Calibration of regional ionospheric delay with uncombined precise point positioning and accuracy assessment

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A new method for the calibration of regional ionospheric delay based on uncombined precise point positioning (U-PPP) is proposed in this study. The performance of the new method was comparatively validated in terms of its accuracy and robustness with respect to the phase-smoothed pseudorange (PSP) method through two short-baseline experiments. Accuracy of the PPP-derived ionospheric delays was further assessed by interpolating them to a user station to perform single-frequency simulated kinematic PPP. Two 24-hr period datasets of four continuous operation reference system (CORS) stations were analyzed, collected during calm and disturbed ionospheric conditions, respectively. The single-frequency GPS observables from a user station, that were *a-priori* corrected by the interpolated ionospheric delays, were utilized to implement single-frequency PPP (SF-PPP). The results show that interpolation accuracy is better than 1 dm and, with the proposed method, is less affected by the ionospheric activity; meanwhile, positioning accuracy of SF-PPP was 4~5 cm (horizontal) and better than 1 dm (vertical). For comparison, two reference SF-PPP solutions were also obtained, in which the ionospheric delays are eliminated either by forming semi-combination observations or by using global ionosphere maps (GIM) model values; in both cases the positioning accuracy was only 4~7 dm (horizontal) and 1 m (vertical). These results provide a further demonstration of the performance of PPP-based regional ionospheric calibration in the parameter domain.

1. Introduction

It is well known that the ionosphere is a dominant error source for single-frequency GPS users (Klobuchar 1987; Goad 1990; Brunini and Azpilicueta 2010). Many models have been developed to mitigate this error, e.g., the Klobuchar model (Klobuchar 1987), grid-based models (Mannucci *et al.* 1998; Walter *et al.* 2000; Dow *et al.* 2005) and tomographic models (Ruffini *et al.* 1998; Shukla *et al.* 2009, 2010; Wen and Liu 2010). One prerequisite for the above ionosphere

models is the extraction of ionospheric observables from dual frequency GPS observations (Wilson and Mannucci 1994; Burrell *et al.* 2006; Rideout and Coster 2006); this is commonly achieved using the phase-smoothed pseudorange (PSP) method (Coco *et al.* 1991; Sardon *et al.* 1994). However, the smoothing process underlying PSP is sensitive to the length of the continuous satellite arc (without cycle slip occurring) and receiver-related model errors, e.g., multipath effects and observational noise (Ciraolo *et al.* 2007; Brunini and Azpilicueta 2009).

Keywords. Uncombined precise point positioning (U-PPP); regional ionospheric delay; phase-smoothed pseudorange (PSP); continuous operation reference system (CORS); interpolation; single-frequency PPP (SF-PPP).

According to recent studies (Ciraolo *et al.* 2007; Brunini and Azpilicueta 2010), separation of satellite and receiver differential code bias (DCB) from ionospheric observables is vitally important in achieving the slant total electron content (sTEC). One of the most popular models is the single-shell model of the ionosphere, which assumes that the ionosphere is compacted into a thin shell at a specific height surrounding the Earth (Mannucci *et al.* 1998; Dow *et al.* 2005). This assumption model reduces a three-dimensional problem to a two dimensional; however, the assumption is unrealistic and might result in modeling errors (Smith *et al.* 2008; Brunini and Azpilicueta 2010). Therefore, several modified ionospheric models, such as tomographic models (Shukla *et al.* 2009, 2010; Das and Shukla 2011), have been exploited.

GPS-based ionosphere studies can benefit both scientific research and practical applications. For instance, some ionospheric models, such as Kolbuchar (Klobuchar 1987), GIM products supplied by IGS (IGS 2006) and regional ionosphere maps (RIM), are usually utilized to correct ionospheric delay for single-frequency PPP (SF-PPP) users. However, the Kolbuchar model can only provide a correction for approximately 50% rms of the ionosphere range error (Klobuchar 1987). Positioning accuracies of 1~2 dm and of the meter level for static and kinematic modes, respectively, can be achieved using IGS GIMs (Le and Tiberius 2007; Zhang *et al.* 2008; Le *et al.* 2009). Furthermore, RIMs, such as the Space Weather Application Center – Ionosphere (SWACI) maps, have been made available and can provide a much higher temporal and spatial resolutions. The SWACI maps can yield a positioning accuracy of 2–3 dm (Le *et al.* 2009). In addition, semi-combination observations are also used in the SF-PPP technique, with a positioning accuracy of several dm (Zhang *et al.* 2008; Ovstedal 2002). The current accuracy of SF-PPP can only fulfill applications needing low-precision navigation and positioning, e.g., cartography (Zhang and Forsberg 2007; Kouba and Héroux 2001; Hu *et al.* 2005), and is mainly restricted by the performance of the ionosphere correction.

In general, the GIM and RIM mentioned above both use the PSP method as a first step for the retrieval of ionospheric observables. In this paper, an alternative method for the calibration of regional ionospheric delay is proposed. Precise line of sight (LOS) ionospheric delays are achieved by primarily extracting ionospheric observables from the dual-frequency GPS data with U-PPP (Keshin *et al.* 2006; Zhang 2009; Zhang *et al.* 2011a, 2011b), followed by separation of the receiver and satellite DCB. The final resultant ionospheric delays

are interpolated satellite-by-satellite and epoch-by-epoch to a SF-PPP user station to serve as that station's ionosphere corrections.

The main improvements of the proposed method are as follows:

- The LOS ionospheric observables are obtained using the U-PPP technique instead of PSP (Sardon *et al.* 1994), which can take full advantage of known reference station coordinates and available external IGS satellite clock and orbits to improve the reliability of estimated resolutions.
- After separation of receiver and satellite DCBs, the LOS ionospheric delays are interpolated satellite-by-satellite and epoch-by-epoch, allowing a better description of local ionospheric variation characteristics and improving the correction accuracy of ionospheric delay (Brunner and Gu 1991).

In the following discussion, we first introduce the principle of U-PPP, and analyze the specific formation of the ionospheric estimator; next, an analysis method is designed to evaluate the accuracy of ionospheric observables derived from either the U-PPP or PSP methods using short-baseline tests, then a method for DCB separation is introduced to obtain 'clean' LOS ionospheric delays. Based on a provincial CORS in northern China, the calibration and interpolation accuracy of the regional ionospheric delay are analyzed for different ionospheric activity conditions. The single-frequency simulated kinematic PPP is implemented by eliminating ionospheric delays with the proposed method; further, the positioning results are compared with results from commonly used GIM products and forming semi-combination observations to validate the effectiveness of the proposed method.

2. Calibration of the U-PPP-based ionospheric delay

The original GNSS dual-frequency observables were adopted in the U-PPP algorithm; LOS ionospheric observables were taken as estimate parameters together with coordinate components, receiver clock correction, zenith tropospheric delay (ZTD) and ambiguities at both frequencies. Considering the moderate variations during short periods (Abdel-Salam 2005), the two kinds of atmospheric delay parameters were modelled as random walk during the parameter estimation process. This strategy accelerates filter convergence and improves both positioning accuracy and reliability (Li 2010; Zhang *et al.* 2010; Cheng *et al.* 2011).

2.1 Mathematical model of the U-PPP

The dual-frequency GPS code and carrier-phase observation equations can be expressed as (Teunissen and Kleusberg 1998):

$$\begin{aligned} p_{r,j}^s &= \rho_r^s + dt_r - dt^s + T_r^s + I_{r,j}^s + b_{r,j} - b_j^s + \varepsilon_p \\ \phi_{r,j}^s &= \rho_r^s + dt_r - dt^s + T_r^s - I_{r,j}^s + \lambda_j \cdot M_{r,j}^s + \varepsilon_\phi \end{aligned} \quad (1)$$

where $p_{r,j}^s$, $\phi_{r,j}^s$ denote the code and phase observables from satellite s to receiver r on frequency j . ρ_r^s is the geometric range between satellite and receiver antennas; dt_r and dt^s refer to the receiver and satellite clock errors; T_r^s denotes the tropospheric delays; b_j^s and $b_{r,j}$ are the satellite and receiver code instrumental delays due to the transmitting and receiving hardware; $I_{r,j}^s = \frac{40.28}{f_j^2}$ sTEC is the dispersive ionospheric effect on GPS group signal propagation at frequency f_j , sTEC is the slant total integrated electron content; λ_j is the wavelength at frequency j ; $M_{r,j}^s$ is the carrier-phase ambiguity including satellite and receiver phase instrumental delays and initial phase bias; and ε_p and ε_ϕ refer to observational noise and multipath effects.

With the satellite positions X_s fixed, equation (1) can be linearized near the approximate receiver position X_r^0 :

$$\begin{aligned} \Delta p_{r,j}^s &= -\boldsymbol{\mu}_r^s \cdot \Delta r + T_r^s + dt_r - dt^s \\ &\quad + I_{r,j}^s - b_j^s + b_{r,j} + \varepsilon_p \\ \Delta \phi_{r,j}^s &= -\boldsymbol{\mu}_r^s \cdot \Delta r + T_r^s + dt_r - dt^s \\ &\quad - I_{r,j}^s + \lambda_j \cdot M_{r,j}^s + \varepsilon_\phi \end{aligned} \quad (2)$$

where $\Delta p_{r,j}^s$ and $\Delta \phi_{r,j}^s$ denote the observed minus calculated observations for the code and carrier-phase, for which some systematic errors (for example, phase wind-up, relativity effects and site displacement) have been *a-priori* corrected; $\boldsymbol{\mu}_r^s$ is the unit direction vector from receiver to satellite; and Δr denotes the increments of X_r^0 . The other symbols have the same meaning as in equation (1).

The precise satellite clock products from IGS can be written as (Dach *et al.* 2009; Dow *et al.* 2009):

$$dt_I^s = dt^s + \frac{f_1^2}{f_1^2 - f_2^2} \cdot b_1^s - \frac{f_2^2}{f_1^2 - f_2^2} \cdot b_2^s \quad (3)$$

where dt^s and dt_I^s denote truth and published values. A systematic discrepancy exists between the two values. Precise clock products from IGS always refer to ionosphere-free code and phase combinations; hence, the products also include the corresponding satellite instrumental biases $b_j^s, j = 1, 2$.

When precise satellite clock products (in equation 3) are used for linearization, an additional combination of b_1^s and b_2^s is introduced to both observation equations in equation (2), which can be absorbed into receiver clock error, ionospheric delay and dual-frequency ambiguity. After elimination of the rank defects arising from the partially linear-dependent unknowns in equation (2), the resulting full-rank observation equations can be expressed as:

$$\begin{aligned} \Delta p_{r,j}^s &= -\boldsymbol{\mu}_r^s \cdot \Delta r + T_r^s + dt_r^b + \mu_j \cdot I_{r,1}^{s,b} + \varepsilon_p \\ \Delta \phi_{r,j}^s &= -\boldsymbol{\mu}_r^s \cdot \Delta r + T_r^s + dt_r^b - \mu_j \cdot I_{r,1}^{s,b} \\ &\quad + \lambda_j \cdot M_{r,j}^{s,b} + \varepsilon_\phi \end{aligned} \quad (4)$$

where dt_r^b , $I_{r,1}^{s,b}$ and $M_{r,j}^{s,b}$ are the estimable receiver clock error, ionospheric observables and carrier-phase ambiguities biased by the b_j^s and $b_{r,j}$, $\mu_j = f_1^2/f_j^2$. It should be noted that equation (4) parameterizes the ionospheric group observables on the L_1 frequency, and the scalar μ_j is used to convert these observables to other frequencies.

The estimable ionospheric delay parameters can be expressed as (Chang *et al.* 2001; Li 2010; Zhang *et al.* 2010):

$$I_{r,1}^{s,b} = \beta - B_r^s = I_{r,1}^s \cdot \alpha - (B^s + B_r) \quad (5)$$

where $\alpha = (f_2^2 - f_1^2)/f_2^2$, $I_{r,1}^s$ is the LOS ionospheric delay at frequency L_1 ; $B^s = b_1^s - b_2^s$ and $B_r = b_{r,1} - b_{r,2}$ are satellite and receiver DCBs, respectively.

Assuming m satellites were simultaneously tracked by station r at epoch i , then by incorporating the linearized equations (as equation 4) for all satellites, the following compact form of the observation equations can be formed (Cheng *et al.* 2011)

$$\begin{aligned} y_i &= \begin{matrix} A_i & \cdot & X_i & + & \varepsilon_y \\ 4m \times 1 & & 4m \times (3m+5) & & (3m+5) \times 1 & & 4m \times 1 \end{matrix} \\ \varepsilon_y &\sim N(0, Q_y) \end{aligned} \quad (6)$$

$$X_i = [\Delta r_r, Z, dt_r^b, I_{r,1}^{s,b}, M_{r,j}^{s,b}]^T \quad (7)$$

where A_i is the designed matrix. The unknown vector X_i in the U-PPP includes three position coordinate parameters, a wet zenith tropospheric delay parameter, a receive clock correction parameter, LOS ionospheric observables on the L_1 frequency, and float ambiguity terms on both frequencies, where $s = 1, \dots, m$, $j = 1, 2$. In the parameter estimation processing, LOS ionospheric observable parameters are modelled as a random walk process as well as the wet zenith tropospheric delay parameter. The remainder of the parameter processing strategy is analogous to that of the traditional

PPP technique (Zumberge *et al.* 1997; Kouba and Héroux 2001; Abdel-Salam 2005).

The quantity Q_y contains information about the accuracy of observables and also the relations among them. We assume the standard deviation of each observable is elevation-dependent and omit the possible cross-correlation and time-correlation. Thus, Q_y takes the form of a diagonal matrix with its diagonal elements being expressed as:

$$Q_{ii} = \frac{\sigma_0^2}{\sin^2(E_r^s)} \quad (8)$$

where E_r^s is the elevation angle of each satellite and σ_0 is the standard deviation of the GPS observation at zenith. For code and carrier phase observations, σ_0 can be chosen as 0.3 and 0.003 m, respectively.

2.2 Dynamic model of U-PPP

The non-linear observation equations in the U-PPP firstly need to be linearized, and the linearization process usually requires that the approximate initial values of the ‘to-be-estimated’ unknowns are sufficiently precise. The Extended Kalman Filter (EKF), which uses the one-step time-updated values for the linearization of the observation equation at the current epoch, is utilized in the parameter estimation (Welch and Bishop 1995). The dynamic model of EKF can be written as:

$$X_{i+1} = \Phi_{i+1,i} \cdot X_i + \omega_i, \quad \omega \sim N(0, Q_\omega) \quad (9)$$

$$\Phi_{i+1,i} = \begin{bmatrix} P_{3 \times 3} & & & & \\ & 1 & & & \\ & & 0 & & \\ & & & I_{m \times m} & \\ & & & & I_{2m \times 2m} \end{bmatrix} \quad (10)$$

$$Q_\omega = \begin{bmatrix} q_p \cdot \Delta t_{3 \times 3} & & & & \\ & q_z \cdot \Delta t & & & \\ & & q_t \cdot \Delta t & & \\ & & & q_I \cdot \Delta t_{m \times m} & \\ & & & & 0_{2m \times 2m} \end{bmatrix} \quad (11)$$

where X_i and X_{i+1} are the unknown vectors (as listed in equation 7) at consecutive epochs i and $i + 1$, $\Phi_{i+1,i}$ is the transition matrix, ω is the normally-distributed process noise with zero-mean and variance-covariance (VC) matrix Q_ω , and Δt is the time interval between adjacent epochs.

In equation (10), the sub-matrix P of $\Phi_{i+1,i}$ corresponds to the positions, and can be chosen either

as the identity or the zero matrix during static or kinematic measurements, which means that the positions are modelled as time-invariant or white noise process. The remaining four sub-matrices in $\Phi_{i+1,i}$ correspond to the transition matrices of the ZTD, receiver clock error, LOS ionospheric observables and ambiguities, which again are either chosen as the identity or zero matrix. In equation (11), the five spectral density matrices of the above-mentioned unknowns are listed, and the actual values of the elements in these matrices depend mainly on the practical applications (Euler and Goad 1991; Goad 1990).

Considering that the coordinates for each reference station are known during generation of the PPP-based ionospheric delay, the positioning parameters are thus constrained as constants. Therefore, the estimation parameters then comprise only the receiver clock error, ZTDs, LOS ionospheric observables and ambiguities on both frequencies.

Based on the above-mentioned dynamic model (equation 9) and observation equation (equation 6), the recursive EKF computational scheme is given in figure 1 (Teunissen 2000). Subscript i refers to the epoch of time, and the respective EKF algorithm involves Kalman Gain (K), covariance update (P_i) and prediction (P_{i+1}), in the time update and observation update steps, as shown in figure 1.

Figure 1. The recursive Extended Kalman Filter (EKF) computational scheme.

At present, some commercial applications for GPS positioning use Differential GPS (DGPS) technology to mitigate the effect of ionospheric delay and other errors (Monteiro *et al.* 2005). The U-PPP, as a positioning technique, has also attracted GPS community. The main differences between DGPS technique and U-PPP include:

- DGPS is a differential positioning technique using at least two simultaneously operating receivers; U-PPP is, however, based upon a stand-alone GPS receiver to perform point positioning.
- The user station of DGPS receives per satellite the differential corrections in the observation domain, which are lump sums of satellite ephemeris errors, ionospheric delays, tropospheric delays and satellite clocks; while the U-PPP technique relies on the satellite orbits/clocks provided by external data source, such as International GNSS Service.
- DGPS adopts code or carrier-phase smoothed code as fundamental observables. Therefore, the achievable positioning accuracy is severely affected by the unmodeled code noise and multipath effects and is of several decimeters' accuracy; In contrast, U-PPP optimally processes GPS code and carrier-phase observables. The obtainable cm-level's positioning accuracy thus benefits largely from carrier-phase, with 100 times less observational noise and multipath than that of the code.

3. Short baseline tests

In this section, we describe tests with short-baselines designed to compare the accuracy of estimated ionospheric observables using both the PPP and PSP methods. Firstly it should be pointed out that, either in the PPP or PSP methods, the derived ionospheric observables can be expressed as (Chang *et al.* 2001; Li 2010; Zhang *et al.* 2010):

$$\bar{I}_{r,1}^s = I_{r,1}^s - \frac{1}{\alpha} (B^s + B_r) \quad (12)$$

where $\bar{I}_{r,1}^s$ is the estimated value, and the other symbols have the same meaning as in equation

(5). Assuming the receivers of a short baseline are r_p and r_q respectively, the ionospheric observables from satellite s to the two receivers are:

$$\begin{aligned} \bar{I}_{p,1}^s &= I_{p,1}^s - \frac{1}{\alpha} (B^s + B_{p,r}), \\ \bar{I}_{q,1}^s &= I_{q,1}^s - \frac{1}{\alpha} (B^s + B_{q,r}). \end{aligned} \quad (13)$$

For a short baseline measurement, the ionospheric delay can be regarded as the same, i.e., $I_{p,1}^s = I_{q,1}^s$, so the single difference (SD) of ionospheric observables between two receivers is:

$$\bar{I}_{p,1}^s - \bar{I}_{q,1}^s = \frac{1}{\alpha} (B_{q,r} - B_{p,r}). \quad (14)$$

From equation (14), only the DCBs of two receivers are included in the SD observations. When $\bar{I}_{r,1}^s$ is not affected by any model errors, such as multipath and measurements noise in short baseline measurements, the term $\bar{I}_{p,1}^s - \bar{I}_{q,1}^s$ for different satellites would be a constant; however, for larger deviations of $\bar{I}_{p,1}^s - \bar{I}_{q,1}^s$ between different satellites, $\bar{I}_{r,1}^s$ is more affected by model errors (Ciraolo *et al.* 2007). Therefore, variations in $\bar{I}_{p,1}^s - \bar{I}_{q,1}^s$ indicate the accuracy of estimated ionospheric observables achieved by the two methods.

Two short baselines were utilized in the tests. Table 1 summarizes their descriptions. The distances of the baselines were 18.2 and 12.0 m respectively. Two 24-hour datasets were adopted to estimate the ionospheric observables. Figure 2 shows the SD of PSP- and PPP-based ionospheric observables from the two baselines, where panels (a) and (b) show the SD ionospheric observables between stations YAR2 and YAR3, and panels (c) and (d) show the SD ionospheric observables between stations MOBK and MOBN. To facilitate the comparison, the range of the vertical axis in each panel pair is kept equal.

In figure 2, the left panels show the SD results from the PSP method, and the right panels show the corresponding results from the PPP estimation. Notably, $\bar{I}_{p,1}^s - \bar{I}_{q,1}^s$ values of each satellite do not coincide with each other, which demonstrates that $\bar{I}_{r,1}^s$ includes some model errors; in panels (b) and (d), the deviations between individual SD observations are obviously less than those in

Table 1. Descriptions of short baselines utilized in the experiments.

Station	Receiver type	Antenna type	Location	Observing sessions	Baseline length
YAR2	ASHTECH UZ-12	AOAD/M-T	115.3°E, 29.0°S	DOY 200/2011	18.25 m
YAR3	LEICA GRX1200GGPRO	LEIAR25			
MOBK	JPS EGGDT	JPSREGANT_SD_E	36.6°E, 55.1°N	DOY 200/2011	12.04 m
MOBN	ASHTECH Z-XII3	ASH701945C_M

Figure 2. The single difference (SD) of PSP-based (left panels) and PPP-based (right panels) ionospheric observables obtained from two baselines. Different colours represent values for different satellites.

panels (a) and (c), which represents that the PPP method can weaken the model errors related to receiver included in $\bar{I}_{r,1}^s$.

4. Calibration and interpolation of regional ionospheric delay

In this section, the main objective is to introduce a mathematical model for separating DCBs from regional LOS ionospheric observables with U-PPP, and to present the algorithm for the interpolation of ‘clean’ LOS ionospheric delays for a user station.

4.1 Separation of DCBs

The DCBs $B_r + B^s$ in equation (5) can be estimated using an ionospheric shell model and a suitable mapping function (Sardon *et al.* 1994; Chang *et al.* 2001). It is usually assumed that all free electrons in the Earth’s ionosphere are concentrated on the ionospheric shell at an altitude H (typically chosen as 350, 400 or 450 km) over the Earth’s surface. Meanwhile, sTEC values are mapped to the vertical TEC (vTEC) at the ionospheric pierce

point (IPP). The corresponding equation can be expressed as (Chang *et al.* 2001):

$$I_{s\text{TEC}} \approx \sec z' \cdot I_{\text{vTEC}} \quad (15)$$

Here, z' is the zenith distance at the IPP, which can be expressed as:

$$z' = \arccos \left[\sqrt{1 - \left(R/R + H \right)^2 \cdot \cos^2(z)} \right], \quad (16)$$

where R is the mean earth radius; H is the height of the single layer (chosen in this paper to be 350 km) and z is the zenith distance at the receiver. Next, characteristics of the spatial distribution of I_{vTEC} are represented by a proper mathematical function. In the present paper, a bi-linear polynomial is utilized for ionosphere modeling at a single station:

$$I_{\text{vTEC}} = a(t) + b(t) \cdot dx + c(t) \cdot dy \quad (17)$$

where t denotes the observation epoch; dx and dy can be expressed as $dx = (\lambda_P - \lambda_R) \cdot \cos(\varphi)$ and $dy = \mu_P - \mu_R$, respectively; λ and ϕ denote geocentric longitude and latitude, respectively, μ is geomagnetic latitude; and the subscripts P and R

Figure 3. Flow chart for the proposed U-PPP-based regional ionospheric calibration and interpolation algorithm.

denote IPP and receiver location. In this paper, unknowns a , b and c are assumed to be time-dependent piecewise functions, which are constant in the interval of $[t_k, t_k + \Delta t]$, namely $a(t) = a_k$, $b(t) = b_k$, $c(t) = c_k$, where Δt is a refreshing interval taken to be 5 minutes (Brunini and Azpilicueta 2010).

Combining equations (5) and (15–17), the observation equation of the satellite and receiver DCBs based on U-PPP is:

$$\begin{aligned} \iota_r^s = & \sec z' \cdot [a_k(t) + b_k(t) \cdot dx \\ & + c_k(t) \cdot dy] \cdot \alpha - B_r^s \end{aligned} \quad (18)$$

where t is in the period of $[t_k, t_k + \Delta t]$, the other symbols have the same meaning as in equation (5).

To solve equation (18) for B_r^s , the Recursive Least Square method (Teunissen 2001) is employed by combining the ionospheric observables of all reference stations. It should be noticed that B_r^s is elevation-independent while the LOS ionospheric delay is elevation-dependent; this dependence ensures the high accuracy of the estimation for B_r^s . The estimation accuracy of satellite DCBs with the PPP method can reach 0.1 ns (Zhang *et al.* 2011a, 2011b). By eliminating B_r^s from equation (5), ‘clean’ LOS ionospheric delays can be obtained.

4.2 Interpolation of ionospheric delay

To achieve a high precision SF-PPP at a user station, ionospheric delays are interpolated satellite-by-satellite and epoch-by-epoch, which yields a better prediction of ionospheric delay at the user station. This method is good for representing local ionospheric variability considering the same ionosphere conditions between one satellite and different reference stations, the position results should

be more reliable with this method. The interpolation processes is as follows:

- In the current epoch, for satellite s , convert ‘clean’ LOS sTEC to vTEC at the IPPs of each reference station using equation (16), meanwhile calculating and saving each IPP location.
- Calculate the IPP location of user station, then calculate the distances of IPPs from user station to each reference station.
- Following the distance-weighting strategy, interpolate vTEC at the IPPs of the user station, then convert calculated vTEC into sTEC using the mapping function in equation (15), for the single-frequency observables correction.

A flow chart for the proposed U-PPP-based regional ionospheric calibration and interpolation method is shown in figure 3.

5. Experimental results and analysis

To validate the performance of proposed algorithm, datasets from two days (DOY 119/2009 and 218/2009) for five reference stations in a provincial CORS in northern China were adopted, and the reference stations are equipped with dual-frequency receivers. One day had calm ionospheric conditions, and the other had disturbed conditions (<http://ftp.gwdg.de/pub/geophys/kp-ap/tab/>). The inter-station distances in the CORS network ranged from 70 to 100 km (see figure 4).

The station AG is a simulated user station with inter-station distances to reference stations of about 50–60 km. Using the two days of data described above, figure 5 shows the discrepancy between ionospheric interpolation and U-PPP-based ionospheric delays at the user station. The satellites PRN 2 and 29 were chosen as examples.

Figure 4. Configuration of experimental reference stations and inter-station distance. AG is a simulated user station.

Figure 5 shows that interpolation of the ionospheric delay for a given satellite did not change significantly between the two days (for example, as shown by PRN 29), which indicates that the proposed interpolation algorithm is less affected by

ionospheric activity. In view of the high precision of U-PPP-based ionosphere delays, the discrepancy in figure 4 reflected interpolation accuracy properly; furthermore, most of the interpolation errors were less than 5 cm, which further demonstrates the feasibility and reliability of the proposed algorithm for regional ionospheric calibration and interpolation.

We now consider the test data for DOY 218/2009 as an example. The simulated kinematic SF-PPP at station AG was implemented, with single-frequency observables corrected by interpolated ionospheric delays. The positioning results and filter convergence characteristics are shown in figure 6; positioning results from semi-combinations and the GIM model are also shown for comparison. Statistics of the results obtained from the three SF-PPP methods are summarized in table 2.

The differences between the three positioning results in figure 6 are merely owing to different methods for ionosphere correction. The proposed algorithm used the U-PPP method to extract the LOS ionospheric delays and interpolated results to the user station satellite-by-satellite and epoch-by-epoch; the semi-combination method used a combination of single frequency pseudorange and

Figure 5. Accuracy of interpolated ionospheric delays from, respectively, PRN 2 and PRN 29 at a simulated user station AG (U-PPP-based ionospheric delays are regarded as the reference values; upper and lower represent Day of Year (DOY) 119/2009 and 218/2009, respectively).

Figure 6. Positioning accuracy of simulated kinematic single-frequency PPP (from left to right: ionospheric delays corrected by regional interpolated values, semi-combinations and GIM model).

Table 2. Summary statistics of component errors with single-frequency PPP, in meters.

Component	Proposed algorithm			Semi-combination			GIM model		
	Means	STD	RMS	Means	STD	RMS	Means	STD	RMS
North	-0.04	0.03	0.04	-0.07	0.39	0.39	-0.06	0.52	0.60
East	0.01	0.05	0.05	-0.01	0.43	0.43	0.07	0.59	0.67
Up	0.03	0.06	0.08	0.06	0.69	0.76	0.17	0.98	1.09

carrier phase to eliminate the ionospheric delay; and the GIM model provided the $v\text{TEC}$ of grid points using the PSP method for LOS ionospheric calibration. Other parameters, such as observation noise and process noise, were given the same values.

From figure 6 and table 2, kinematic positioning results from the regional interpolated ionosphere have a better convergence characteristics, such that positioning resolutions can reach sub-meter accuracy in about 30 minutes; after filter convergence, positioning accuracy was 4~5 cm (horizontal) and better than 1 dm (vertical). By contrast, positioning accuracies from semi-combination observations and the GIM model correction were both approximately 4~7 dm (horizontal) and 1 m (vertical), respectively. Positioning accuracy for the semi-combination method was slightly better than that from the GIM model, perhaps because of stronger influence of ionospheric disturbances in the latter case.

6. Conclusions and perspectives

In ionosphere studies, the high precision calibration of ionospheric delays is a prerequisite. A new algorithm is proposed in this study, and includes: calibration of U-PPP-based regional biased

ionospheric observables; satellites and receiver DCB separation, ionospheric interpolation to a user station satellite-by-satellite and epoch-by-epoch, and single-frequency kinematic PPP implementation with interpolated ionospheric delay. A method for assessing the accuracy of estimated ionospheric observables was presented, and results show that the PPP-based calibration of ionospheric observables improves the calibration accuracy and is more resistant to receiver-related model errors.

Two 24-hour datasets for four reference stations in a provincial CORS in northern China were analyzed with the proposed method. The results for the new method show that interpolation accuracy is better than 1 dm and is less affected by ionospheric activity; positioning accuracy of simulated kinematic single-frequency PPP is 4~5 cm (horizontal) and better than 1 dm (vertical). The positioning accuracy was therefore significantly improved when compared with the existing semi-combination model and GIM model, this broaden the scope of potential applications of single-frequency PPP.

The single-frequency PPP is flexible in operation and free from inter-station distances between base stations and user station. It also has the advantage of optimally utilizing the GPS code and carrier-phase observables and gives a better positioning accuracy than the DGPS. Based on the existing

network-RTK technique, the proposed method has the potential to serve single-frequency users.

In order to further improve the accuracy and reliability of regional ionospheric calibration, some improvements can be considered. Firstly, the single shell ionospheric model is utilized for converting sTEC to vTEC, according to Brunini and Azpilicueta (2010), during the single-shell ionospheric modelling with a single GPS site, the total errors caused by above-mentioned mapping function, the shell height and the vTEC mathematic representation are not more than ± 2 TECu when the ionosphere condition is not very active. However, the single shell model may not give reliable estimate of vTEC when the satellite elevation angle is less than 50 degree. If a larger and denser regional network is chosen, it could be replaced by advanced tomography (Shukla et al. 2009, 2010; Wen and Liu 2010).

Secondly, taking the spatial correlation of the ionosphere into consideration, the distance weighting strategy was used to retrieve the ionospheric delays at the simulated user position. The maximum baseline length that could be used in this strategy is inevitably affected by the ionospheric conditions due to the severe spatial decorrelation of the ionosphere; the greater the ionospheric disturbance, the shorter the maximum baseline length. In our test, considering that the reference stations and the simulated user site are all located in mid-latitude regions, and that the distances are only 50–60 km, the maximum baseline length employed in our distance weighing strategy is therefore no more than 60 km. As for the low latitude region where the Equatorial Ionization (EIA) is quite prevalent, the valid maximum baseline length could be shorter when using this strategy. A better method for reconstruction ionospheric delays could be used (Foster and Evans 2008).

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