu = \mu_0, \quad \rho = \rho_0. \quad (13)$$

Using equation (13), equation (12) reduces to

$$\frac{d^2 V}{dz^2} - m_0^2 V(z) = 0, \quad (14)$$

where m_0^2 and the shear wave velocity c_0 are defined in Appendix. The solution of equation (14) which vanishes at $z \rightarrow -\infty$ may be written as:

$$V(z) = D_1 e^{m_0 z}.$$

Hence, the displacement component in the homogeneous upper half-space is

$$v = v_0 \text{ (say)} = D_1 e^{m_0 z} J_1(kr) e^{i\omega t}. \quad (15)$$

3.2 Solution for lower half-space

The heterogeneity of the lower half-space is characterized by

$$\mu = \mu_2(1 + az)^2, \quad \rho = \rho_2(1 + bz)^2 \quad (16)$$

With the help of (16), the equation of motion (12) takes the form

$$\frac{d^2 V}{dz^2} + \frac{2a}{1 + az} \frac{dV}{dz} - k^2 \left(1 - \frac{c^2}{c_2^2} \frac{1 + 2bz}{1 + 2az} \right) V(z) = 0, \quad (17)$$

where c_2 is the shear wave velocity in the lower half-space.

On substituting $V(z) = (\phi(z)/(1 + az))$ into (17) to eliminate the term (dV/dz) , we get:

$$\frac{d^2 \phi}{dz^2} - k^2 \left(1 - \frac{c^2}{c_2^2} \frac{1 + 2bz}{1 + 2az} \right) \phi(z) = 0. \quad (18)$$

Now introducing dimensionless quantities $\gamma = \sqrt{1 - \frac{bc^2}{ac_2^2}}$ and $\eta = \frac{\gamma k(1 + 2az)}{a}$ in equation (18), it reduces to

$$\frac{d^2 \phi}{d\eta^2} + \left(-\frac{1}{4} + \frac{R}{\eta} \right) \phi(\eta) = 0, \quad (19)$$

where R is defined in Appendix.

The solution of equation (19) may be written as:

$$\phi(\eta) = D_4 W_{R, \frac{1}{2}}(\eta) + D_5 W_{R, -\frac{1}{2}}(-\eta),$$

where $W_{R, \frac{1}{2}}(\eta)$ is the Whittaker function (Whittaker and Watson 1990). Since this solution vanishes at $z \rightarrow \infty$, i.e., for $\eta \rightarrow \infty$, it may be written as:

$$\phi(\eta) = D_4 W_{R, \frac{1}{2}}(\eta).$$

Expanding the Whittaker's function upto linear terms we get the displacement component for the torsional wave in the lower half-space as:

$$\begin{aligned} v = v_2 \text{ (say)} &= \frac{D_4}{1 + az} e^{-\frac{\gamma k(1 + 2az)}{2a}} \frac{\gamma k(1 + 2az)}{a} \\ &\times \left[1 + \frac{\gamma k A}{a} (1 + 2az) \right] J_1(kr) e^{i\omega t}, \end{aligned} \quad (20)$$

where A is defined in Appendix.

4. Solution for layer

The dynamical equations of motion in an anisotropic porous medium under the effect of initial stress P without body force are (Biot 1965)

$$\left. \begin{aligned} &\frac{\partial \tau_{rr}}{\partial r} + \frac{1}{r^2} \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} - P \frac{\partial w'_\theta}{\partial z} \\ &= \frac{\partial^2}{\partial t^2} (\rho_{rr} u + \rho_{r\theta} U), \\ &\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r^2} \frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{2\tau_{r\theta}}{r} - P \frac{\partial w'_z}{\partial r} \\ &= \frac{\partial^2}{\partial t^2} (\rho_{rr} v + \rho_{r\theta} V), \\ &\frac{\partial \tau_{rz}}{\partial r} + \frac{1}{r} \frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} + \frac{\tau_{rz}}{r} - P \frac{\partial w'_\theta}{\partial r} \\ &= \frac{\partial^2}{\partial t^2} (\rho_{rr} w + \rho_{r\theta} W), \end{aligned} \right\} \quad (21)$$

and

$$\left. \begin{aligned} \frac{\partial \tau}{\partial r} &= \frac{\partial^2}{\partial t^2} (\rho_{r\theta} u + \rho_{\theta\theta} U), \\ \frac{\partial \tau}{\partial \theta} &= \frac{\partial^2}{\partial t^2} (\rho_{r\theta} v + \rho_{\theta\theta} V), \\ \frac{\partial \tau}{\partial z} &= \frac{\partial^2}{\partial t^2} (\rho_{r\theta} w + \rho_{\theta\theta} W), \end{aligned} \right\} \quad (22)$$

where τ_{rr} , $\tau_{\theta\theta}$, τ_{zz} , τ_{rz} , $\tau_{r\theta}$ and $\tau_{\theta z}$ are the respective stress components, (u, v, w) are the respective displacement components of the solid, (U, V, W) are the displacement components of the liquid and τ is the stress in the liquid,

$$\begin{aligned} w'_r &= \frac{1}{2r} \left(\frac{\partial w}{\partial \theta} - r \frac{\partial v}{\partial z} \right), \\ w'_\theta &= \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial r} \right), \\ w'_z &= \frac{1}{2r} \left(\frac{\partial(rv)}{\partial r} - \frac{\partial v}{\partial \theta} \right) \end{aligned} \quad (23)$$

are the components of the rotational vector w' .

The stress-strain relations for porous layer are given by:

$$\left. \begin{aligned} \tau_{rr} &= (A + P)e_{rr} + (A - 2N + P)e_{\theta\theta} \\ &\quad + (F + P)e_{zz} + Q_\varepsilon, \\ \tau_{\theta\theta} &= (A - 2N)e_{rr} + Ae_{\theta\theta} + Fe_{zz} + Q_\varepsilon, \\ \tau_{zz} &= Fe_{rr} + Fe_{\theta\theta} + Ce_{zz} + Q_\varepsilon, \\ \tau_{r\theta} &= 2Ne_{r\theta}, \quad \tau_{\theta z} = 2Ge_{\theta z}, \quad \tau_{rz} = 2Ge_{zr}, \end{aligned} \right\} \quad (24)$$

where A , F , C are elastic constants of the medium, N and G are the shear moduli along r and z directions respectively and Q_ε is the measure of the coupling between the change of volume of the solid and liquid. The relation between stress vector τ and the fluid pressure P' is

$$-\tau = fP', \quad (25)$$

where f = porosity of the layer. The mass coefficients ρ_{rr} , $\rho_{r\theta}$ and $\rho_{\theta\theta}$ are related to the densities ρ , ρ_s and ρ_w of the layer, the solid and the water respectively by

$$\begin{aligned} \rho_{rr} + \rho_{r\theta} &= (1 - f)\rho_s, \\ \rho_{r\theta} + \rho_{\theta\theta} &= f\rho_w. \end{aligned} \quad (26)$$

Hence, the mass density of the aggregate is

$$\rho' = \rho_s + f(\rho_w - \rho_s). \quad (27)$$

Biot has shown that the following inequalities hold for the mass coefficients

$$\rho_{rr} > 0, \quad \rho_{\theta\theta} > 0, \quad \rho_{r\theta} < 0, \quad \rho_{rr}\rho_{\theta\theta} - \rho_{r\theta}^2 > 0. \quad (28)$$

The torsional wave is characterized by:

$$\begin{aligned} u &= 0, \quad w = 0, \quad v = v(r, z, t), \\ U &= 0, \quad W = 0, \quad V = V(r, z, t) \end{aligned} \quad (29)$$

which give the strain components as:

$$\left. \begin{aligned} e_{rr} &= 0, \quad e_{\theta\theta} = 0, \\ e_{zz} &= 0, \quad e_{zr} = 0, \\ 2e_{\theta z} &= \frac{\partial v}{\partial z}, \\ 2e_{r\theta} &= \frac{\partial v}{\partial r} - \frac{v}{r}. \end{aligned} \right\} \quad (30)$$

Using the equations (29) and (30) in equation (24), we get the following two non-zero stress components

$$\tau_{\theta z} = G \frac{\partial v}{\partial z}, \quad \tau_{r\theta} = N \left(\frac{\partial v}{\partial r} - \frac{v}{r} \right). \quad (31)$$

Inserting the relations (31) in equations (21 and 22) one gets

$$\begin{aligned} \left(N - \frac{P}{2} \right) \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} \right) + G \frac{\partial^2 v}{\partial z^2} \\ = \frac{\partial^2}{\partial t^2} (\rho_{rr} v + \rho_{r\theta} V) \end{aligned} \quad (32)$$

and

$$\frac{\partial^2}{\partial t^2} (\rho_{r\theta} v + \rho_{\theta\theta} V) = 0. \quad (33)$$

The last equation may be written as:

$$\rho_{r\theta} v + \rho_{\theta\theta} V = k_1 \text{ (say)}. \quad (34)$$

Eliminating V from equations (32 and 34), one gets

$$\left(N - \frac{P}{2} \right) \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} \right) + G \frac{\partial^2 v}{\partial z^2} = d' \frac{\partial^2 v}{\partial t^2} \quad (35)$$

where d' is defined in the Appendix.

Equation (35) shows that shear wave velocity along the radial direction is

$$\sqrt{\frac{N - \frac{P}{2}}{d'}} = \sqrt{\frac{1 - \zeta}{d}} c_1^2,$$

where c_1 , ζ and d are defined in the Appendix, $\gamma_{11} = \frac{\rho_{rr}}{\rho'}$, $\gamma_{12} = \frac{\rho_{r\theta}}{\rho'}$ and $\gamma_{22} = \frac{\rho_{\theta\theta}}{\rho'}$ are dimensionless parameters for the material of the porous layer.

Again consider the solution of equation (35) is of the form

$$v = V(z) J_1(kr) e^{i\omega t}, \quad (36)$$

equation (35) with the help of (36) takes the form

$$\frac{d^2 V}{dz^2} + q^2 V = 0, \quad (37)$$

where q and p are defined in Appendix. The solution of (37) is given by

$$V(z) = D_2 \cos(qz) + D_3 \sin(qz).$$

Hence, the displacement component in the initially stressed anisotropic porous layer is

$$v = v_1 (\text{say}) = [D_2 \cos(qz) + D_3 \sin(qz)] J_1(kr) e^{i\omega t}. \quad (38)$$

5. Boundary conditions

The following boundary conditions must be satisfied

- (1) At the interface of the layer and the upper half-space, the displacement component and stress component are continuous, i.e.,

$$(i) \quad v_0 = v_1 \quad \text{at } z = -H. \quad (39)$$

$$(ii) \quad \mu_0 \frac{\partial v_0}{\partial z} = \mu_1 \frac{\partial v_1}{\partial z} \quad \text{at } z = -H. \quad (40)$$

- (2) At the interface irregularity of the layer and the lower half-space the displacement component and the stress component are continuous, i.e.,

$$(i) \quad v_1 = v_2 \quad \text{at } z = \varepsilon F(r). \quad (41)$$

$$(ii) \quad (t^{(n)})_1 = (t^{(n)})_2 \quad \text{at } z = \varepsilon F(r) \quad (42)$$

where $t^{(n)} = \tau_{\theta r} n_r + \tau_{\theta z} n_z$. Since the irregularity surface is flat, the above boundary condition reduces to

$$(\tau_{\theta z})_1 = (\tau_{\theta z})_2 \quad \text{at } z = \varepsilon F(r).$$

where $(\tau_{\theta z})_1, (\tau_{\theta z})_2$ are defined in the Appendix, $(l, 0, n)$ are the components of the unit normal to the interface at $z = 0$.

$$l = 0, \quad n = 1 \quad \text{at } z = -H$$

$$l = -\frac{\varepsilon F'}{\sqrt{1 + \varepsilon^2 F'^2}},$$

$$n = \frac{1}{\sqrt{1 + \varepsilon^2 F'^2}} \quad \text{at } z = \varepsilon F(r)$$

where $F' = \frac{dF}{dr}$. Using equations (15), (20) and (38) in the above four boundary conditions, the following equations have been obtained

$$D_1 e^{-m_0 H} - D_2 \cos(qH) + D_3 \sin(qH) = 0, \quad (43)$$

$$D_1 [\mu_0 m_0 e^{-m_0 H}] - D_2 [\mu_1 q \sin(qH)] - D_3 [\mu_1 q \cos(qH)] = 0, \quad (44)$$

$$D_2 \cos(q\varepsilon F(r)) + D_3 \sin(q\varepsilon F(r)) - D_4 A_1 = 0, \quad (45)$$

$$D_2 [-\mu_1 q \sin(q\varepsilon F(r))] + D_3 [\mu_1 q \cos(q\varepsilon F(r))] - D_4 [\mu_2 A_2] = 0, \quad (46)$$

where A_1 and A_2 are defined in the Appendix.

Eliminating the arbitrary constants D_1, D_2, D_3 and D_4 from the above four equations we get:

$$\begin{aligned} \tan(qH) = & [\mu_1 q \{ \mu_2 A_2 \cos(q\varepsilon F(r)) \\ & + \mu_1 q A_1 \sin(q\varepsilon F(r)) \} \\ & + \mu_0 m_0 \{ \mu_2 A_2 \sin(q\varepsilon F(r)) \\ & - \mu_1 q A_1 \cos(q\varepsilon F(r)) \}] \\ & / [\mu_1 q \{ \mu_2 A_2 \sin(q\varepsilon F(r)) \\ & - \mu_1 q A_1 \cos(q\varepsilon F(r)) \} \\ & - \mu_0 m_0 \{ \mu_2 A_2 \cos(q\varepsilon F(r)) \\ & + \mu_1 q A_1 \sin(q\varepsilon F(r)) \}], \end{aligned} \quad (47)$$

which is the dispersion equation giving the velocity of torsional surface wave in an initially stressed porous layer sandwiched between a homogeneous and a non-homogeneous half-space under the effect of irregularity.

6. Particular cases

If the medium is non-porous, i.e., $f \rightarrow 0$, then $\rho_s \rightarrow \rho'$ which implies $\gamma_{11} + \gamma_{12} \rightarrow 1$ and $\gamma_{12} + \gamma_{22} \rightarrow 0$ and so $d = \gamma_{11} - \frac{\gamma_{12}^2}{\gamma_{22}} \rightarrow 1$.

Again if $f \rightarrow 1$, then $\rho_w \rightarrow \rho'$ and the medium becomes fluid. In this case $d \rightarrow 0$ and so shear wave velocity in the medium does not exist.

Case I: If there is no irregularity at the lower interface, i.e., $\varepsilon \rightarrow 0$, the dispersion equation (47) becomes

$$\tan(qH) = \frac{\frac{q}{k} \left[\mu_0 \mu_1 \left(\frac{m_0}{k} \right) - \mu_1 \mu_2 \left(\frac{A_2}{A_1 k} \right) \right]}{\mu_1^2 \left(\frac{q^2}{k^2} \right) + \mu_0 \mu_2 \left(\frac{m_0}{k} \right) \left(\frac{A_2}{A_1 k} \right)}$$

where $(A_2/A_1 k)$ is defined in the Appendix.

This is the dispersion equation torsional surface waves in an initially stressed anisotropic porous layer sandwiched between a homogeneous and a non-homogeneous half-space.

Case II: If $\varepsilon \rightarrow 0$ and $a \rightarrow 0, b \rightarrow 0$, i.e., the lower half-space is homogeneous, then velocity equation (47) of torsional surface wave reduces to

$$\tan(qH) = \frac{\frac{q}{k} \left[\mu_0 \mu_1 \left(\frac{m_0}{k} \right) + \gamma_1 \mu_1 \mu_2 \right]}{\mu_1^2 \left(\frac{q^2}{k^2} \right) - \gamma_1 \mu_0 \mu_2 \left(\frac{m_0}{k} \right)},$$

where γ_1 is defined in the Appendix.

Case III: If $a \rightarrow 0$, $b \rightarrow 0$ and $P \neq 0$, i.e., in case of initially stressed anisotropic porous layer sandwiched between two homogeneous half-spaces under the effect of irregularity, the dispersion equation takes the form

$$\begin{aligned} \tan(qH) = & \left[\mu_1 q \left\{ -\mu_2 \gamma_1 \cos(q\varepsilon F(r)) \right. \right. \\ & + \mu_1 \frac{q}{k} \sin(q\varepsilon F(r)) \left. \right\} \\ & + \mu_0 m_0 \left\{ -\mu_2 \gamma_1 \sin(q\varepsilon F(r)) \right. \\ & \left. \left. - \mu_1 \frac{q}{k} \cos(q\varepsilon F(r)) \right\} \right] \\ & / \left[\mu_1 q \left\{ -\mu_2 \gamma_1 \sin(q\varepsilon F(r)) \right. \right. \\ & \left. \left. - \mu_1 \frac{q}{k} \cos(q\varepsilon F(r)) \right\} \right. \\ & \left. - \mu_0 m_0 \left\{ -\mu_2 \gamma_1 \cos(q\varepsilon F(r)) \right. \right. \\ & \left. \left. + \mu_1 \frac{q}{k} \sin(q\varepsilon F(r)) \right\} \right]. \end{aligned}$$

Case IV: If $a \rightarrow 0$, $b \rightarrow 0$ and $P = 0$, i.e., if both the half-spaces are homogeneous and there is no initial stress in the layer, then we get the velocity equation of torsional wave as:

$$\begin{aligned} \tan(q_1 H) = & \left[\mu_1 q_1 \left\{ -\mu_2 \gamma_1 \cos(q_1 \varepsilon F(r)) \right. \right. \\ & + \mu_1 \frac{q_1}{k} \sin(q_1 \varepsilon F(r)) \left. \right\} \\ & + \mu_0 m_0 \left\{ -\mu_2 \gamma_1 \sin(q_1 \varepsilon F(r)) \right. \\ & \left. \left. - \mu_1 \frac{q_1}{k} \cos(q_1 \varepsilon F(r)) \right\} \right] \\ & / \left[\mu_1 q_1 \left\{ -\mu_2 \gamma_1 \sin(q_1 \varepsilon F(r)) \right. \right. \\ & \left. \left. - \mu_1 \frac{q_1}{k} \cos(q_1 \varepsilon F(r)) \right\} \right. \\ & \left. - \mu_0 m_0 \left\{ -\mu_2 \gamma_1 \cos(q_1 \varepsilon F(r)) \right. \right. \\ & \left. \left. + \mu_1 \frac{q_1}{k} \sin(q_1 \varepsilon F(r)) \right\} \right]. \end{aligned}$$

where q_1 is defined in the Appendix.

Case V: If $a \rightarrow 0$, $b \rightarrow 0$, $\varepsilon \rightarrow 0$, $\zeta \rightarrow 0$, $d \rightarrow 1$ and $N = G$, i.e., if both the half-spaces are homogeneous and the layer is isotropic, non-porous and free from initial stress, then the dispersion equation (47) in the absence of irregularity reduces to

$$\tan \left[kH \sqrt{\frac{c^2}{c_1^2} - 1} \right] = \frac{\mu_0 \mu_1 q_2 m_0 + \mu_1 \mu_2 q_2 k \gamma_1}{\mu_1^2 q_2^2 - \mu_0 \mu_2 m_0 k \gamma_1}, \quad (48)$$

where q_2 , m_0 are defined in the Appendix.

Case VI: Now if we neglect the upper half-space, the dispersion equation (48) takes the form

$$\tan \left[kH \sqrt{\frac{c^2}{c_1^2} - 1} \right] = \frac{\mu_2 \sqrt{1 - \frac{c^2}{c_2^2}}}{\mu_1 \sqrt{\frac{c^2}{c_1^2} - 1}}. \quad (49)$$

Case VII: Again if we neglect the lower half-space, the dispersion equation (48) reduces to

$$\tan \left[kH \sqrt{\frac{c^2}{c_1^2} - 1} \right] = \frac{\mu_0 \sqrt{1 - \frac{c^2}{c_0^2}}}{\mu_1 \sqrt{\frac{c^2}{c_1^2} - 1}}. \quad (50)$$

Equations (49) and (50) are the well known classical result of Love wave and hence validating the solution of the problem is discussed.

7. Numerical results and discussion

Based on the dispersion equation (47), numerical computations are performed to demonstrate the effect for different values of $\frac{a}{k}$, $\frac{b}{k}$, $\frac{h}{H}$, d , $\frac{N}{G}$, ζ and ε on the propagation of torsional waves in the porous layer sandwiched between a homogeneous and a non-homogeneous half-space. The numerical data has been taken from (Gubbins 1990). In the figures, the variation of $\frac{c}{c_1}$ against kH for different values of those is displayed. The values of $(\mu_1/\mu_0) = 0.2$, $(\mu_2/\mu_0) = 0.8064$, $(c_1^2/c_0^2) = 0.2$ and $(c_1^2/c_2^2) = 0.4$ have been kept fixed in all the figures. From all these figures it has been noticed that the phase velocity (c/c_1) increases with decreases in

Figure 1. Geometry of the problem.

Table 1. Values of various dimensionless parameters.

	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6	Figure 7	Figure 8	Figure 9
a/k	—	0.1	0.1	0.1	0	0.1	0.1	0.1
b/k	0.4	—	0.4	0.4	0.4	0.4	0.4	0.4
h/H	0.3	0.3	—	0.3	0.3	0.3	0.3	0.3
d	0.6	0.6	0.6	—	0.6	0.6	0.6	0.6
N/G	2	2	2	2	—	2	2	2
ζ	0.4	0.4	0.4	0.4	0.4	—	—	0.4
ε	0.01	0.01	0.01	0.01	0.01	0.01	0.01	—

Figure 2. Effect of inhomogeneity parameter (a/k) associated with rigidity of the lower half-space on the propagation of torsional surface wave for $(b/k) = 0.4$, $(h/H) = 0.3$, $d = 0.6$, $(N/G) = 2$, $\zeta = 0.4$, $\varepsilon = 0.01$.

Figure 3. Effect of inhomogeneity parameter (b/k) associated with density of the lower half-space on the propagation of torsional surface wave for $(a/k) = 0.1$, $(h/H) = 0.3$, $d = 0.6$, $(N/G) = 2$, $\zeta = 0.4$, $\varepsilon = 0.01$.

dimensionless wave number kH under the assumed condition (figure 1 and table 1).

Figure 2 shows the effect of inhomogeneity parameter (a/k) involved in the rigidity of the lower half-space on the torsional wave. The value of (a/k) for curves 1, 2, 3 and 4 has been taken as 0.1, 0.2, 0.3 and 0.4 respectively. From this figure it has been found that for a particular dimensionless wave number kH , the dimensionless phase velocity (c/c_1) increases as the value of (a/k) increases.

In figure 3, study has been made to get the effect of inhomogeneity parameter (b/k). For curves 1, 2 and 3, the value of (b/k) has been taken as 0.02, 0.04 and 0.06 respectively. It has been observed that an increase in dimensionless parameter (b/k) in the lower half-space decreases the phase velocity for the same frequency.

Figure 4 manifests the effect of the ratio (h/H) on the propagation of torsional wave in the porous medium. The value of the ratio (h/H) has been taken as 1, 3, 5 and 7 for curves 1, 2, 3 and 4 respectively. From this figure, as the dimensionless ratio (h/H) increases, it has been observed that the phase velocity decreases at the same frequency.

Figure 4. Torsional wave dispersion curves at different sizes of irregularity (h/H) for $(a/k) = 0.1$, $(b/k) = 0.4$, $d = 0.6$, $(N/G) = 2$, $\zeta = 0.4$, $\varepsilon = 0.01$.

Figure 5 gives a variation of velocity of torsional surface waves for the variation of porosity d . For curves 1, 2, 3 and 4, the value of d has been taken as

0.50, 0.52, 0.54 and 0.56 respectively. It says that as the porosity factor increases the phase velocity decreases.

Figure 6 gives the variation of the phase velocity of torsional surface wave against dimensionless wave number kH for different values of anisotropic factor (N/G). The value of (N/G) for curves 1, 2, 3 and 4 has been taken as 1.00, 1.05, 1.10 and 1.15, respectively. It is observed that the phase velocity of torsional wave decreases with an increase in anisotropic factor (N/G).

Figure 7 discusses the influence of compressive initial stress $\zeta(>0)$ in the porous layer. The value of ζ for curves 1, 2, 3 and 4 has been taken as 0.0, 0.2, 0.4 and 0.6 respectively. The figure shows that the

Figure 5. Velocity dispersion curves of torsional surface wave in a porous layer for various porosity in terms of parameter d and for $(a/k) = 0.1$, $(b/k) = 0.4$, $(h/H) = 0.3$, $(N/G) = 2$, $\zeta = 0.4$, $\varepsilon = 0.01$.

Figure 6. Effect of anisotropy (N/G) in the porous layer on the propagation of torsional surface wave for $(a/k) = 0$, $(b/k) = 0.4$, $(h/H) = 0.3$, $d = 0.6$, $\zeta = 0.4$, $\varepsilon = 0.01$.

effect of compressive initial stress is very prominent on the propagation of torsional surface waves. The presence of such stress reduces the phase velocity.

In figure 8, the effect of tensile initial stress $\zeta(<0)$ on torsional wave propagation has been shown. For curves 1, 2, 3 and 4, the value of ζ has been taken as -0.2 , -0.4 , -0.6 and -0.8 respectively. It also shows that such stress has remarkable effect on torsional wave propagation. This figure confirms that the phase velocity is reduced by the presence of tensile initial stress.

Figure 9 discusses the effect of different sizes of irregularity parameter ε . The value of ε for curves 1, 2, 3 and 4 has been taken as 0.01, 0.05, 0.09 and 0.13 respectively. From this figure we have noticed

Figure 7. Effect of compressive initial stress ζ in the porous layer on the propagation of torsional surface wave for $(a/k) = 0.1$, $(b/k) = 0.4$, $(h/H) = 0.3$, $d = 0.6$, $(N/G) = 2$, $\varepsilon = 0.01$.

Figure 8. Effect of tensile initial stress ζ in the porous layer on the propagation of torsional surface wave for $(a/k) = 0.1$, $(b/k) = 0.4$, $(h/H) = 0.3$, $d = 0.6$, $(N/G) = 2$, $\varepsilon = 0.01$.

Figure 9. Torsional wave dispersion curves at different sizes of irregularity ε for $(a/k) = 0.1$, $(b/k) = 0.4$, $(h/H) = 0.3$, $d = 0.6$, $(N/G) = 2$, $\zeta = 0.4$.

that the irregularity parameter has notable effect on the propagation of torsional wave. As the value of ε increases, the phase velocity decreases at a particular frequency.

8. Conclusions

Torsional surface wave propagation in an initially stressed porous layer sandwiched between a homogeneous and a non-homogeneous half-space has been studied and the dispersion equation is obtained analytically. In the absence of the upper or lower half-space, it has been observed that for a homogeneous layer the dispersion equation reduces to the result of Love wave. Effect of non-homogeneity, compressive and tensile initial stress, porosity, anisotropic factor and irregularity on the dimensionless phase velocity has been shown graphically. We have observed that

- Dimensionless phase velocity (c/c_1) of torsional surface wave increases with the decreases of dimensionless wave number kH in all the figures.
- In the liquid layer, torsional surface wave does not exist.
- The phase velocity of the torsional wave increases when the inhomogeneity parameter (a/k) increases.
- The phase velocity increases with the decreases of dimensionless parameters (b/k) , (h/H) , porosity factor d , anisotropic factor (N/G) , compressive and tensile initial stress ζ and different sizes of irregularity ε .

The study on seismic waves gives important information about the layered Earth structure and has been used to determine the epicentre of

earthquake. Seismologists are able to learn about Earth's internal structure by measuring the arrival of seismic waves at stations around the world because these waves travel at different speeds through different materials. Knowing how fast these waves travel through the Earth, seismologists can calculate the time when the earthquake occurred and its location by comparing the times when shaking was recorded at several stations. If a wave arrives late, it passed through a hot, soft part of the Earth. This study shows that phase velocity dispersion curve is affected by inhomogeneity and since the Earth's crust and mantle are non-homogeneous in nature, the findings may play a vital role in understanding the cause of damage due to earthquakes. Some regions of the layers of the Earth are porous without filling of any significant materials and the velocity depends on porosity. So the existence of torsional waves in such a layer attracts the attention of seismologists. The initial stresses and irregularity also exert significant influence on phase velocity. Since the earth has an irregular surface space, it is more realistic to consider the irregularity effect. Hence the results may be useful for the study of seismic waves which are generated by artificial explosion and earthquakes. The information obtained from these results has importance to find the location of earthquakes as well as their energy, mechanism, etc. and may provide valuable information about the selection of proper structural materials for present day construction work.

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Appendix

1. In equation (4), the function $F(r)$ and irregularity ε are defined as:

$$F(r) = h \quad \text{for } |r| \leq m, \\ = 0 \quad \text{for } |r| > m$$

and

$$\varepsilon = \frac{h}{2m}, \quad \varepsilon \ll 1.$$

2. The strain components are defined as:

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad i, j = 1, 2, 3.$$

3. The shear wave velocity in the half-space is $c_s = \sqrt{(\mu/\rho)}$ and the torsional wave velocity is $c = (\omega/k)$.
4. $c_0 = \sqrt{\frac{\mu_0}{\rho_0}}$, $c_2 = \sqrt{\frac{\mu_2}{\rho_2}}$ are the shear wave velocities in the half-space.
5. $R = \frac{c^2(a-b)k}{4c_2^2 a^2 \gamma}$, $A = \frac{1-R}{2}$, $m_0^2 = k^2 \left(1 - \frac{c^2}{c_0^2}\right)$, $q = k\sqrt{pd \left(\frac{c^2}{c_1^2} - \frac{1-\zeta}{d}\right)}$, $p = \frac{N}{G}$.
6. $d' = \rho_{rr} - \frac{\rho_{r\theta}^2}{\rho_{\theta\theta}}$. $c_1 = \sqrt{\frac{N}{\rho'}} =$ the shear wave velocity in the porous layer, $\zeta = \frac{P}{2N} =$ dimensionless parameter due to initial stress P , $d = \frac{d'}{\rho'} = \gamma_{11} - \frac{\gamma_{12}^2}{\gamma_{22}}$.
7. $(\tau_{\theta z})_1 = l\mu_1 \left(\frac{\partial v_1}{\partial r} - \frac{v_1}{r}\right) + n\mu_1 \frac{\partial v_1}{\partial z}$,
 $(\tau_{\theta z})_2 = l\mu_2 \left(\frac{\partial v_2}{\partial r} - \frac{v_2}{r}\right) + n\mu_2 \frac{\partial v_2}{\partial z}$.
8. $A_1 = \frac{1}{1+a\varepsilon F(r)} e^{-\frac{\gamma k(1+2a\varepsilon F(r))}{2a}} \left\{ \frac{\gamma k(1+2a\varepsilon F(r))}{a} \right\}$
 $\times \left\{ 1 + \frac{\gamma k A}{a} (1 + 2a\varepsilon F(r)) \right\}$,
 $A_2 = 2\gamma k e^{-\frac{\gamma k}{2a}(1+2a\varepsilon F(r))} \left[\left\{ 1 + \frac{\gamma k A}{a} (1 + 2a\varepsilon F(r)) \right\} \right.$
 $\times \left\{ -\frac{1+2a\varepsilon F(r)}{2(1+a\varepsilon F(r))^2} - \frac{\gamma k(1+2a\varepsilon F(r))}{2a(1+a\varepsilon F(r))} + \frac{1}{1+a\varepsilon F(r)} \right\}$
 $\left. + \frac{\gamma k A(1+2a\varepsilon F(r))}{a(1+a\varepsilon F(r))} \right]$
 and
 $\frac{A_2}{A_1 k} = \frac{2a}{k} \left[-\frac{\gamma k}{2a} + \frac{1}{2} + \frac{\gamma k A}{a + \gamma k A} \right]$.
9. $\gamma_1 = \sqrt{1 - \frac{c^2}{c_2^2}}$, $q_1 = k\sqrt{pd \left(\frac{c^2}{c_1^2} - \frac{1}{d}\right)}$, $q_2 = k\sqrt{\frac{c^2}{c_1^2} - 1}$,
 $m_0 = k\sqrt{1 - \frac{c^2}{c_0^2}}$.

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