Influence of storm magnitude and watershed size on runoff nonlinearity

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The inherent nonlinear characteristics of the watershed runoff process related to storm magnitude and watershed size are discussed in detail in this study. The first type of nonlinearity is referred to rainfall-runoff dynamic process and the second type is with respect to a Power-law relation between peak discharge and upstream drainage area. The dynamic nonlinearity induced by storm magnitude was first demonstrated by inspecting rainfall-runoff records at three watersheds in Taiwan. Then the derivation of the watershed unit hydrograph (UH) using two linear hydrological models shows that the peak discharge and time to peak discharge that characterize the shape of UH vary event-to-event. Hence, the intention of deriving a unique and universal UH for all rainfall-runoff simulation cases is questionable. In contrast, the UHs by the other two adopted nonlinear hydrological models were responsive to rainfall intensity without relying on linear proportion principle, and are excellent in presenting dynamic nonlinearity. Based on the two-segment regression, the scaling nonlinearity between peak discharge and drainage area was investigated by analyzing the variation of Power-law exponent. The results demonstrate that the scaling nonlinearity is particularly significant for a watershed having larger area and subjecting to a small-size of storm. For three study watersheds, a large tributary that contributes relatively great drainage area or inflow is found to cause a transition break in scaling relationship and convert the scaling relationship from linearity to nonlinearity.

1. Introduction

The conventional unit hydrograph (UH) theory proposed by Sherman (1932) has been widely applied for transferring rainfall into watershed runoff. However, the linear system assumptions in the UH theory are not always suitable to depict the runoff response characteristics in natural watersheds which is usually nonlinear and time variant to rainfall (Minshall 1960; Chow 1964; Wang *et al.* 1981). For decades, such runoff nonlinear behaviours have been progressively functioned and developed into hydrological models (e.g., Ding

1974; Rodriguez-Iturbe and Valdes 1979; Chen and Singh 1986; Jakeman *et al.* 1990; Young 2001). Robinson *et al.* (1995) indicated that the nonlinearity in runoff response does not really disappear; it was dominated by hillslope at small drainage area and by channel network hydrodynamics at large one. Several factors, including the spatial variability of flow velocities (Saco and Kumar 2004; Paik and Kumar 2004), antecedent soil moisture condition (Beven 2001), and precipitation intensity (Szilagyi 2007) may be able to explain such runoff nonlinearity. In addition to above dynamic nonlinearity, the runoff peak discharge and its arrival

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time were observed to be nonlinearly correlative to the scale of upstream drainage area (Huang and Willgoose 1993). The statistical Power-law relation was proposed by Sivaplan *et al.* (2002) to quantify this dependence and was regarded as 'scaling relationship' to distinguish from the dynamic sense.

From the view of Power-law scaling relationship, Goodrich *et al.* (1997) investigated hydrological data in Walunt Gulch Basin of Arizona and discovered that the scale-dependency becomes more nonlinear as basin scale increasing. A transition break between mean annual runoff volume and drainage area exists, in which a two-segment Power-law regression was adopted. The primary reason of the occurrence of transition break is because of an increasing importance of ephemeral channel losses and partial storm area coverage (Goodrich *et al.* 1997). Also for Walunt Gulch Basin, Mantilla *et al.* (2006) revealed that the network structure and flow dynamics play fundamental role in determining Power-law exponent. In Goodwin Creek Experimental Watershed (GCEW) of Mississippi, Ogden and Dawdy (2003) indicated that the intercept and exponent in Power-law relation vary event-to-event. Furey and Gupta (2005) concluded that the event-to-event variability in the intercept and exponent for GCEW was due to the changes in the depth, duration, and spatial variability of excess-rainfall. Furey and Gupta (2007) further diagnosed and predicted the exponents and the intercepts of Power-law scaling relation in GCEW. By employing the GAGES-II (Falcone 2011) database that contains basin characteristics for 9322 streamflow gages in 18 USGS Water Resource Regions across the United States, Kroll *et al.* (2014) provided an initial analysis of the spatial scaling properties of average, low, and high streamflow series and associated statistics. Medhi and Tripathi (2015) further identified the relationships between the flood scaling exponent and the basin attributes in these 18 water resource regions. Based on numerical flow modeling, the variation of Power-law exponent for a self-similar network has also been analyzed (Gupta *et al.* 1996; Menabde *et al.* 2001; Morrison and Smith 2001). Lee *et al.* (2009) reported that the transition break is due to a significant inflow from a large tributary to result in the Power-law relation transferring from linearity to nonlinearity. In the most recent, Gupta *et al.* (2010) observed excellent fit of Power-law scaling relationship for the flood of June 2008 in the eastern Iowa River basin.

The first part of this study intends to ascertain the exhibition of dynamic nonlinearity at three steep watersheds in Taiwan. The simplest way applied herein was to regress statistic relation between recorded rainfall intensity and peak discharge. For a more detailed analysis, two linear

hydrological methods, including the UH theory model and the linear reservoir model, were applied to derive the UHs for every single storm to check whether the linear hydrological models can adequately simulate the inherent dynamic nonlinearity or not. Furthermore, the study also investigates the dynamic nonlinearity imitated in the other two nonlinear hydrological models, including variable instantaneous unit hydrograph (VIUH) and kinematic-wave-based geomorphologic IUH (KW-GIUH).

Besides, even the factors affecting the scaling nonlinearity were widely studied, seldom works discussed the mutual effects of storm magnitude and watershed size on the variation of Power-law exponent in two-segment regression (Goodrich *et al.* 1997). Hence by taking advantage of the physically-based KW-GIUH model, the second part of the study was to discover the transition break in scaling relationship at three study watersheds and to analyze the variation of Power-law exponent under a large- and small-size storms. To expound the research content, the following sections are organized as: (1) the analytical methods, including the theory of Power-law scaling relationship and the four hydrological models, are described in section 2; (2) section 3 gives a brief description of the study watersheds; (3) the dynamic nonlinearity from the hydrological records are discussed in section 4; (4) the performance of scaling nonlinearity by linear and nonlinear hydrological models are studied in section 5; and (5) the conclusion is finally presented in section 6.

2. Analytical methods

2.1 Runoff nonlinearity analysis

The magnitude of peak discharge (Q_p) is affected by rainfall characteristic and watershed geomorphologic features, which can be expressed as:

$$Q_p = f(R, \text{slope}, \text{land cover}, \text{stream network}) \quad (1)$$

where R is the rainfall characteristics. Considering the interrelation between the geomorphologic features (Gray 1961; Mesa and Gupta 1987; Lee *et al.* 2009), equation (1) can be simplified as:

$$Q_p = f(R, A) \quad (2)$$

where A is the drainage area. The rainfall characteristics (R) can be defined as the rainfall volume or mean rainfall intensity corresponding to a specified duration that likely contributes to the peak discharge. As shown in figure 1, the mean effective rainfall intensity ($\bar{R}_{tc}$) that occurs within a period of concentration time (t_c) prior to the time to peak

Figure 1. The diagram of variable $\bar{R}_{tc}$, which is defined as the mean effective rainfall intensity occurring within the concentration time period before the arrival of peak discharge.

discharge is defined as a characteristic index of the storm event. Thus, a Power-law function relating Q_p and $\bar{R}_{tc}$ can be written as:

$$Q_p = \alpha_1 \bar{R}_{tc}^{\theta_1} \quad (3)$$

where α_1 is the coefficient, θ_1 is the scaling exponent reflecting the degree of nonlinearity between Q_p and $\bar{R}_{tc}$.

In general, the relationship between peak discharge (Q_p) and drainage area (A) at a stream network position is expressed as (Huang and Willgoose 1993; Lee *et al.* 2009):

$$Q_p = \alpha_2 A^{\theta_2} \quad (4)$$

where α_2 is the coefficient, θ_2 is the scaling exponent. The physical meaning of θ_2 can be a measure of the degree to which a basin retards, filters, or diminishes the transformation from rainfall into runoff (Huang and Willgoose 1993), or the fraction of basin area contributing to peak discharge (Gupta and Waymire 1989). The value of θ_2 tends to depend on drainage area and aridity (Benson 1962; Alexander 1972) and be affected by the spatio-temporal variability of rainfall (Jothityangkoon and Sivapalan 2001). By using the hydrological data in GCEW, Furey and Gupta (2005) showed that the θ_2 increases as the rainfall duration increases. Gupta and Dawdy (1995) found that regional floods exhibit simple scaling if θ_2 is independent of return period, else region floods exhibit multiscaling. The empirical values of θ_2 have been found to lie in the range of $0.5 \leq \theta_2 \leq 1.0$ (Leopold *et al.* 1964; Benson 1962, 1964; Alexander 1972; Medhi and Tripathi 2015). The θ_2 approximate to 1.0 when the runoff equilibrium

state is achieved (Sivapalan *et al.* 2002). In general, the runoff equilibrium state can only occur in regions that have small drainage area or subject to a high-intensity rainfall with long duration. In this study, equation (3) was used to examine the behaviour of hydrological dynamic nonlinearity and equation (4) was for investigating the variation of scaling nonlinearity.

2.2 Hydrological models

Two linear hydrological models were applied to derive the UH from every single storms and to analyze the variation of peak discharge and the time to peak of resulted UHs between storms. Then two nonlinear hydrological models were used to illustrate how the UHs were varied systematically under various rainfall intensities. In this section, the structure of each model is described respectively as follows.

2.2.1 Unit hydrograph theory

The UH is the unit pulse-response function of a linear hydrological system. It is defined as the direct runoff hydrograph resulting from a unit (1 cm) depth of effective rainfall falling over the watershed area at a uniform rate during a specified period (Sherman 1932). The linear proportion and linear superposition are two essential hypotheses for the UH theory. Based on these two hypotheses, the observed rainfall distribution can be convoluted with the UH to calculate the discharge at the watershed outlet. The mathematical operation is written in a matrix form as:

$$[\mathbf{I}]_{(p+r-1) \times r} [\mathbf{U}]_{r \times 1} = [\mathbf{Q}]_{(p+r-1) \times 1} \quad (5)$$

where $[\mathbf{I}]_{(p+r-1) \times r}$ is the effective rainfall matrix (L), $[\mathbf{U}]_{r \times 1} = [u_1, u_2, \dots, u_r]$ is the unit hydrograph matrix (L^3/T), $[\mathbf{Q}]_{p+r-1} = [q_1, q_2, \dots, q_{p+r-1}]$ is the direct runoff matrix (L^3/T), p is the number of effective rainfall pulses, and r is the number of direct runoff ordinates.

Equation (5) is an over-determined system and an exact solution of $[\mathbf{U}]_{r \times 1}$ is unavailable. The least-squares method (LSM) that considers minimizing the sum of squared residuals is an alternative way to solve the problem. The supposition of using LSM is that the data are relatively error-free or that the error is likely minor (Ramírez 2000). Linear programming (LP) is another feasible approach to the over-determined problem (Singh 1976; Mays and Coles 1980). The LP goes along a pathway in solution domain to search the best outcome of the objective function. For equation (5), the objective function of the problem is to find an optimal $[\mathbf{U}]_{r \times 1}$ that can minimize the

sum of absolute residuals between the estimated discharge $[\hat{\mathbf{Q}}] (= [\hat{q}_1, \hat{q}_2, \dots, \hat{q}_{p+r-1}])$ and the recorded discharge $[\mathbf{Q}]$. Thus, the mathematical form can be expressed as:

$$\text{Min} \sum_{i=1}^{p+r-1} \frac{q_i}{\sum_{j=1}^{p+r-1} q_j} \cdot (q_i - \hat{q}_i) \quad (6)$$

where $q_i / \sum_{j=1}^{p+r-1} q_j$ is the weighting for the i th residual between q_i and $\hat{q}_i$. Equation (6) is subjected to three constraints: (a) all u_i are non-negative; (b) the sum of u_i is equal to the total amount of the UH; and (c) u_i should be gradually increasing before time to peak and gradually decreasing after time to peak.

2.2.2 Linear reservoir

Nash (1957) proposed a conceptual model to simulate the hydrological response of a watershed by having n identical linear reservoirs in series. The storage (S) (L^3) of the linear reservoir is assumed to be proportional to the outflow (Q) (L^3/T) as:

$$S = KQ \quad (7)$$

where K is the storage coefficient (T). By integrating equation (7) with the continuity equation, the IUH of an n th reservoir resulting from the unit-volume inflow through the system is given as:

$$Q_n(t) = \frac{t^{n-1}}{K^n \Gamma(n)} e^{(-t/K)} \quad (8)$$

where $\Gamma(n)$ is the gamma function, and t is the time (T). The appropriate values of n and K can be calibrated using hydrological records.

2.2.3 Variable instantaneous unit hydrograph

Ding (1974) proposed a 2-parameter variable instantaneous unit hydrograph (VIUH) from the generalized Muskingum model and S-hydrograph method. The dynamic equation of the VIUH can be written as:

$$S^m = K^m Q + K^m X(I - Q) \quad (9)$$

where K is a storage coefficient ($L^{-3(1-1/m)} T^{-1/m}$), m is an exponent, and X is a weighting coefficient in the range $0 \leq X \leq 0.5$. The conventional Muskingum method is one of the special cases of equation (9) for $m = 1$.

For a unit-depth input, the VIUH can be derived from equation (9) with the continuity equation as:

$$u(t) = \frac{m}{K} i_e(0)^{1-1/m} v^{m-1} (1 - v^m) \quad (10)$$

where

$$t = \frac{K}{i_e(0)^{1-1/m}} \int_0^v \frac{dv}{1 - v^m} \quad (11)$$

in which $i_e(0)$ is the effective rainfall intensity (L/T) occurring instantaneously at $t = 0$. Equation (10) indicates that the ordinate of the VIUH is nonlinearly related to $i_e(0)$ with a power of $1-1/m$. The values of m and K should be calibrated using hydrological records.

2.2.4 Kinematic-wave-based geomorphologic IUH

Based on the Strahler (1950) stream-ordering system, a watershed of order Ω can be divided into different runoff states. For an instantaneous unit of effective rainfall uniformly distributed over the watershed, the IUH can be derived based on a probability concept as follows (Rodriguez-Iturbe and Valdes 1979)

$$u(t) = \sum_{w \in W} [f_{x_{oi}}(t) * f_{x_i}(t) * f_{x_j}(t) * \dots * f_{x_{\Omega}}(t)]_w \cdot P(w) \quad (12)$$

where $u(t)$ is the geomorphologic instantaneous unit hydrograph (GIUH), $f_{x_k}(t)$ is the travel time probability density function in state x_k with a mean value of travel time T_{x_k} , $*$ denotes a convolution integral, $P(w)$ represents the probability of a raindrop adopting a specified path of w , and W is the path space given as $W = (x_{oi}, x_i, x_j, \dots, x_{\Omega})$.

Lee and Yen (1997) used the kinematic-wave theory to analytically determine the travel times of various runoff states (T_{x_k}) to develop a kinematic-wave-based GIUH (KW-GIUH) model. In KW-GIUH model, an i th-order subwatershed is assumed as a V-shaped model, which consists of two identical rectangular overland-flow planes and a central channel of constant cross-section and slope (Lee and Yen 1997; Yen and Lee 1997). The runoff travel time for the i th-order overland plane is (Henderson and Wooding 1964)

$$T_{x_{oi}} = \left(\frac{n_o \bar{L}_{oi}}{\bar{S}_{oi}^{1/2} i_e^{\beta-1}} \right)^{1/\beta} \quad (13)$$

where n_o is the overland roughness coefficient ($T/L^{1/3}$), $\bar{L}_{oi}$ and $\bar{S}_{oi}$ are the mean i th-order overland length (L) and slope (L/L), respectively; i_e is rainfall excess (L/T), and β is a constant that can be recognized as 5/3 from Manning's equation. The travel time for the i th-order channel is (Lee and Yen 1997)

$$T_{x_i} = \frac{B_i}{2 i_e \bar{L}_{oi}} \left[\left(h_{coi}^{\beta} + \frac{2 i_e n_c \bar{L}_{oi} \bar{L}_{ci}}{\bar{S}_{ci}^{1/2} B_i} \right)^{1/\beta} - h_{coi} \right] \quad (14)$$

where B_i is the i th-order channel width (L), n_c is the roughness coefficient of the channel ($T/L^{1/3}$), $\bar{S}_{c_i}$ is the mean i th-order channel slope (L/L), and h_{co_i} is the water depth at the entrance of the i th-order channel (L). The geomorphologic factors required for KW-GIUH can be acquired from a digital elevation model (DEM), and the roughness coefficient n_o and n_c for gauged or ungauged basins can be obtained by calibration or adapting from a neighbouring basin with similar geomorphologic and hydrological conditions.

3. Description of study watersheds

Three watersheds, Pao-Chiao (PC), San-Hsia (SH), and Pi-An-Chiao (PAC), in Taiwan were adopted in this study. Figure 2 shows the location, stream network and hydrological gauging stations of each watershed. The PC watershed is one of the subwatersheds of Hsin-Tine River basin in Taipei County. It has a drainage area of 110.85 km² and a watershed mean slope of 0.2711. The elevation ranges from 13 m at the outlet to 838 m at the crest line. The nearby rain-gauging station is Shih-Ting station. The SH watershed is one of the subwatersheds of Ta-Han River basin in New Taipei County. The drainage area is 122.88 km² and the neighbouring

rain-gauging station is Ta-Pao station. The watershed mean slope is 0.3050. The elevation ranges from 29 m at the outlet to 1711 m at the crest line. The PAC watershed is one of the subwatersheds of Hou-Lung River basin in central Taiwan, and has a drainage area of 110.73 km². The nearby rain-gauging station is Ta-Hu station. The watershed mean slope is 0.3079, and the elevation ranges from 227 to 2246 m. As comparing the mean slopes in PC, SH and PAC watersheds to those in Walunt Gulch (0.0080) and GCEW (0.0061), the former three are more steep and are about 33.7–38.5 and 44.4–50.5 times the slope of latter two basins.

The mean annual rainfall of the study watersheds is approximately 2500 mm, but the plum rains and typhoons occurring between May and October usually account for 80% of the annual rainfall. The hourly rainfall and runoff data measured by the Taiwan Water Resources Agency were used for study. In these watersheds, more than 70% of the area is forestland, and the remaining areas are mainly cultivated, grass, and buildings. A 40-m resolution DEM dataset provided by the Taiwan Aerial Survey Office, Forestry Bureau, was used to extract the geomorphologic factors (Lee 1998). The detailed results are summarized in table 1.

Figure 2. The location, stream network and hydrological gauging stations of three study watersheds in Taiwan.

Table 1. *Geomorphologic factors of the study watersheds.*

Watershed	Geomorphologic factor	Stream order (i)			
		1	2	3	4
PC	N_i	66	16	3	1
	$\bar{L}_{c_i}$ (km)	0.84	1.47	5.11	18.16
	$\bar{A}_i$ (km ²)	1.05	3.60	19.45	110.85
	P_{OA_i}	0.623	0.148	0.113	0.116
	$\bar{S}_{c_i}$ (m/m)	0.0975	0.0529	0.0199	0.0037
SH	N_i	69	16	3	1
	$\bar{L}_{c_i}$ (km)	0.92	2.08	3.88	17.83
	$\bar{A}_i$ (km ²)	1.15	4.99	18.15	125.88
	P_{OA_i}	0.629	0.197	0.078	0.096
	$\bar{S}_{c_i}$ (m/m)	0.1613	0.0924	0.0372	0.0131
PAC	N_i	32	8	2	1
	$\bar{L}_{c_i}$ (km)	1.24	3.15	8.27	4.01
	$\bar{A}_i$ (km ²)	1.94	8.79	50.57	110.73
	P_{OA_i}	0.560	0.227	0.163	0.050
	$\bar{S}_{c_i}$ (m/m)	0.0974	0.0549	0.0134	0.0069

Note: N_i = number of i th-order channel; $\bar{L}_{c_i}$ = mean length of i th-order channel; $\bar{A}_i$ = mean area of i th-order subwatershed; P_{OA_i} = ratio of i th-order subwatershed area to total watershed area; $\bar{S}_{c_i}$ = mean slope of i th-order channel.

Table 2. *Storm information and the analysis results of UH theory and linear reservoir model.*

Watershed	Storm event	Rainfall condition			Direct runoff			Conventional UH			Linear reservoir		
		Effective duration	Concentration time	Total depth	Mean effective rainfall	Base time	Peak discharge	t_p	u_p	n	k	t_p	u_p
		t_d (hr)	t_c (hr)	(mm)	$\bar{R}_{t_c}$ (mm/hr)	t_b (hr)	Q_p (m ³ /s)	(hr)	(m ³ /s)			(hr)	
PC	07/14/1987*	6	14.99	82	2.03	27	131.12	5	46.54	2.4	2.2	3	0.1445
	07/27/1987	18	5.66	184	17.51	30	467.07	3	56.06	2.8	1.75	3	0.1620
	09/08/1987	67	5.17	427	24.17	82	495.29	3	44.34	1.1	4.0	1	0.1782
	08/18/1990	51	5.72	369	21.6	65	546.96	3	63.83	1.25	4.0	1	0.1519
	08/21/1994	17	8.27	151	8.61	52	239.42	3	31.97	1.4	4.5	2	0.1161
	08/29/1997	23	5.47	391	24.12	43	588.71	3	52.99	2.1	2.2	2	0.1576
	08/22/2000	58	5.13	307	14.9	43	378.86	3	51.50	2.5	1.8	2	0.1611
SH	07/19/1981	25	5.12	399	13.62	28	369.32	2	80.91	1.2	2.8	1	0.2215
	06/03/1983	29	7.48	130	3.62	12	216.03	3	79.83	3.1	1.1	2	0.2357
	08/07/1984	20	5.35	238	9.13	20	265.45	2	61.41	1.2	3.5	1	0.1820
	07/27/1987	16	6.28	114	7.68	29	331.34	2	81.52	1.25	2.8	1	0.2131
	08/29/1990	24	3.74	316	22.37	22	761.29	3	94.41	2.8	1.1	2	0.2582
	10/09/1994*	43	5.53	277	11.23	62	482.73	2	84.40	1.4	1.7	1	0.2978
	08/17/1997	44	6.61	232	12.47	42	356.82	1	61.22	1.1	2.8	1	0.2370
PAC	08/22/1986	20	8.94	133	4.9	37	211.06	2	62.01	1.3	2.2	1	0.2538
	06/25/1987	29	8.18	209	10.34	85	380.72	1	52.42	1.5	2.0	1	0.2420
	08/17/1991	25	8.24	157	7.18	40	221.31	3	44.15	2.8	1.5	3	0.1874
	10/09/1994	26	9.16	174	7.79	57	186.94	1	27.55	2.5	2.0	3	0.1542
	05/22/1995	33	10.46	121	4.65	54	141.92	7	35.25	5.1	1.2	5	0.1608
	03/21/1997*	64	13.99	148	2.13	50	64.92	3	29.79	2.0	2.8	3	0.1311
	10/16/1998	16	11.93	296	7.88	33	352.22	1	60.43	1.0	3.2	2	0.1673

*Reference storm event.

4. Dynamic nonlinearity resulting from storm magnitude

4.1 Empirical analysis based on hydrological records

The storm events that produce single flow peak in each watershed were considered for analysis, and are summarized in table 2. The Horton's equation (Horton 1939) was applied to calculate infiltration loss and effective rainfall. The runoff concentration time, which was needed to determine $\bar{R}_{tc}$, was defined as (Lee *et al.* 2009)

$$t_c = \text{Max}_{1 \leq i \leq \Omega} \left\langle T_{x_{oi}} + \sum_{j=i}^{\Omega} T_{x_j} \right\rangle \quad (15)$$

where $T_{x_{oi}}$ and T_{x_j} are defined in equations (13 and 14). By cooperating equations (13, 14, and 15), an iteration is required for determining t_c and $\bar{R}_{tc}$. Table 2 shows the resulted t_c , $\bar{R}_{tc}$, and other information of the adopted storms.

By regressing $\bar{R}_{tc}$ and Q_p listed in table 2 through equation (3), the runoff dynamic nonlinearity can be examined, as shown in figure 3. The values of coefficient of determination R^2 are 0.96, 0.76, and 0.87 for PC, SH, and PAC watersheds respectively, which suggest that the Power-law regression fits well with the relation between Q_p and $\bar{R}_{tc}$. The values of scaling exponent (θ_1) demonstrates that Q_p is nonlinearly correlated to $\bar{R}_{tc}$ in PC ($\theta_1 = 0.599$) and SH ($\theta_1 = 0.632$) watersheds, but is linear correlative in PAC watershed ($\theta_1 = 1.061$).

4.2 Numerical analysis based on linear hydrological models

In this section, the UH of each single storm in table 2 was derived by UH theory and linear reservoir method. To examine the capability of linear hydrological models in simulating the dynamic nonlinearity, the peak discharge (u_p) and the time to peak discharge (t_p) of the resulted UH were regressed with $\bar{R}_{tc}$ in a dimensionless form similar to equation (3) as:

$$\frac{u_p}{u_p^*} = a \left(\frac{\bar{R}_{tc}}{\bar{R}_{tc}^*} \right)^b \quad (16)$$

$$\frac{t_p}{t_p^*} = c \left(\frac{\bar{R}_{tc}}{\bar{R}_{tc}^*} \right)^d \quad (17)$$

where u_p^* and t_p^* are the peak discharge and time to peak discharge of a reference storm with $\bar{R}_{tc} = \bar{R}_{tc}^*$, *i.e.*, the storm has the minimum total rainfall depth

(a) PC watershed

(b) SH watershed

(c) PAC watershed

Figure 3. Dynamic nonlinearity between mean effective rainfall $\bar{R}_{tc}$ and peak discharge Q_p .

among all adopted storms and is marked by an asterisk in table 2; a , b , c , and d are the regressed coefficients.

The resulted values of u_p and t_p of UHs are summarized in table 2. In table 2, the u_p and t_p that characterize the shape of the single UH are distinct and varied from event-to-event, which confirms that the intention of deriving a unique and universal UH for different storm events is questionable. As reported by Blažková (1992) in Spautka basin, three UH groups were identified by using Nash's linear reservoir model. The results in Blažková (1992) were induced by various spatial rainfall patterns, initial flow, and skewness of rainfall time pattern. Figure 4 shows the graphs of u_p/u_p^* vs. $\bar{R}_{tc}/\bar{R}_{tc}^*$ and t_p/t_p^* vs. $\bar{R}_{tc}/\bar{R}_{tc}^*$ of

two linear models. The regressed curves given by equations (16 and 17) and the R^2 values are also displayed. The regression coefficients a , b , c , and d are listed in table 3. In figure 4, the data points generated from the linear models distribute disorderly and without following Power-law trend as measured by low R^2 values. Two regressed curves in a sub-figure of figure 4 are similar, which implies that the linear proportion and linear superposition used in the linear models are unable to reflect the hydrodynamic nonlinearity in the rainfall-runoff process. This defect can be one of the major reasons to lead to unsatisfactory runoff simulation.

Figure 4. Variation between peak discharge u_p , time to peak discharge t_p , and mean rainfall intensity $\bar{R}_{tc}$ of the UHs generated by conventional UH and linear reservoir model in the study watersheds.

Table 3. Regressed coefficients and R^2 values of power-law equation for three study watersheds.

Watershed	Hydrologic model	$\frac{u_p}{u_p^*} = a \left(\frac{\bar{R}_{tc}}{R_{tc}} \right)^b$ & $\frac{u_p}{u_p^*} = a \left(\frac{i_e}{i_e^*} \right)^b$			$\frac{t_p}{t_p^*} = c \left(\frac{\bar{R}_{tc}}{R_{tc}} \right)^d$ & $\frac{t_p}{t_p^*} = c \left(\frac{i_e}{i_e^*} \right)^d$		
		a	b	R^2	c	d	R^2
PC	Conventional UH	0.8814	0.0923	0.139	0.9306	-0.1984	0.836
	Linear reservoir	0.9168	0.0742	0.240	1.0664	-0.2993	0.342
	VIUH	1.0078	0.3382	0.998	0.9860	-0.2168	0.670
	KW-GIUH	1.0036	0.3807	0.998	0.9860	-0.2168	0.670
SH	Conventional UH	0.9078	0.0566	0.038	0.7711	-0.1252	0.037
	Linear reservoir	0.9188	0.0698	0.065	0.6649	-0.0846	0.020
	VIUH	0.9996	0.3477	0.997	1.1914	-0.3437	0.576
	KW-GIUH	1.0567	0.3122	0.960	0.9075	-0.1901	0.665
PAC	Conventional UH	1.0889	0.2699	0.181	1.5769	-0.8615	0.365
	Linear reservoir	1.0815	0.2414	0.276	1.1278	-0.4183	0.130
	VIUH	1.0055	0.3311	0.999	0.9860	-0.2168	0.670
	KW-GIUH	1.0035	0.3692	0.994	0.9860	-0.2168	0.670

Table 4. Parameters and simulated performance of the VIUH and KW-GIUH models.

Watershed	Rainfall event	Variable IUH					KW-GIUH					Remark
		Parameter		Performance			Parameter		Performance			
		m	$1/K$	CE	EQ_p (%)	ET_p (hr)	n_o	n_c	CE	EQ_p (%)	ET_p (hr)	
PC	07/14/1987*	1.54	0.023	0.59	−1.05	−1	2.5	0.08	0.43	0.01	−2	Figure 5(a) Figure 5(a)
	07/27/1987	1.54	0.018	0.67	−1.62	−2	2.1	0.08	0.93	1.17	−1	
	09/08/1987	1.54	0.014	0.97	−0.27	1	4.5	0.08	0.96	1.37	2	
	08/18/1990	1.54	0.018	0.98	−0.24	−1	3.6	0.08	0.98	0.99	0	
	08/21/1994	1.54	0.017	0.85	−1.28	1	2.8	0.08	0.69	1.50	−1	
	08/29/1997	1.54	0.018	0.95	−1.55	−1	3.5	0.08	0.92	0.84	0	
	08/22/2000	1.54	0.020	0.89	1.12	−1	1.7	0.08	0.98	0.72	−1	
	Mean	1.54	0.018				3.0	0.08				
SH	07/19/1981	1.54	0.022	0.89	−0.91	−1	2.0	0.08	0.77	0.51	0	Figure 5(b) Figure 5(b)
	06/03/1983	1.54	0.030	0.53	−0.78	−2	1.1	0.07	0.82	−0.46	−1	
	08/07/1984	1.54	0.021	0.85	1.27	0	1.3	0.08	0.71	−0.49	1	
	07/27/1987	1.54	0.024	0.82	−0.50	0	1.6	0.07	0.95	1.37	1	
	08/29/1990	1.54	0.025	0.85	0.19	−1	1.5	0.07	0.84	0.55	0	
	10/09/1994*	1.54	0.026	0.94	0.61	0	1.6	0.08	0.94	1.18	1	
	08/17/1997	1.54	0.022	0.94	−0.25	0	2.7	0.08	0.85	1.43	1	
	Mean	1.54	0.024				1.7	0.08				
PAC	08/22/1986	1.54	0.021	0.92	−0.84	0	1.8	0.07	0.83	0.31	1	Figure 5(c) Figure 5(c)
	06/25/1987	1.54	0.019	0.74	2.45	0	3.0	0.07	0.70	1.30	1	
	08/17/1991	1.54	0.020	0.84	0.88	0	2.1	0.07	0.94	1.15	0	
	10/09/1994	1.54	0.017	0.97	0.26	1	3.2	0.07	0.95	1.64	1	
	05/22/1995	1.54	0.018	0.65	1.41	−2	2.4	0.07	0.92	−0.17	−1	
	03/21/1997*	1.54	0.017	0.84	−2.14	0	2.0	0.07	0.93	0.49	1	
	10/16/1998	1.46	0.017	0.92	−0.08	1	5.8	0.07	0.80	1.21	2	
	Mean	1.53	0.018				2.9	0.07				

*Reference storm event.

(a) PC watershed

(b) SH watershed

(c) PAC watershed

Figure 5. Examples of rainfall-runoff simulation using VIUH and KW-GIUH models.

4.3 Numerical analysis based on nonlinear hydrological models

According to equations (9–14), the IUH by VIUH and KW-GIUH models is a function of effective rainfall intensity (i_e). When effective rainfall intensity (i_e) is changed, the peak discharge (u_p) and time to peak discharge (t_p) of the IUH varies simultaneously. Similar to equations (16 and 17), the values of u_p and t_p of the IUH can be related to i_e in a dimensionless form as:

$$\frac{u_p}{u_p^*} = a \left(\frac{i_e}{i_e^*} \right)^b \quad (18)$$

$$\frac{t_p}{t_p^*} = c \left(\frac{i_e}{i_e^*} \right)^d \quad (19)$$

where u_p^* and t_p^* are the peak discharge and time to peak discharge of the IUH generated by a reference rainfall intensity i_e^* . Herein the unit-depth input ($i_e^* = 10$ mm/hr) was adopted.

To show how the VIUH and KW-GIUH models illustrate the hydrodynamic nonlinearity in the study watersheds, the storm events in table 2 were also applied to calibrate the required parameters. For each model, the required parameters, which are m and K for VIUH as well as n_o and n_c for KW-GIUH, were calibrated on the premise that the relative error of peak discharge (EQ_p) can maintain within $\pm 15\%$ and the coefficient of efficiency (CE) can exceed 0.70. In addition, the error of time to peak discharge (ET_p) was also assessed for reference. The evaluated criteria of EQ_p , ET_p , and CE are defined as:

$$EQ_p(\%) = \frac{(Q_p)_{\text{sim}} - (Q_p)_{\text{rec}}}{(Q_p)_{\text{rec}}} \times 100 \quad (20)$$

$$ET_p = (T_p)_{\text{sim}} - (T_p)_{\text{rec}} \quad (21)$$

$$CE = 1 - \frac{\sum_{t=1}^n [Q_{\text{sim}}(t) - Q_{\text{rec}}(t)]^2}{\sum_{t=1}^n [Q_{\text{rec}}(t) - \bar{Q}_{\text{rec}}]^2} \quad (22)$$

where $(Q_p)_{\text{sim}}$ and $(Q_p)_{\text{rec}}$ are the simulated and recorded peak discharges, $(T_p)_{\text{sim}}$ and $(T_p)_{\text{rec}}$ are the simulated and recorded time to peak discharge, $\bar{Q}_{\text{rec}}$ is the mean value of recorded discharge. The calibrated parameters and the resulted EQ_p , ET_p , and CE are shown in table 4, in which most of the cases are well simulated only with few exceptions. The examples of simulated hydrographs by each model in the study watersheds are shown in figure 5. In figure 5, the simulated hydrographs by VIUH and KW-GIUH are in good agreement with records in the limbs of rising, peak, and recession.

Table 5 and figure 6 show the characteristics of IUHs corresponding to different rainfall intensities generated by using VIUH and KW-GIUH. In performing the models, the mean values of m and K for VIUH and n_o and n_c for KW-GIUH were adopted (the bold-faced numbers shown in table 4). As shown in figure 6, it is obvious that as i_e/i_e^* increases, u_p/u_p^* increases and t_p/t_p^* decreases nonlinearly. The regressed curves given by equations (18 and 19) are well fitted to data points with acceptable R^2 , particularly for the graph of u_p/u_p^* vs. i_e/i_e^* that has $R^2 \geq 0.96$. The regressed coefficients a , b , c , and d are listed in table 3. The values of b and d for the results generated by VIUH and KW-GIUH range from 0.3122 to 0.3807 and -0.1901 to -0.3437 , respectively. It can conclude that the runoff dynamic nonlinearity is obviously presented.

Figure 7 shows the IUHs generated from VIUH and KW-GIUH for $i_e = 1, 10, 50$, and 100 mm/hr, respectively. The corresponding u_p and t_p of the IUHs can be referred to table 5 for details. In figure 7, the high-intensity rainfall produces an IUH with a large peak discharge and a short time to peak, whereas a low-intensity rainfall generates opposite results. In figure 7, the variations of u_p and t_p induced by various i_e from VIUH and

Table 5. Peak discharge and time to peak of the VIUH and KW-GIUH models under various effective rainfall intensities.

Watershed	Effective rainfall i_e (mm/hr)	VIUH		KW-GIUH	
		u_p (hr ⁻¹)	t_p (hr)	u_p (hr ⁻¹)	t_p (hr)
PC	1	0.045	7	0.049	7
	10	0.100	3	0.121	3
	20	0.127	3	0.158	3
	30	0.147	2	0.182	2
	40	0.163	2	0.208	2
	50	0.176	2	0.229	2
	100	0.219	2	0.288	2
SH	1	0.061	5	0.072	7
	10	0.134	2	0.177	3
	20	0.173	2	0.234	2
	30	0.197	2	0.274	2
	40	0.213	2	0.301	2
	50	0.230	1	0.319	2
	100	0.304	1	0.357	2
PAC	1	0.044	7	0.048	7
	10	0.096	3	0.120	3
	20	0.121	3	0.153	3
	30	0.138	2	0.182	2
	40	0.154	2	0.206	2
	50	0.166	2	0.224	2
	100	0.203	2	0.275	2

Figure 6. Variation between peak discharge u_p , time to peak discharge t_p , and rainfall intensity i_e of the UHs generated by VIUH and KW-GIUH models in the study watersheds.

KW-GIUH are closer to each other, although the peak discharges of the IUHs generated by KW-GIUH are usually higher than those from VIUH model. The results imply that both nonlinear models characterize the runoff dynamic nonlinearity in similar tendency. Although the VIUH model is more convenient for practical application because of fewer parameters used (only m and K), the values of these parameters are hard to be determined based on their physical meaning, and can only be calibrated through hydrological records. However, the KW-GIUH model integrates a large amount of geomorphologic factors and the parameters, n_o and n_c , can be chosen according to watershed land

cover, which enable the KW-GIUH model to be applicable in gauged and ungauged watersheds.

5. Scaling nonlinearity resulting from watershed size and storm magnitude

The rational formula, $Q_p = CiA$, is the simplest method to estimate the peak discharge in a small-scale watershed, where C is the runoff coefficient and i is the rainfall intensity. By comparing the rational formula with equation (4), the rational formula can be a special case for $\alpha_2 = Ci$ and $\theta_2 = 1$. For $\theta_2 = 1$, equation (4) points out that the runoff

Figure 7. IUHs of VIUH and KW-GIUH models under various effective rainfall intensities.

at outlet reaches to an equilibrium contributing state. However, the case of $\theta_2 = 1$ is rarely happened in a large size of watershed during storms. In this section, the mutual effects of watershed size and storm magnitude on the change of θ_2 are discussed as follows.

5.1 Watershed size on scaling nonlinearity

Because the KW-GIUH model incorporates watershed area and detailed geomorphologic factors into model structure, the model is promising to reflect

the watershed scaling effect on runoff variation. The method applied here is to assign a series of target points on the stream network from the upstream of the mainstream to the watershed outlet. The geomorphologic factors at each target point were then extracted by using a DEM to initiate the KW-GIUH model. Afterward, the runoff routing was carried out, and the relationship between resulted peak discharge and drainage area was analyzed.

As shown in figure 8, the mainstream in PC watershed with a length of 28.21 km was discretized into

Figure 8. (a) Scaling relationship between peak discharge Q_p and drainage area A and (b) the largest tributary (the grey-shaded area $A' = 30.77 \text{ km}^2$) joins the mainstream at Point b in PC watershed.

570 points. By performing runoff simulation with a constant rainfall intensity $i_e = 10 \text{ mm/hr}$ and rainfall duration $t_d = 1 \text{ hr}$, 570 runoff peaks at the target points were obtained. A graph that relates these 570 peak discharges (Q_p) and its drainage areas (A) is shown in figure 8. In figure 8(a), the data points are found to distribute in several groups. By following the method in Goodrich *et al.* (1997) and Lee *et al.* (2009), the two-segment Power-law approach was applied. As shown in figure 8(b), because a large tributary with contributing area $A' = 30.77 \text{ km}^2$ joins the mainstream at Point b , the total drainage area increases from

31.98 km^2 at Point a to 62.75 km^2 at Point b . Consequently, the peak discharges increase evidently from $11.82 \text{ m}^3/\text{s}$ (at Point a) to $25.04 \text{ m}^3/\text{s}$ (at Point b) for the test rainstorm and result in a manifest transition break. The lower segment is for $A \leq 31.98 \text{ km}^2$, and the upper segment is for $A \geq 62.75 \text{ km}^2$. The scaling relationships for these two segments are

$$Q_p = 0.56 A^{0.90} \quad A \leq 31.98 \text{ km}^2 \quad (23)$$

$$Q_p = 6.53 A^{0.33} \quad A \geq 62.75 \text{ km}^2 \quad (24)$$

The lower segment (equation 23) can be suited for small-scale drainage area in PC watershed, in which the relationship between Q_p and A is nearly linear ($\theta_2 = 0.90$). On the contrary, the $\theta_2 = 0.33$ in the upper segment (equation 24) reveals a nonlinear relationship between Q_p and A , and is applicable for a larger drainage area in PC watershed.

The same processes were also executed for the SH and PAC watersheds, in which the number of target points for the SH watershed is 512, corresponding to a mainstream length of 25.01 km. There are 611 target points for the PAC watershed, corresponding to a mainstream length of 29.11 km. Figure 9(a) shows a graph relating peak discharge and drainage area for the SH watershed, where the extent of the largest tributary is $A' = 17.69 \text{ km}^2$, as can be seen in figure 9(b). The scaling relationships for the two segments are

$$Q_p = 0.54 A^{0.98} \quad A \leq 73.23 \text{ km}^2 \quad (25)$$

$$Q_p = 2.20 A^{0.65} \quad A \geq 90.92 \text{ km}^2 \quad (26)$$

For the PAC watershed, the graph relating Q_p and A is displayed in figure 10(a and b) shows the location and extent of the largest tributary with $A' = 43.28 \text{ km}^2$. The scaling relationships are

$$Q_p = 0.32 A^{0.98} \quad A \leq 57.87 \text{ km}^2 \quad (27)$$

$$Q_p = 1.38 A^{0.70} \quad A \geq 101.15 \text{ km}^2 \quad (28)$$

For SH watershed, the scaling exponents in equations (25 and 26) change from 0.98 to 0.65, and for PAC watershed it changes from 0.98 to 0.70 in equations (27 and 28). Such variations not only prove the existence of the transition break in scaling graphs for different watersheds, but also confirm that a tributary with a relative large drainage area flowing into the mainstream is able to transfer the scaling relationship between Q_p and A from linearity to nonlinearity.

Figure 9. (a) Scaling relationship between peak discharge Q_p and drainage area A and (b) the largest tributary (the grey-shaded area $A' = 17.69 \text{ km}^2$) joins the mainstream at Point b in SH watershed.

Figure 10. (a) Scaling relationship between peak discharge Q_p and drainage area A and (b) the largest tributary (the grey-shaded area $A' = 43.28 \text{ km}^2$) joins the mainstream at Point b in PAC watershed.

5.2 Storm magnitude and watershed size on scaling nonlinearity

Storm is usually spatiotemporal variation across a watershed in nature. To investigate the influence of storm magnitude on the Q_p – A scaling relationship, one relative large-scale and another small-scale storms were chosen from table 2 for runoff simulation. In the PC watershed, two storm events occurring on August 18, 1990 and August 21, 1994 were selected as the large- and small-scale storms, whose hydrographs are shown in figure 5(a).

Figure 11(a) shows the Q_p – A relationship along the mainstream generated by using the KW-GIUH model, in which the circles are the results for the large-scale storm and the triangles are those for the small-scale storm. For the large-scale storm, the two-segment regression equations are obtained as follows:

$$Q_p = 5.71 A^{0.99} \quad A \leq 31.98 \text{ km}^2 \quad (29)$$

$$Q_p = 9.42 A^{0.87} \quad A \geq 62.75 \text{ km}^2 \quad (30)$$

Figure 11. Scaling relationship between peak discharge Q_p and drainage area A under different storm magnitude conditions.

and those for the small-scale storm are

$$Q_p = 4.01 A^{0.86} \quad A \leq 31.98 \text{ km}^2 \quad (31)$$

$$Q_p = 14.99 A^{0.57} \quad A \geq 62.75 \text{ km}^2 \quad (32)$$

The variation of scaling exponent θ_2 in equations (29 and 31) indicates that the scaling nonlinearity of Q_p – A becomes more obvious for a small-scale storm. Because a small-scale storm can only partially cover the drainage area or has a short duration, the runoff equilibrium state is rarely reached. Consequently, the scaling exponent was found to be decreased. Moreover, the exponents in equations (30 and 32) show that the scaling nonlinearity is particularly significant for a large watershed subject to small-scale storm.

By using the same approach, figure 11(b and c) shows the Q_p – A relationship by applying large/small storms in the SH and PAC watersheds. The hydrographs of these storms at the outlet are referred to figure 5(b and c). In figure 11(b), the nonlinearity between Q_p and A in the SH watershed is obvious ($\theta_2 = 0.80$ to 0.64) for a drainage area larger than 90.92 km^2 . However, the nonlinearity in the PAC watershed is less obvious, even in the small-scale storm ($\theta_2 = 0.92$). Perhaps this is because that the storm occurring on October 9, 1994 (right-hand side of figure 5c) lasted over 10 hr before the peak discharge occurs, and allows the runoff approximating to equilibrium state. Consequently, the Q_p – A scaling relationship is approximately linear.

6. Conclusion

The dynamic and scaling nonlinearity of watershed runoff response produced by storm magnitude and watershed size are discussed. By inspecting the hydrological records at three watersheds in Taiwan, the peak discharge is found to be nonlinearly related to $\bar{R}_{tc}$, which is defined as the mean effective rainfall intensity occurring within a concentration time period before the occurrence of peak discharge. The shape of UHs derived from UH theory and linear reservoir model varies from event-to-event, which confirms that the intention of obtaining a single and universal UH from linear models for different rainstorm simulations is questionable. However, the IUHs from the other two nonlinear hydrological models, *i.e.*, VIUH and KW-GIUH, are responsive to rainfall intensity. In performing the nonlinear models, the variations of peak discharge and time to peak discharge can be clearly observed in the generated IUHs.

The physically-based KW-GIUH model was adopted to analyze the scaling nonlinearity caused by watershed size and storm magnitude. As executing KW-GIUH model with designed rainfall, the relation between resulted peak discharges and corresponding drainage areas along the mainstream

was regressed using a two-segment method. The results show that when a large tributary joins the mainstream, the increment of drainage area or inflow is contributed to cause a transition break in the Q_p - A graph. This transition break indicates the scaling relationship transferring from linearity to nonlinearity. Since a multi-segment regression may be applied to obtain a series of Power-law exponents, the change of Power-law exponents can be used to exhibit the systematic variation of Q_p - A scaling relationship.

By considering the mutual effects of watershed size and storm magnitude, the scaling nonlinearity is found less obvious in small watershed. It is because of the short runoff concentration time and easily-attained equilibrium state. Consequently, the method used to determine peak discharge for small watershed is usually linearly related to drainage area. However, scaling nonlinearity becomes more obvious for a large-scale watershed subjecting to a small-scale storm.

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