Two-dimensional deformation of a uniform half-space due to non-uniform movement accompanying a long vertical tensile fracture

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The solution of the static deformation of a homogeneous, isotropic, perfectly elastic half-space caused by uniform movement along a long vertical tensile fracture is well known. In this paper, we study the problem of static deformation of a homogeneous, isotropic, perfectly elastic half-space caused by a non-uniform movement along a long vertical tensile fracture of infinite length and finite depth. Four movement profiles are considered: linear, parabolic, elliptic and cubic. The deformation corresponding to the four non-uniform movement profiles is compared numerically with the deformation due to a uniform case, assuming the source potency to be the same. The equality in source potency is achieved in two ways: One, by varying the depth of fracture and keeping the surface discontinuity constant and the other way, by keeping the depth of fracture constant and varying the surface discontinuity. It is found that the effect of non-uniformity in movement in the near field is noteworthy. The far field is not affected significantly by the non-uniformity in movement. In the first case, horizontal displacement is significantly affected rather than vertical displacement. In the second case, non-uniformity in movement changes the magnitude of the displacement at the surface. Also, the displacements around a long vertical tensile fracture for different movement profiles are plotted in three dimensions.

1. Introduction

Recent studies reveal that a large number of earthquake sources cannot be represented by double-couple source mechanism which models a shear fault. According to Sipkin (1986), the non-double-couple mechanism might be due to tensile failure under high fluid pressure. The problem of deformation caused by a tensile dislocation in a layered media has been studied by several investigators (e.g., Maruyama 1964; Davis 1983; Yang and Davis 1986; Bonafede and Danesi 1997; Bonafede and Rivalta 1999a, b; Singh and Singh 2000, 2004; Singh *et al.* 2002; He *et al.* 2003; Kumar *et al.* 2005, etc.). However, most of these studies have assumed the movement to be uniform. The assumption of uniform movement makes the edges of the fault plane singular where the displacement is indeterminate and the stress is infinite. For this reason, uniform movement models cannot be used in the near field deformation.

The aim of the present paper is to study the deformation of a homogeneous, isotropic, elastic half-space caused by non-uniform movement along a long vertical tensile fracture of finite width. It is an attempt to consider the movement different from uniform one. The corresponding problem of strike-slip and dip-slip faults in a uniform half-space has been discussed by several investigators. Chinnery and Petrak (1968) used numerical

Keywords. Two-dimensional; vertical tensile fracture; static deformation; non-uniform movement.

J. Earth Syst. Sci. **122**, No. 4, August 2013, pp. 1055–1063

integration to compute the elastic field due to a vertical strike slip fault with slip that varies exponentially over the face of the fault. Wang and Wu (1983) obtained closed-form analytical expressions for the displacements and the stresses due to non-uniform slip along a strike-slip fault. Singh *et al.* (1994) obtained the deformation of a homogeneous, isotropic, perfectly elastic half-space caused by non-uniform slip along a vertical strike-slip and dip-slip fault. Rani and Singh (2007) have obtained residual elastic field in two welded half-spaces due to non-uniform slip along a long strike slip fault.

A lot of work has been done to obtain the displacement field numerically caused by elastic cracks on planer faults for the three-dimensional case. Volkov (1986) used the finite difference method for elastic cracks in the free and semi-infinite media. Stephan and Wendland (1986) presented a solution for the three-dimensional crack problems using the calculus of pseudo-differential operators. Niu *et al.* (2005) proposed a semi-analytical algorithm to analyze the three dimensional elasticity problems such as very thin-walled structures using finite element method.

We have obtained closed-form analytical expressions for the displacements caused by a non-uniform movement along a long vertical tensile fracture in a uniform half-space. Four movement profiles are considered: linear, parabolic, elliptic and cubic. It is assumed that the movement b decreases from a value b_0 to zero at depth L. The deformation corresponding to the four non-uniform movement profiles is compared numerically with the deformation due to a uniform movement, assuming the source potency to be the same. The equality in the source potency is achieved in two ways. In the first case, it is achieved by varying the depth of fracture L and keeping the surface discontinuity same for all the profiles. In the second case, depth of the fracture is same for all the profiles but the surface discontinuity b_0 for a particular profile is so chosen that the source potency is same for all the profiles. The analytical expressions for the displacements are used to compare the variation of the elastic field with the distance from the fracture for non-uniform case with that for the uniform-movement. Also, the displacement field has been plotted in three dimensions for different movement profiles. In case of uniform movement, the displacements are singular at the lower edge of the fracture. However, this singularity is not present in the displacement field for linear and parabolic cases.

The deformation due to tensile failure has several important geophysical applications, such as modelling of the deformation field due to dyke injection in the volcanic region, mine collapse and fluid-driven cracks. Moreover, the models of non-uniform movement can be used to study the near field deformation. The study may be useful in applications where movements are approximated using linear, quadratic or cubic interpolants as described by Ionescu and Volkov (2009).

2. Theory

Consider a homogeneous, isotropic, perfectly elastic half-space occupying the region $x_3 \geq 0$. A vertical tensile fracture of infinite length and finite depth occupies the region $-\infty < x_1 < \infty$, $x_2 = 0$, $0 \leq x_3 \leq L$ (figure 1). Let the movement on the fracture be denoted by b, which is non-uniform in general. We are considering a two-dimensional approximation in which b is independent of x_1. Using the expressions of the displacements given by Rani *et al.* (1991) and Singh and Rani (1991) due to the movement b on a vertical tensile fracture with dislocation in the x_2-direction at any point of a homogeneous, isotropic, perfectly elastic half-space ($x_3 \geq 0$) can be expressed as:

$$u_i = \int_0^L b(h) G_i(x_2, x_3, h)\, dh, \quad (i = 2, 3). \tag{1}$$

The Green's functions appearing in equation (1) are given by

$$G_2 = \frac{x_2}{4\pi(1-\sigma)} \Big[(3-2\sigma)\frac{1}{R^2} - \frac{2(x_3-h)^2}{R^4} + (5-6\sigma)\frac{1}{S^2} - \big\{ 2\big(3x_3^2 + 4x_3 h - h^2\big) + 8(1-\sigma)h(x_3+h) \big\} \frac{1}{S^4} + 16 x_3 h (x_3+h)^2 \frac{1}{S^6} \Big] \tag{2}$$

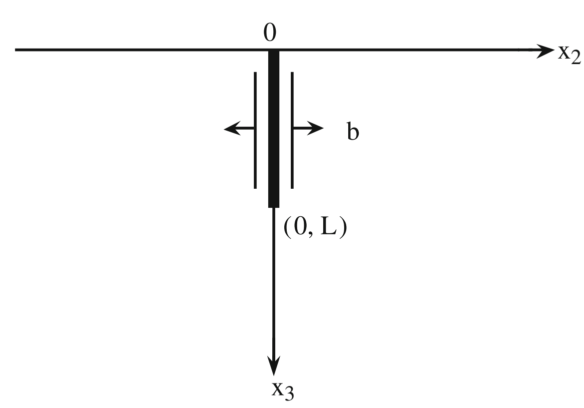

Figure 1. Geometry of a vertical tensile fracture of infinite length and finite depth occupying the region $-\infty < x_1 < \infty$, $x_2 = 0$, $0 \leq x_3 \leq L$.

$$G_3 = \frac{1}{4\pi(1-\sigma)}\left[(1+2\sigma)\frac{(x_3-h)}{R^2} - \frac{2(x_3-h)^3}{R^4} + \{2\sigma(3x_3+5h) - x_3 - 7h\}\frac{1}{S^2} - 2\{4\sigma h(x_3+h) + 3(x_3^2+2x_3h-h^2)\} \times \frac{(x_3+h)}{S^4} + 16x_3h\frac{(x_3+h)^3}{S^6}\right] \quad (3)$$

where

$$R^2 = x_2^2 + (x_3-h)^2, \qquad S^2 = x_2^2 + (x_3+h)^2, \quad (4)$$

and σ being the Poisson's ratio. The expressions for the displacements for various movement profiles have been obtained from equation (1) by integrating analytically.

2.1 *Uniform case*

The case of uniform movement $b(h) = b_0$ over a vertical tensile fracture has been discussed by Singh and Singh (2000). We give their results for ready reference.

$$u_2 = \frac{b_0}{4\pi(1-\sigma)}\left[2(1-\sigma)\left\{\frac{2Y}{B^2} + \tan^{-1}\left(\frac{1-Z}{Y}\right) + \tan^{-1}\left(\frac{1+Z}{Y}\right)\right\} - Y(Z-1)\left(\frac{1}{A^2} - \frac{1}{B^2}\right) - \frac{4YZ(Z+1)}{B^4}\right] \quad (5)$$

$$u_3 = \frac{b_0}{4\pi(1-\sigma)}\left[(1-2\sigma)\ln\left(\frac{A}{B}\right) - \frac{(Z-1)^2}{A^2} + \{Z^2 - 3 + 4\sigma(Z+1)\}\frac{1}{B^2} - 4Z(Z+1)^2\frac{1}{B^4}\right] \quad (6)$$

where

$$Y = \frac{x_2}{L}, \qquad Z = \frac{x_3}{L}, \qquad A^2 = Y^2 + (Z-1)^2, \quad B^2 = Y^2 + (Z+1)^2. \quad (7)$$

2.2 *Linear case*

Let the movement on the fracture vary according to the law

$$b(h) = b_0\left(1 - \frac{h}{L}\right), \quad (0 \le h \le L) \quad (8)$$

where b_0 is the surface discontinuity and L is the depth of fracture. Inserting the expression for $b(h)$ in equation (1) and integrating, closed-form expressions for the displacements are obtained as follows:

$$u_2 = \frac{b_0}{4\pi(1-\sigma)}\left[(2\sigma-1)Y\ln\left(\frac{A}{B}\right) + \frac{2YZ}{B^2} + 2(1-\sigma)(1-Z)\left\{\tan^{-1}\left(\frac{Z+1}{Y}\right) - \tan^{-1}\left(\frac{Z-1}{Y}\right)\right\}\right] \quad (9)$$

$$u_3 = \frac{b_0}{4\pi(1-\sigma)}\left[\frac{2Z(Z+1)}{B^2} - 4(1-\sigma) + (1-2\sigma)(1-Z)\ln\left(\frac{A}{B}\right) + 2(1-\sigma)Y \times\left\{\tan^{-1}\left(\frac{1-Z}{Y}\right) + \tan^{-1}\left(\frac{1+Z}{Y}\right)\right\}\right]. \quad (10)$$

2.3 *Parabolic case*

Let the movement on the fracture be given by

$$b(h) = b_0\left(1 - \frac{h^2}{L^2}\right), \quad (0 \le h \le L). \quad (11)$$

The expressions for the displacements obtained from equations (1), (2), (3) and (11) are as follows:

$$u_2 = \frac{b_0}{4\pi(1-\sigma)}\left[4(1-\sigma)Y + \frac{4YZ}{B^2} + 16(1-\sigma)YZ\ln A_0 + 2(2\sigma-1)YZ\ln A - 2(7-6\sigma)YZ\ln B + 4\{Y^2 - 3Z^2 - 2\sigma(Y^2-Z^2)\}\tan^{-1}\left(\frac{Z}{Y}\right) + 2\{Z^2 - 1 - \sigma(Z^2-Y^2-1)\}\tan^{-1}\left(\frac{Z-1}{Y}\right) + 2\{1 - 2Y^2 + 5Z^2 - \sigma(1-3Y^2+3Z^2)\} \times \tan^{-1}\left(\frac{Z+1}{Y}\right)\right] \quad (12)$$

$$u_3 = \frac{b_0}{4\pi(1-\sigma)}\left[-2(2+Z) - 4\sigma(Z-1) + \frac{4Z(Z+1)}{B^2} - 8\{Y^2 - \sigma(Y^2-Z^2)\}\ln A_0 + \{3Y^2 - Z^2 + 1 - 2\sigma(Y^2-Z^2+1)\}\ln A - \{1 - Z^2 - 5Y^2 - 2\sigma(1+3Z^2-3Y^2)\}\ln B + 4YZ\left\{(1-\sigma)\tan^{-1}\left(\frac{1-Z}{Y}\right) + (3\sigma-1)\tan^{-1}\left(\frac{1+Z}{Y}\right)\right\} - 8YZ(2\sigma-1)\tan^{-1}\left(\frac{Z}{Y}\right)\right] \quad (13)$$

where

$$A_0^2 = Y^2 + Z^2. \tag{14}$$

2.4 *Elliptic case*

The results corresponding to the elliptic movement profile

$$b\,(h) = b_0 \left(1 - \frac{h^2}{L^2}\right)^{1/2}, \quad (0 \le h \le L) \tag{15}$$

which are valid at the surface ($x_3 = 0$) are as follows:

$$u_2 = \pm \frac{b_0}{2} \frac{1}{\sqrt{1+Y^2}} \tag{16}$$

$$u_3 = \frac{b_0}{\pi} \left[\frac{Y^2}{2\sqrt{1+Y^2}} \ln \left| \frac{\sqrt{1+Y^2}+1}{\sqrt{1+Y^2}-1} \right| - 1 \right] \tag{17}$$

where upper sign is for $Y > 0$ and lower sign for $Y < 0$.

2.5 *Cubic case*

Let the movement on the fracture vary according to the law

$$b\,(h) = b_0 \left(1 - \frac{h^2}{L^2}\right)^{3/2}, \quad (0 \le h \le L)\,. \tag{18}$$

The expressions for the displacements obtained from equations (1)–(3) and (18) valid at the surface ($x_3 = 0$) are given below:

$$u_2 = b_0 \left[Y^3 \mp \frac{3Y^4 - 1 + 2Y^6}{2\left(1+Y^2\right)^{3/2}} \right] \tag{19}$$

$$u_3 = \frac{b_0}{\pi} \left[-1 - 3Y^2 + \frac{3}{2} Y^2 \sqrt{1+Y^2} \times \ln \left| \frac{\sqrt{1+Y^2}+1}{\sqrt{1+Y^2}-1} \right| \right] \tag{20}$$

where upper sign is for $Y > 0$ and lower sign for $Y < 0$.

3. Numerical results and discussion

We wish to compare the deformation due to non-uniform movement profiles with the corresponding deformation due to uniform movement. For all the movement profiles considered, the movement decreases from a value b_0 at the surface to zero at the depth L. If the surface discontinuity b_0 and the depth of fracture L are assumed to be the same for all the cases, then the source potency $\int_0^L b(h)dh$ per unit length of the fracture is different for different profiles. It is not justified in comparing deformation of sources of different potency. Therefore, equality in source potency must be assured. The equality in source potency for different movement profiles is achieved in two ways either by adjusting the surface discontinuity or the depth. We consider both the cases.

Case I: In this case, we have assumed that the surface discontinuity b_0 is the same for all movement profiles, but the depth of fracture L is so adjusted that equality in source potency is achieved. This yields relationship (figure 2).

$$L_1 = \frac{1}{2} L_2 = \frac{2}{3} L_3 = \frac{\pi}{4} L_4 = \frac{3\pi}{16} L_5 = L\,(\text{say}) \tag{21}$$

where L_1 is the depth of fracture for the uniform movement model and L_2, L_3, L_4 and L_5 are, respectively, the depths of fracture for the linear, parabolic, elliptic and cubic profiles. We measure the displacements in unit of the surface discontinuity b_0, the distances in unit of the depth of fracture $L_1 = L$ for the uniform movement model. The dimensionless quantities Y and Z for non-uniform movement cases are to be suitably modified in accordance with the relation (equation 21). For example, in equation (12), Y should be replaced by

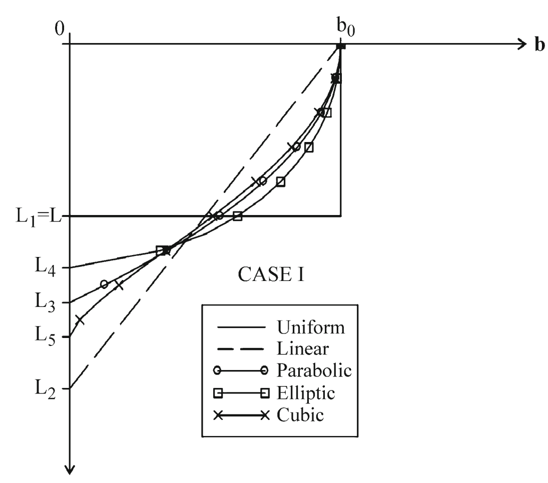

Figure 2. Five movement profiles considered: uniform, linear, parabolic, elliptic and cubic. For non-uniform movement profile, the discontinuity decreases monotonically with depth from the surface value b_0 to zero at the buried edge of the fracture. For various movement profiles, the depth of fracture is taken as uniform case L_1; linear L_2; parabolic L_3; elliptic L_4 and cubic L_5.

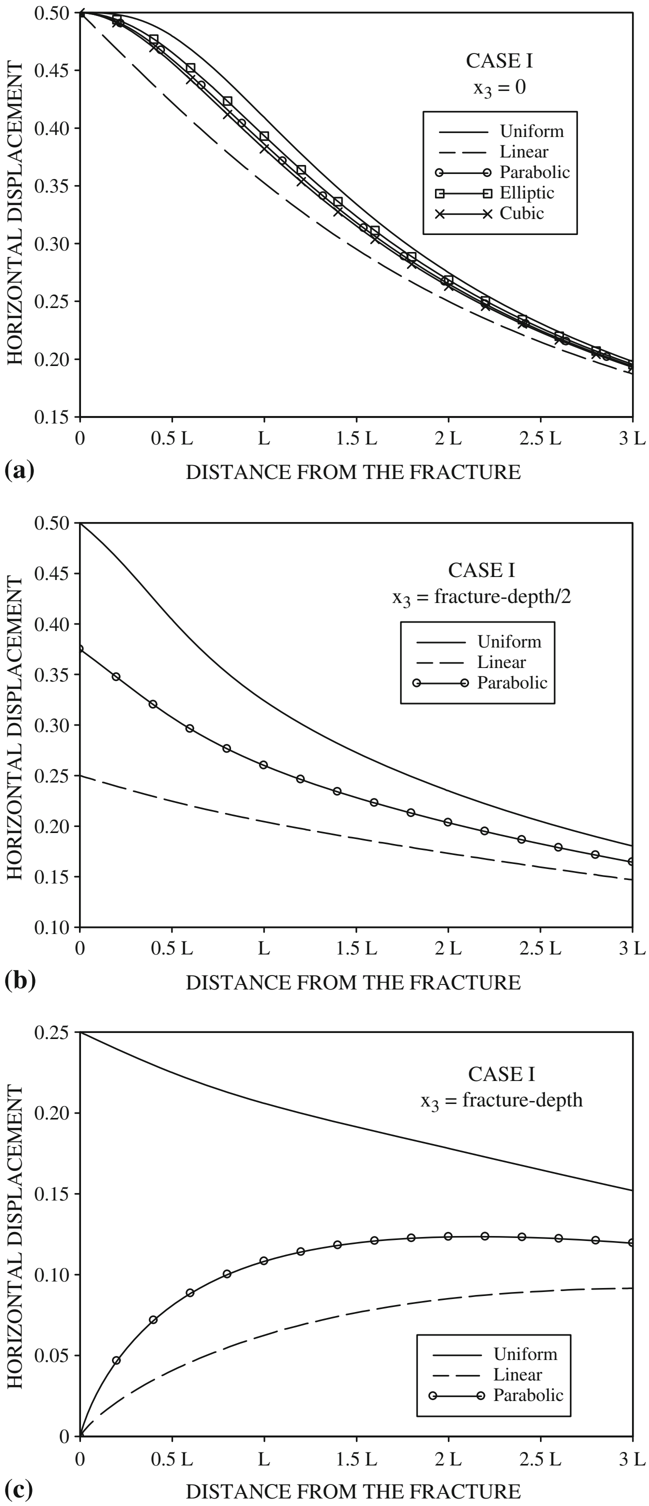

Figure 3. Variation of the horizontal displacement (u_2/b_0) with the distance from the fracture in units of the depth of fracture $L_1 = L$ for various movement profiles (**a**) $x_3 = 0$, (**b**) $x_3 =$ fracture-depth/2, and (**c**) $x_3 =$ fracture-depth. Note that the depth of the fracture for different movement profiles is different. We observe that non-uniform movement profiles have a significant change at depth rather than at the surface.

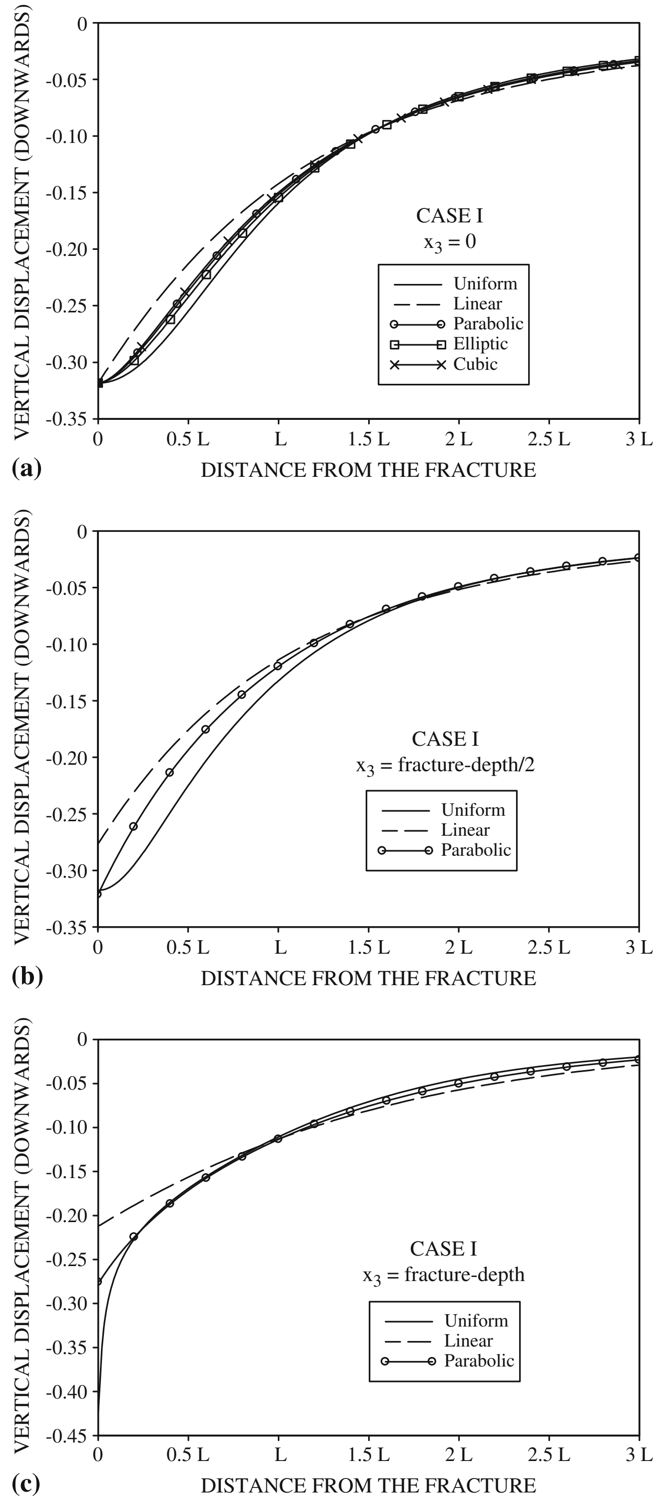

Figure 4. Variation of the vertical displacement (u_3/b_0) with the distance from the fracture in units of depth $L_1 = L$ for various movement profiles (**a**) $x_3 = 0$, (**b**) $x_3 =$ fracture-depth/2, and (**c**) $x_3 =$ fracture-depth. There is a significant change near the fracture but far field remains unaffected.

$(2/3)Y$ and Z by $(2/3)Z$. In all our computations, we have assumed that the material is Poissonion (σ=0.25). Figure 3(a–c) shows the variation of the horizontal displacement (u_2) with the distance from a long, vertical, tensile fracture for three values of x_3. We observe that the surface displacement for all the movement profiles have a little variation. But, the subsurface deformation in the near field depends on the details of the movement under the ground. Figure 4(a–c) displays the variation of vertical displacement (downwards) with the distance from the fracture for different movement profiles. The subsurface deformation in the near field is affected by the movement profile chosen.

Case II: Here, we assume the depth of fracture L is same for all the movement profiles, but the surface discontinuity b_0 is so adjusted that parity in source potency is achieved. This yields relationship (figure 5)

$$b_1 = \frac{1}{2}b_2 = \frac{2}{3}b_3 = \frac{\pi}{4}b_4 = \frac{3\pi}{16}b_5 = b_0\,(\text{say}) \qquad (22)$$

where b_1 is the magnitude of surface discontinuity for the uniform movement model and b_2, b_3, b_4 and b_5 are, respectively the magnitude of surface discontinuity for the linear, parabolic, elliptic and cubic profiles. Variation of the horizontal displacement (u_2) and the vertical displacement (u_3) with the distance from a long, vertical, tensile fracture are shown in figures 6 and 7, respectively. In this case, we observe that there is a significant change in the deformation due to non-uniform movement as compared to uniform movement at the surface.

The horizontal and vertical displacements around a long, vertical, tensile fracture for uniform,

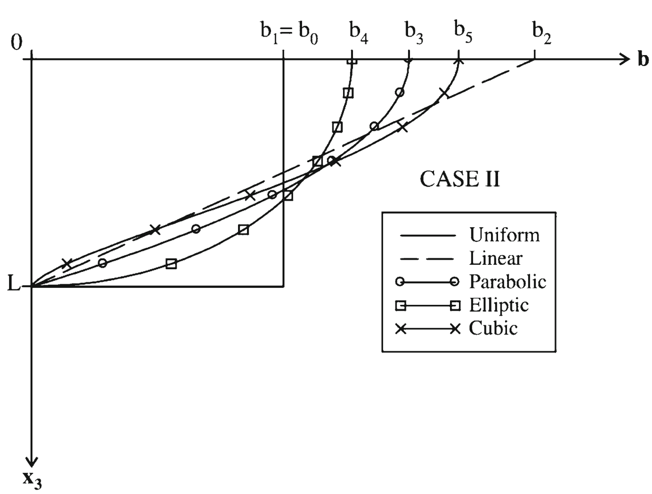

Figure 5. Various movement profiles having the depth of the fracture L to be same but the magnitude of surface discontinuity for uniform case as b_1; linear b_2; parabolic b_3; elliptic b_4 and cubic b_5 are so chosen that the source potency is equal to b_0L which is the source potency for uniform movement.

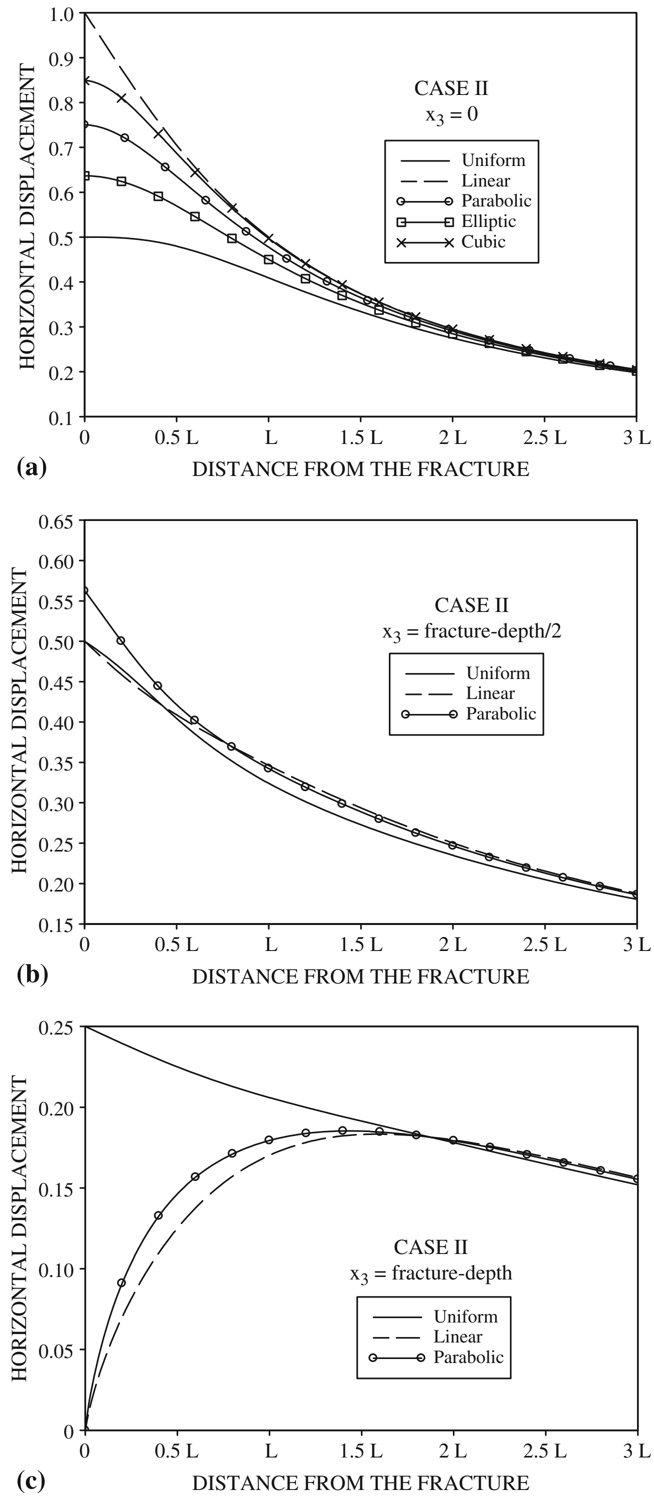

Figure 6. Variation of the horizontal displacement (u_2/b) with the distance from the fracture in unit of L for various movement profiles at (**a**) $x_3 = 0$, (**b**) x_3 = fracture-depth/2, and (**c**) x_3 = fracture-depth.

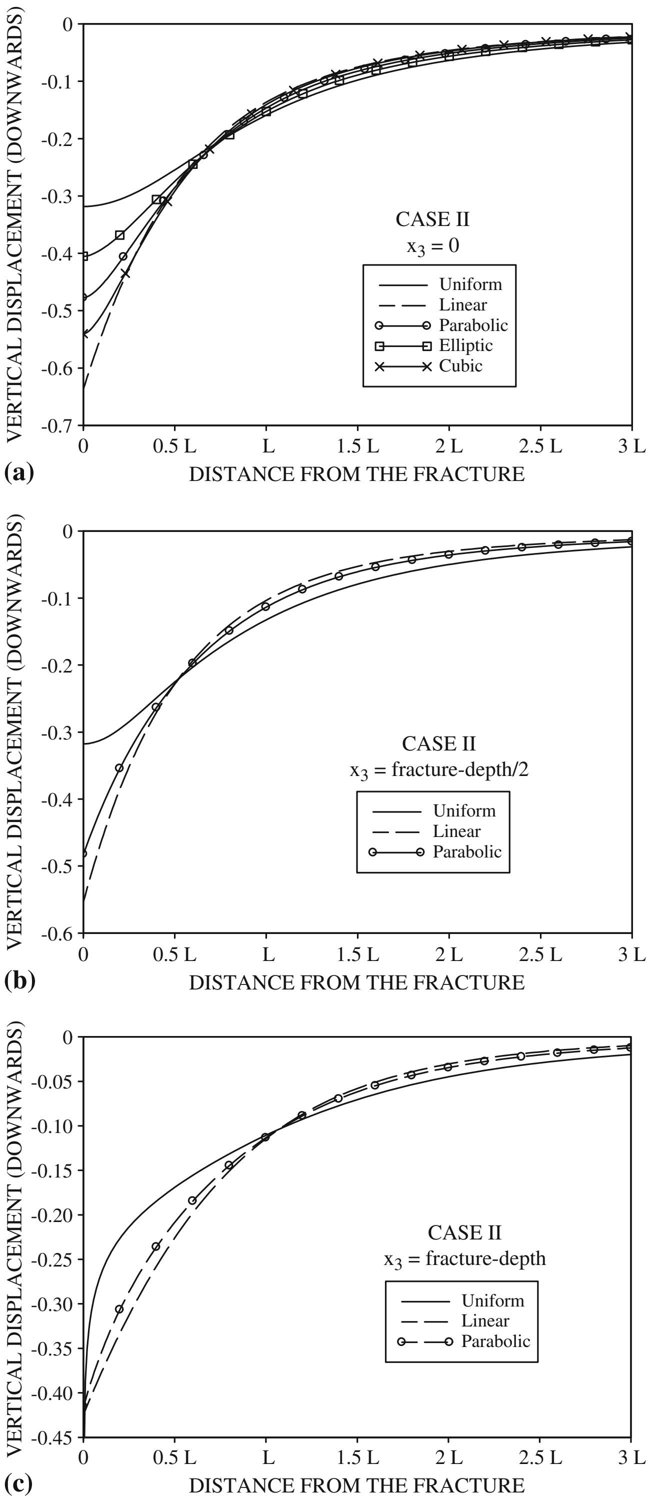

Figure 7. Variation of the vertical displacement (u_3/b) with the distance from the fracture in unit of L for various movement profiles at (**a**) $x_3 = 0$, (**b**) $x_3 =$ fracture-depth/2, and (**c**) $x_3 =$ fracture-depth.

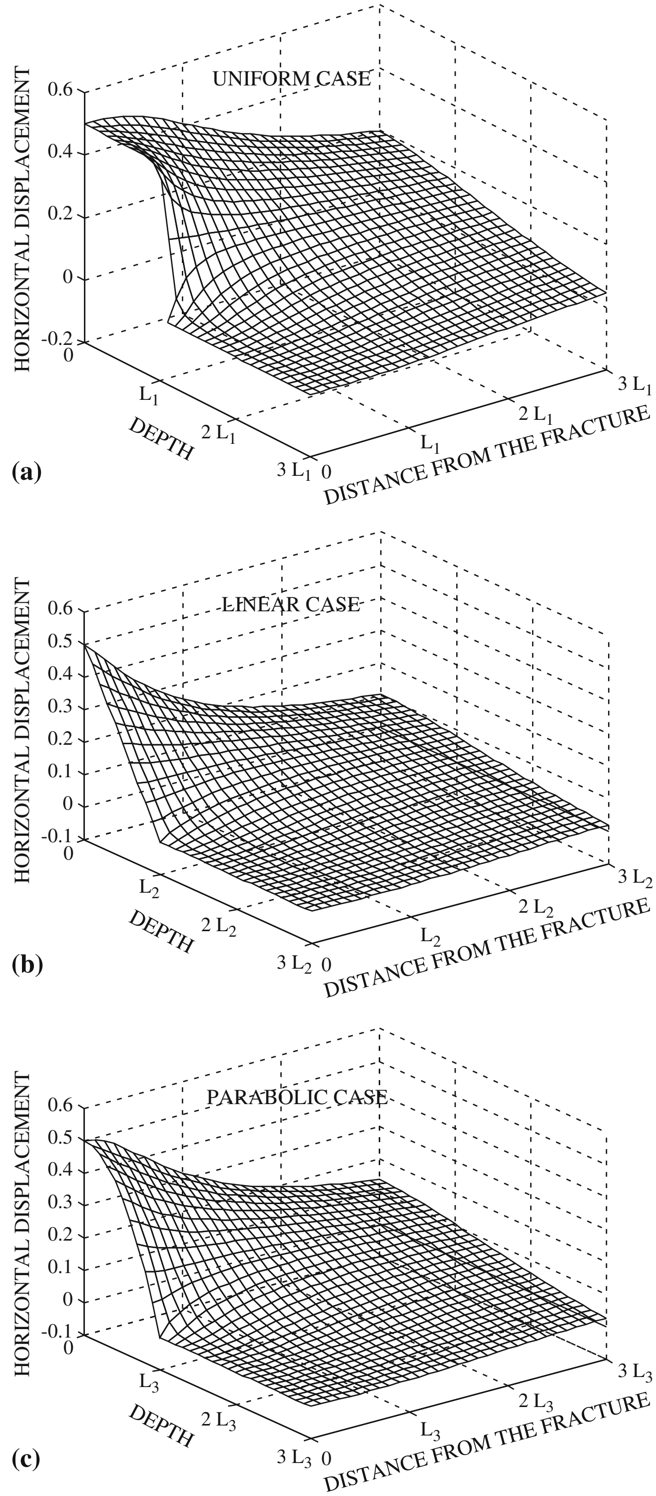

Figure 8. 3-D plot of the horizontal displacement (u_2/b_0) for (**a**) uniform case, (**b**) linear case, and (**c**) parabolic case.

linear and parabolic cases have been plotted in three dimensions in figures 8–9. The displacements are measured in units of b_0. We observe that the displacements in case of uniform movement, at the lower edge of the fracture ($Y \to 0$, $Z \to 1$ in equations (5 and 6)) are singular. However, this singularity is not present in the displacement field for linear and parabolic cases. 3-D plots show that the subsurface deformation near the fracture for different movement profiles is significantly

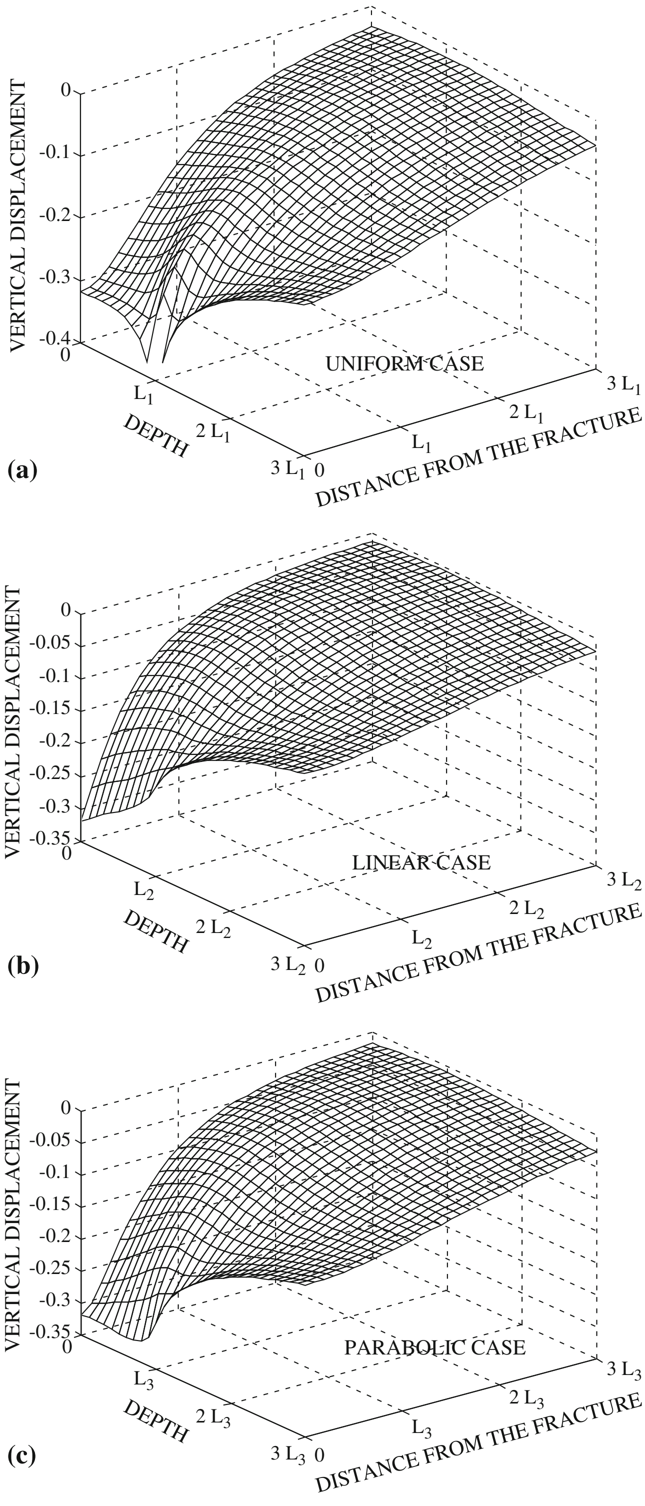

Figure 9. 3-D plot of the vertical displacement (u_3/b_0) for (**a**) uniform case, (**b**) linear case, and (**c**) parabolic case.

different. But the far field cannot see the details of the movement on the fracture.

Acknowledgements

The authors are thankful to the University Grant Commission, New Delhi for the financial support in the form of Junior Research Fellowship sanctioned to R C Verma and Major Research Project sanctioned to S Rani. They are also thankful to the reviewers for their suggestions in improving the manuscript.

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MS received 18 August 2012; revised 8 February 2013; accepted 12 February 2013