Statistical models of interoccurrence times of Iranian earthquakes on the basis of information criteria

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By analyzing the seismic catalogue of Iran, the probability distributions of interoccurrence times of earthquakes were investigated for different seismotectonic settings. Several probability distributions were applied to data from major seismotectonic provinces in different cut-off magnitudes and the distribution parameters were determined through the method of maximum likelihood. With the help of goodness-of-fit tests (AIC and BIC criteria based on information theory, Kolmogorov–Smirnov test) and the coefficient of determination, we have found that the gamma statistics and generalized normal statistics coexist in interoccurrence time statistics. Our results imply that a transition from a generalized normal regime to a gamma regime occurs if the threshold magnitude in certain seismotectonic regions (Alborz–Azarbayegan, Zagros, and Central-East Iran) is changed.

1. Introduction

Earthquakes are phenomena exhibiting great complexity characterized by many empirical statistical laws (Rundle *et al* 2003; Hasumi *et al* 2009). Despite this complexity, by neglecting the spatial scale of earthquake rupture zones and the temporal scale of the duration of each earthquake, one can consider the occurrence of earthquakes to be a point process in space and time (Vere-Jones 1970; Ogata 1999; Daley and Vere-Jones 2002; Shcherbakov *et al* 2005). It is then possible to study the statistical properties of this process and test models that may explain the observed seismic activity (Shcherbakov *et al* 2005). For the understanding of long-term variation in earthquake occurrences, earthquake predictions, and seismic

risk estimates, it is necessary to study the frequency or probability distributions of interoccurrence times of earthquakes (Wang and Kuo 1998).

Early studies led to the conclusion that main sequence events were significantly non-Poissonian, regardless of whether worldwide catalogues (Aki 1956) or local ones (Knopoff 1964; Isacks *et al* 1967; Singh and Sanford 1972) were considered. Utsu (1984) applied four models (Weibull, gamma, lognormal and exponential) to several seismic regions of Japan and concluded that the four models represent the distributions of interoccurrence times fairly well, though different models are best suited for different sets of data. Recently, a unified scaling law of interoccurrence times was proposed using the Southern California catalogue (Bak *et al* 2002) and global earthquake

Keywords. Information criteria; interoccurrence time model; seismotectonic province; generalized gamma family; lognormal–gamma transition.

Figure 1. Major seismotectonic provinces of Iran (Mirzaei *et al* 1998). Selected active faults are also added.

catalogues (Corral 2004). In these works, inter-occurrence times were analyzed for the events with a magnitude (m) above a certain threshold where the following two presumptions apply:

- earthquakes can be considered as a point process in space and time;
- there is no distinction between foreshocks, main-shocks, and aftershocks (Hasumi *et al* 2009).

Abe and Suzuki (2005) showed that the survivor function of the interoccurrence time can be described by a power law. It has been reported that the sequence of aftershocks and successive independent earthquakes is a Poisson process (Nanjo *et al* 2005; Abaimov *et al* 2007). Hasumi *et al* (2009) studied the effect of changing the threshold magnitude on the interoccurrence time statistics by analyzing the Japan Meteorological Agency (JMA) seismic catalogue. They showed that the probability distributions of interoccurrence times of successive earthquakes can be described by the superposition of the Weibull distribution and the log-Weibull distribution.

In this paper, we have paid special attention to seismotectonic features more than the previous works by Bak *et al* (2002), Lennartz *et al* (2008), Hasumi *et al* (2009) and others. The probability distributions of interoccurrence times of

earthquakes are studied and the change in inter-occurrence time statistics produced by varying of the cut-off magnitude (m_c) is traced; first, in major seismotectonic provinces of Iran (figure 1) and then, in cells of size L degrees in longitude

Figure 2. The spatial areas of Iran covered with 12 cells of size $5^\circ \times 5^\circ$, with no weight assigned to seismotectonic differences.

and L degrees in latitude which cover the spatial areas of Iran, with no weight assigned to seismotectonic differences (figure 2). Comparison of results displays the effect of region selection in the interoccurrence times of earthquakes.

To determine the distribution function of the interoccurrence times, it was observed that in previous works, gamma, generalized gamma, Weibull, exponential, etc., distributions have been used. All of these are specific cases of generalized gamma (GG) distribution. Consequently, we focused on the GG distribution and subfamilies of it in this study.

Other procedures discussed in this study include fitting distributions to the data, estimation of distribution parameters using the maximum likelihood estimation method and selection of a suitable distribution via the Akaike and Bayesian information criteria, the Kolmogorov–Smirnov test, and the coefficient of determination.

2. Data analysis and applicable distributions

In this study, the initial goal is to determine the probability distributions for interoccurrence times of earthquakes with different values of m_c in the major seismotectonic provinces of Iran. A seismotectonic province is defined as an area that, under the present-day geodynamic regimes, has a comparable tectonic setting and unified seismicity pattern (Ye *et al* 1993, 1995). Mirzaei *et al* (1998) delineated five major seismotectonic provinces in Iran based on available seismicity, geological, tectonic and geophysical information: Alborz–Azarbayejan, Kopeh Dagh, Makran, Zagros and Central-East Iran provinces. Figure 1 shows these major seismotectonic provinces of Iran (Mirzaei *et al* 1998).

The highly seismic regions of Alborz–Azarbayejan covering north and northwest of Iran, which constitute a part of northern limit of Alpine-Himalayan orogenic belt, has experienced intense seismic activity throughout the province during historical and instrumental time periods. In northeast, the continental collision zone of Kopeh Dagh has the lowest level of seismic activity in Iran, but it is characterized by the occurrence of highest portion of large magnitude earthquakes. In the intraplate environment of Central-East Iran, unlike Zagros and Alborz–Azarbayejan, seismicity is mainly concentrated on several seismogenic fault zones surrounding relatively stable microcontinental fragments. East part shows more intense seismicity, in which major seismic activity is concentrated in a few patches. In southwest of

Iran, the continental–continental collision zone of Zagros is seismically very active. It is the most seismic region of Iran (over 50% of teleseismically recorded earthquakes in Iran have occurred in Zagros). Zagros is characterized by the occurrence of mainly low and moderate intensity earthquakes; there is no record of a great earthquake ($M_s \geq 8.0$) in this region. In southeast Iran, the oceanic–continental subduction zone of Makran exhibits a strong segmentation between east and west in the seismic behaviour. The plate boundary in eastern Makran ruptured in the great thrust earthquake of 1945, $M_w = 8.1$, and currently experiences low and moderate intensity earthquakes. In contrast, western Makran exhibits no documented great earthquake. The modern instrumentation shows obvious gap in seismicity along the plate boundary in western boundary.

The earthquake catalogue used in the current study contains occurrence times and hypocentre locations of Iranian earthquakes and is derived from the seismic catalogue published by Mirzaei *et al* (2002) for earthquakes during 1975–2000. The International Seismological Centre catalogue (www.isc.ac.uk) was used to update the catalogue data up until the year 2008.

In our catalogue, m_b or M_s or both magnitude scales were assigned to earthquakes. To make the uniform catalogue of earthquakes, m_b was converted into M_s . The M_s – m_b relationships shown in table 1 were applied to those events for which no M_s was assigned, and earthquakes with known M_s value were excluded from the conversion and added to the catalogue directly. This earthquake catalogue includes earthquakes with a magnitude of $M_s \geq 4.0$ – a value used because complete recording of earthquakes with that magnitude in the major seismotectonic provinces of Iran began in 1975 (Mirzaei *et al* 1997).

The method of our spatiotemporal analysis is similar to the work of Bak *et al* (2002), Lennartz *et al* (2008) and Hasumi *et al* (2009). However, different information criteria were used to select the appropriate models and more weight has been given to seismotectonic features than all previous works. For each province, earthquakes with magnitude, M_s above a certain cut-off magnitude, m_c , were considered. No events were eliminated; all shocks were considered equally. The interoccurrence times were analyzed and then the data fitting was performed in the time domain. The interoccurrence times of earthquakes were defined using

$$T_i = t_i - t_{i-1}, \quad (1)$$

where T_i is the time coordinate of the i th earthquake within the province with $m > m_c$.

Table 1. M_s – m_b relationships for different earthquake regions of Iran (Mirzaei et al 1997).

Earthquake region	M_s – m_b relationship	Magnitude range
Alborz–Azarbayejan and Kopeh Dagh	$M_s = 2.01, m_b = 5.28$	$4.0 \leq m_b < 6.2$
Central-East Iran	$M_s = 1.79, m_b = 4.32$	$4.0 \leq m_b < 6.2$
Zagros	$M_s = 2.00, m_b = 5.28$	$4.1 \leq m_b < 6.2$
Makran	$M_s = 1.58, m_b = 3.11$	$4.0 \leq m_b < 5.9$

Table 2. Probability density functions of the generalized gamma and subfamilies of it.

Distribution	Probability density function		
Generalized gamma	$P_{GG}(y \alpha, \tau, \lambda) = \frac{\tau}{\lambda \Gamma(\alpha)} \left(\frac{y}{\lambda}\right)^{\alpha\tau-1} e^{-\left(\frac{y}{\lambda}\right)^\tau}$	$y \geq 0$	$\tau, \alpha, \lambda > 0$
Exponential	$P_{Exp}(y \lambda) = \frac{1}{\lambda} e^{-\left(\frac{y}{\lambda}\right)}$	$y \geq 0$	$\lambda > 0$
Gamma	$P_G(y \alpha, \lambda) = \frac{1}{\lambda \Gamma(\alpha)} \left(\frac{y}{\lambda}\right)^{\alpha-1} e^{-\left(\frac{y}{\lambda}\right)}$	$y \geq 0$	$\alpha, \lambda > 0$
Weibull	$P_W(y \tau, \lambda) = \frac{\tau}{\lambda} \left(\frac{y}{\lambda}\right)^{\tau-1} e^{-\left(\frac{y}{\lambda}\right)^\tau}$	$y \geq 0$	$\tau, \lambda > 0$
Generalized normal	$P_{GN}(y \alpha, \lambda) = \frac{2}{\lambda \Gamma(\alpha)} \left(\frac{y}{\lambda}\right)^{2\alpha-1} e^{-\left(\frac{y}{\lambda}\right)^2}$	$y \geq 0$	$\alpha, \lambda > 0$
Half-normal	$P_{HN}(y \sigma) = \frac{\sqrt{2}}{\sigma \Gamma(0.5)} e^{-\frac{1}{2}\left(\frac{y}{\sigma}\right)^2}$	$y \geq 0$	$\sigma > 0$
Rayleigh	$P_R(y \sigma) = \frac{y}{\sigma^2} e^{-\frac{1}{2}\left(\frac{y}{\sigma}\right)^2}$	$y \geq 0$	$\sigma > 0$
Maxwell–Boltzmann	$P_{MB}(y \lambda) = \frac{2}{\lambda \Gamma(1.5)} \left(\frac{y}{\lambda}\right)^2 e^{-\left(\frac{y}{\lambda}\right)^2}$	$y \geq 0$	$\lambda > 0$

$\Gamma(\cdot)$ is the gamma function, α and τ are shape parameters and λ and σ are the scale parameters.

Our main goal in this study is to determine the distribution function of the interoccurrence time.

We focussed on the GG distribution and subfamilies of it such as exponential ($\alpha = \tau = 1$), gamma ($\tau = 1$), Weibull ($\alpha = 1$), lognormal ($\alpha \rightarrow \infty$) and generalized normal ($\tau = 2$). The generalized normal itself is a flexible family and includes half-normal ($\alpha = 1/2, \lambda^2 = 2\sigma^2$), Rayleigh ($\alpha = 1, \lambda^2 = 2\sigma^2$), and Maxwell-Boltzmann ($\alpha = 3/2$). (For more details, see Khodabin and Ahmadabadi 2010.) The probability density functions of these distributions are displayed in table 2. The maximum likelihood estimation (MLE) method was applied to estimate the parameters of above distributions.

In order to detect whether a specific distribution is preferred, three different goodness-of-fit tests were used: Akaike and Bayesian information criteria, and the Kolmogorov–Smirnov test.

Akaike and Bayesian information criteria were used first. Akaike's information criterion (AIC) is a measure of the goodness-of-fit for an estimated statistical model (Akaike 1973, 1974). The AIC is not a test of the model in the sense of hypothesis testing; rather it is a test between models – a tool for model selection. Given a dataset, several competing models may be ranked according to their AIC, with the one having the lowest AIC being the best. The Bayesian information criterion

(BIC), or Schwarz criterion, is used for model selection among a class of parametric models with different numbers of parameters. Choosing a model to optimize BIC is a form of regularization and is closely related to AIC. Generally, the AIC and BIC criteria are given by:

$$\text{AIC} = 2K - 2\log(L(\hat{\theta})). \quad (2)$$

$$\text{BIC} = K \log n - 2\log(L(\hat{\theta})). \quad (3)$$

where K is the number of parameters in the statistical model, n is the number of data, and $L(\hat{\theta})$ is the maximized value of the likelihood function for the estimated model ($L(\hat{\theta}) = \prod_{i=1}^n f_{\theta}(x_i)$).

The Kolmogorov–Smirnov (K–S) test was performed next. This test is used to decide if a sample comes from a hypothesized continuous distribution. It is based on the empirical cumulative distribution function, $F_n(x)$, assuming a random sample $x_1, \dots, x_n$ from a continuous distribution with a cumulative distribution function, $F(x)$. The K–S statistic value (D) is based on the largest vertical difference between $F(x)$ and $F_n(x)$. The preferred distribution is the one with the smallest value of D_n (D'Agostino and Stephens 1986; Rayner and Best 1989). It is defined as:

$$D_n = \sup_x |F_n(x) - F(x)|. \quad (4)$$

Table 3. Estimated parameters and AIC, BIC, K-S and R^2 values for different distribution functions and cut-off magnitudes in major seismotectonic provinces of Iran.

Province	m_c	Distribution*	Parameters				AIC	BIC	K-S	R^2
			α	λ	τ	σ				
Alborz–Azarbayegan	4.0	Gamma	0.339	115.441			2504.673	2512.152	0.052	0.994
		GG	0.267	138.390	1.212		2505.549	2516.768	0.051	0.995
		GN	0.146	194.457	2.000		2511.105	2518.585	0.049	0.995
		Weibull		20.212	0.465		2527.471	2534.950	0.084	0.981
		Exponential		39.101			2904.340	2908.080	0.247	0.948
		Half-normal				74.350	3133.502	3137.242	0.370	0.850
		Rayleigh				52.573	4514.115	4517.855	0.518	0.709
		Maxwell		60.706			6012.882	6016.622	0.587	0.642
	4.5	GN	0.132	536.021			1001.840	1007.148	0.082	0.986
		GG	0.076	601.855	3.318		1002.929	1010.891	0.079	0.988
		Gamma	0.286	393.065			1008.285	1013.593	0.113	0.976
		Weibull		50.359	0.397		1025.829	1031.137	0.139	0.954
		Exponential		112.603			1204.012	1206.666	0.278	0.948
		Half-normal				194.539	1261.249	1263.903	0.336	0.886
		Rayleigh		194.539		137.560	1797.395	1800.049	0.467	0.768
		Maxwell		158.840			2373.432	2376.086	0.523	0.704
	5.0	GN	0.119	1372.390			464.540	468.063	0.152	0.945
		GG	0.098	1439.825	2.372		466.429	471.713	0.000	0.947
		Gamma	0.253	1080.403			467.785	471.307	0.173	0.930
		Weibull		112.050	0.350		479.250	482.772	0.200	0.893
		Exponential		273.274			570.501	572.262	0.278	0.967
		Half-normal				472.510	594.011	595.772	0.310	0.909
		Rayleigh				334.115	846.123	847.884	0.454	0.786
		Maxwell		385.803			1114.571	1116.332	0.534	0.719
Makran	4.0	Gamma	0.553	224.351			1104.362	1109.511	0.043	0.998
		GN	0.226	418.661			1105.713	1110.863	0.078	0.992
		GG	0.414	291.289	1.242		1105.970	1113.694	0.080	0.997
		Weibull		98.247	0.681		1107.063	1112.212	0.053	0.996
		Exponential		124.166			1131.395	1133.970	0.144	0.981
		Half-normal				198.830	1169.539	1172.113	0.276	0.918
		Rayleigh				140.594	1398.817	1401.392	0.437	0.802
		Maxwell		162.344			1664.948	1667.523	0.512	0.739
	4.5	Gamma	0.464	645.865			453.226	456.337	0.084	0.987
		Weibull		199.613	0.597		453.816	456.927	0.102	0.987
		GG	0.822	291.166	0.674		455.173	459.839	0.146	0.986
		GN	0.181	1378.172			458.183	461.294	0.174	0.971
		Exponential		299.506			471.149	472.705	0.248	0.956
		Half-normal				587.023	499.060	500.615	0.389	0.828
		Rayleigh				415.088	613.856	615.412	0.505	0.652
		Maxwell		479.303			741.950	743.505	0.589	0.553
Kopeh Dagħ	4.0	GN	0.201	368.968			1249.041	1254.548	0.047	0.997
		GG	0.144	408.619	2.676		1250.838	1259.098	0.051	0.994
		Gamma	0.470	222.581			1253.055	1258.562	0.052	0.997
		Weibull		75.830	0.605		1261.003	1266.510	0.070	0.993
		Exponential		104.521			1312.658	1315.411	0.189	0.976
		Half-normal				165.552	1355.737	1358.491	0.277	0.918
		Rayleigh		165.552		117.063	1682.142	1684.896	0.421	0.807
		Maxwell		135.172			2052.618	2055.371	0.490	0.746

Finally, the coefficient of determination (R^2) was applied. R^2 is a statistic value that provides information about the goodness-of-fit of a model. This

coefficient is the square of the correlation coefficient between the observed and modelled data values. R^2 will have a value between 0 and 1.

Table 3. (Continued).

Province	m_c	Distribution*	Parameters				AIC	BIC	K-S	R^2
			α	λ	τ	σ				
Kopeh Dagh	4.5	GN	0.185	1071.669			505.835	509.213	0.083	0.992
		Gamma	0.424	671.884			507.428	510.806	0.081	0.991
		GG	0.158	1127.434	2.288		507.800	512.867	0.085	0.991
		Weibull		194.111	0.561		511.260	514.638	0.088	0.987
		Exponential		285.072			534.219	535.908	0.207	0.964
		HN				460.637	550.672	552.361	0.321	0.903
		Rayleigh				325.720	678.452	680.141	0.450	0.789
		Maxwell		376.109			821.429	823.118	0.521	0.730
Central-East Iran	4.0	GN	0.146	179.065			2506.209	2513.727	0.058	0.995
		GG	0.172	168.283	1.739		2507.894	2519.171	0.060	0.995
		Gamma	0.331	114.324			2510.470	2517.988	0.066	0.994
		Weibull		18.604	0.454		2529.177	2536.695	0.078	0.987
		Exponential		37.868			2940.019	2943.778	0.291	0.922
		Half-normal				68.444	3141.450	3145.209	0.384	0.843
		Rayleigh				48.398	4549.826	4553.585	0.521	0.722
		Maxwell		55.885			6078.635	6082.394	0.578	0.668
	4.5	GN	0.159	429.160			1243.197	1248.822	0.049	0.997
		Gamma	0.365	267.210			1244.746	1250.370	0.052	0.996
		GG	0.202	388.035	1.631		1244.869	1253.306	0.076	0.998
		Weibull		54.885	0.492		1253.126	1258.750	0.084	0.988
		Exponential		97.594			1374.881	1377.693	0.276	0.943
		Half-normal				171.295	1445.818	1448.630	0.355	0.877
		Rayleigh				121.124	1930.937	1933.749	0.490	0.761
		Maxwell		139.862			2462.786	2465.598	0.547	0.706
	5.0	Gamma	0.382	640.426			581.944	585.686	0.115	0.986
		GN	0.164	1081.538			582.381	586.123	0.108	0.989
		GG	0.270	832.506	1.322		583.687	589.301	0.125	0.988
		Weibull		141.435	0.510		584.103	587.846	0.142	0.981
		Exponential		244.573			625.953	627.825	0.322	0.931
		Half-normal				437.570	655.475	657.346	0.403	0.882
		Rayleigh				309.409	837.762	839.633	0.488	0.781
		Maxwell		357.274			1038.285	1040.156	0.543	0.725
Zagros	4.0	GN	0.176	54.656			5915.847	5931.009	0.015	0.999
		GG	0.250	46.103	1.489		5921.347	5930.340	0.026	0.999
		Gamma	0.410	32.672			5926.701	5936.363	0.050	0.996
		Weibull		8.549	0.542		5988.929	5998.591	0.075	0.988
		Exponential		13.391			6659.163	6663.994	0.223	0.965
		Half-normal				22.910	7145.879	7150.710	0.310	0.905
		Rayleigh				16.200	10326.360	10331.191	0.454	0.789
		Maxwell		18.706			13858.644	13863.475	0.512	0.728
	4.5	GN	0.152	190.997			2453.719	2461.561	0.049	0.994
		GG	0.211	163.879	1.511		2454.166	2464.810	0.053	0.995
		Gamma	0.347	119.917			2457.249	2464.643	0.074	0.989
		Weibull		23.173	0.475		2491.942	2499.336	0.100	0.975
		Exponential		41.564			2819.434	2823.131	0.239	0.965
		Half-normal				74.433	3003.272	3006.969	0.319	0.886
		Rayleigh				52.632	4258.952	4262.649	0.459	0.747
		Maxwell		60.774			5627.848	5631.545	0.534	0.674

A value of R^2 near to 1 indicated that most of the variation of the response data explained by the different input values, whereas a value of R^2

near to zero indicated that little of the variation of the response data can be explained by the different input values (Ross 2010). The most

Table 3. (Continued).

Province	m_c	Distribution*	Parameters				AIC	BIC	K-S	R^2
			α	λ	τ	σ				
Zagros	5.0	GN	0.158	562.596			945.935	950.913	0.163	0.961
		GG	0.120	607.119	2.560		947.616	955.082	0.162	0.962
		Gamma	0.349	397.454			952.463	957.440	0.188	0.946
		Weibull		85.401	0.477		970.715	975.692	0.198	0.925
		Exponential		138.543			1057.750	1060.239	0.194	0.984
		Half-normal				223.785	1094.293	1096.781	0.282	0.920
		Rayleigh				158.240	1448.682	1451.170	0.445	0.790
		Maxwell		182.720			1836.883	1839.372	0.519	0.718

*GN: Generalized normal distribution.

general definition of the coefficient of determination is:

$$R^2 = \frac{\sum_{i=1}^n (f_i - \bar{x}_i)^2}{\sum_{i=1}^n (x_i - \bar{x}_i)^2}, \quad (5)$$

where n is the number of data. A dataset has values x_i , each of which has an associated modelled value f_i , and $\bar{x}_i$ is the mean of the observed data.

3. Results

The total number of earthquakes was 1567 in Alborz–Azarbayejan, 213 in Kopeh Dagh, 966 in Central-East Iran, 3620 in Zagros and 228 in Makran. Cut-off magnitude analysis started from $m_c = 4.0$, and was increased gradually by a rate of 0.1. An increase in m_c signifies a decrease in the number of events in a province. Therefore, in order to limit statistical errors, interoccurrence times were analyzed using at least 50 events.

Different goodness-of-fit tests were used to detect whether a specific distribution was preferred for all regions in all cut-off magnitudes. As an example, the estimated distribution parameters and also the results of AIC, BIC and K-S tests and R^2 for the GG distribution and subfamilies of it at different cut-off magnitudes for five major seismotectonic provinces of Iran are displayed in table 3, where distributions are rearranged based on the results of AIC and BIC tests. To give some examples of plots of the interoccurrence time distributions, together with fitting, using different distribution functions, the cumulative distributions of the interoccurrence times for these provinces, at the same cut-off magnitudes as in table 3, are displayed in figures 3–7.

The AIC and BIC criteria are minimax-optimal; they are established based on the likelihood

Figure 3. Distribution of interoccurrence time at Alborz–Azarbayejan province for different m_c and distribution functions: (a) $m_c = 4.0$; (b) $m_c = 4.5$; (c) $m_c = 5.0$. ECDF: Empirical cumulative distribution function.

function, which is stronger than the coefficient of determination or non-parametric statistics such as the K-S statistic. Therefore, in cases that a specified distribution was not the most prominent distribution in all tests, the results of the AIC and BIC tests were considered for selection of the appropriate distribution. Table 4 shows a summary of the selected distributions based on goodness-of-fit test results for different seismotectonic provinces at all cut-off magnitudes.

Further, to compare the result of region selection with respect to seismotectonic features by just covering the region with a grid with $L^\circ \times L^\circ$ cells, for instance, it is assumed that $L = 5^\circ$. Generally, for L less than 5° , most of the cells should be omitted from our analysis because of inadequate data. On the other hand, for L more than 5° , the greater part of most of the cells was located outside our study area. By selecting $L = 5^\circ$, 12 cells extend over the spatial areas of this study, as shown in figure 2. With respect to the existence of sufficient earthquake data in each cell, the regions $45^\circ\text{--}50^\circ\text{E}$ and $25^\circ\text{--}30^\circ\text{N}$; $60^\circ\text{--}65^\circ\text{E}$ and $30^\circ\text{--}35^\circ\text{N}$ and $60^\circ\text{--}65^\circ\text{E}$ and $35^\circ\text{--}40^\circ\text{N}$ were not analyzed. In table 5, selected distributions for other nine cells are listed for a variety of cut-off magnitudes.

Figure 4. Distribution of interoccurrence time at Koppeh Dagh province for different m_c and distribution functions: (a) $m_c = 4.0$; (b) $m_c = 4.5$. ECDF: Empirical cumulative distribution function.

Figure 5. Distribution of interoccurrence time at Central-East Iran province for different m_c and distribution functions: (a) $m_c = 4.0$; (b) $m_c = 4.5$; (c) $m_c = 5.0$. ECDF: Empirical cumulative distribution function.

4. Discussion

After focussing on the interoccurrence time distributions of major seismotectonic provinces of Iran, through more testing it became apparent to the researchers that the most suitable distribution for different m_c and variety regions is the generalized normal distribution or the gamma distribution (table 4). In general, there is a possibility that the preferred distribution is not unique but dependent on the test used. However, the current results show that in some cases the generalized normal is the

Figure 6. Distribution of interoccurrence time at Makran province for different m_c and distribution functions: (a) $m_c = 4.0$; (b) $m_c = 4.5$. ECDF: Empirical cumulative distribution function.

most appropriate distribution while for the others, the most suitable distribution is the gamma distribution.

It can be observed that with a gradual decrease in m_c , the fitting accuracy of the distribution becomes worse in Alborz–Azarbayejan seismotectonic province, and the dominant distribution changes from the generalized normal regime to the gamma regime. The generalized normal distribution showed the best fit to the interoccurrence time data of Kopeh Dag and Zagros seismotectonic provinces in all threshold magnitudes. In the Central-East Iran seismotectonic province, the most appropriate distribution changes from the generalized normal distribution to the gamma distribution by decreasing m_c . In all cut-off magnitudes, the gamma distribution was the most suitable for interoccurrence time distribution in the Makran seismotectonic province. It was thus concluded that interoccurrence time statistics exhibit a generalized normal gamma transition in Alborz–Azarbayejan, Zagros and Central-East Iran seismotectonic provinces.

Although, suitable distribution was selected based on the Akaike and Bayesian information criteria, seismicity characteristics have no influences in our choice. We expect various seismicity

Figure 7. Distribution of interoccurrence time at Zagros province for different m_c and distribution functions: (a) $m_c = 4.0$; (b) $m_c = 4.5$; (c) $m_c = 5.0$. ECDF: Empirical cumulative distribution function.

behaviours in different seismotectonic provinces. So, the different suitable distribution of the interoccurrence times and a variety of behaviours in cut-off magnitude changes are not something unexpected.

In this study of the interoccurrence time statistics of cells ($5^\circ \times 5^\circ$) with no seismotectonic feature consideration, in all cells and cut-off magnitudes, the selected distribution based on goodness-of-fit tests result is the gamma or the generalized normal distributions. By changing m_c , the interoccurrence time statistics exhibit transitions

Table 4. *Selected distributions and estimated parameters for major seismotectonic province of Iran.*

Province	m_c	Distribution	Parameters		m_c	Distribution	Parameters	
			α	λ			α	λ
Alborz–Azarbayejan	4.0	Gamma	0.339	115.441	4.7	Generalized normal	0.124	861.896
	4.1	Gamma	0.341	140.619	4.8	Generalized normal	0.119	1181.906
	4.2	Generalized normal	0.148	273.461	4.9	Generalized normal	0.190	1022.436
	4.3	Generalized normal	0.143	377.988	5.0	Generalized normal	0.185	1071.669
	4.4	Generalized normal	0.136	424.843	5.1	Generalized normal	0.113	1664.108
	4.5	Generalized normal	0.132	536.021	5.2	Generalized normal	0.107	1827.592
	4.6	Generalized normal	0.124	689.443				
Kopeh Dagħ	4.0	Generalized normal	0.201	368.968	4.3	Generalized normal	0.193	713.947
	4.1	Generalized normal	0.198	426.112	4.4	Generalized normal	0.190	1022.436
	4.2	Generalized normal	0.197	593.884	4.5	Generalized normal	0.185	1071.669
Central–East Iran	4.0	Generalized normal	0.146	179.065	4.7	Generalized normal	0.172	561.973
	4.1	Generalized normal	0.142	193.766	4.8	Generalized normal	0.172	691.758
	4.2	Generalized normal	0.144	228.289	4.9	Gamma	0.368	560.255
	4.3	Generalized normal	0.151	291.588	5.0	Gamma	0.382	640.426
	4.4	Generalized normal	0.159	388.216	5.1	Gamma	0.368	707.336
	4.5	Generalized normal	0.159	429.160	5.2	Gamma	0.365	676.112
	4.6	Generalized normal	0.174	497.905				
Zagros	4.0	Generalized normal	0.176	54.656	4.9	Generalized normal	0.150	461.040
	4.1	Generalized normal	0.170	68.368	5.0	Generalized normal	0.158	562.596
	4.2	Generalized normal	0.165	75.940	5.1	Generalized normal	0.154	672.751
	4.3	Generalized normal	0.158	105.184	5.2	Generalized normal	0.146	827.309
	4.4	Generalized normal	0.155	139.848	5.3	Generalized normal	0.145	945.232
	4.5	Generalized normal	0.152	190.997	5.4	Generalized normal	0.129	1287.511
	4.6	Generalized normal	0.143	278.096	5.5	Generalized normal	0.132	1473.453
	4.7	Generalized normal	0.147	319.650	5.6	Generalized normal	0.114	2088.970
Makran	4.0	Gamma	0.553	224.351	4.4	Gamma	0.549	446.939
	4.1	Gamma	0.525	286.885	4.5	Gamma	0.464	645.865
	4.2	Gamma	0.517	363.958	4.6	Gamma	0.489	714.229
	4.3	Gamma	0.544	380.356				

between the gamma distribution and the generalized normal distribution in some cells, in the similar fashion to what was observed in the seismotectonic provinces. However, we have not found any regular pattern in cells where these transitions occurred.

Finally, the results were compared with those of the previous studies. The unified scaling law shows a generalized gamma distribution which is approximately a gamma distribution, because τ in Corral (2004) is close to unity ($\tau = 0.98 \pm 0.05$). However, we have demonstrated that the generalized normal distribution is more appropriate than the gamma distribution, because the AIC and BIC values obtained from the generalized normal distribution are smaller than those of the gamma distribution in most seismotectonic provinces and m_c . In addition, Hasumi *et al* (2009) demonstrated that the probability distribution changes by varying the m_c ; in the present study, however, a transition was observed from a generalized normal regime

to a gamma regime, which has not been reported previously.

5. Conclusion

In this paper, a new feature of interoccurrence time statistics has been proposed by analyzing the earthquake catalogue for different tectonic conditions of Iran. The findings support the view that generalized normal statistics and gamma statistics coexist in the interoccurrence time statistics and with a gradual decrease in m_c , the fitting accuracy of the dominant distribution becomes worse in some regions. In the Alborz–Azerbaijan seismotectonic province, the predominant distribution changed from the gamma distribution to the generalized normal distribution when m_c increased, while in the Central–East Iran seismotectonic province, this change occurred with a decrease in m_c . In Kopeh Dagħ and Zagros seismotectonic

Table 5. Selected distributions and estimated parameters for different cells.

Region	m_c	Distribution	Parameters		m_c	Distribution	Parameters	
			α	λ			α	λ
25°–30°N	4.0	Generalized normal	0.179	141.282	4.4	Generalized normal	0.165	301.191
50°–55°E	4.1	Gamma	0.402	107.892	4.5	Gamma	0.366	269.226
	4.2	Gamma	0.380	138.694	4.6	Generalized normal	0.154	694.273
	4.3	Gamma	0.375	183.017	4.7	Generalized normal	0.153	848.554
25°–30°N	4.0	Generalized normal	0.136	162.224	4.5	Generalized normal	0.119	498.344
55°–60°E	4.1	Generalized normal	0.135	197.410	4.6	Generalized normal	0.131	533.654
	4.2	Generalized normal	0.124	270.966	4.7	Generalized normal	0.132	643.777
	4.3	Generalized normal	0.124	363.649	4.8	Generalized normal	0.131	766.487
	4.4	Generalized normal	0.120	390.195	4.9	Generalized normal	0.111	1059.250
25°–30°N	4.0	Gamma	0.513	305.371	4.2	Generalized normal	0.211	737.331
60°–65°E	4.1	Gamma	0.505	342.680	4.3	Gamma	0.458	618.082
30°–35°N	4.0	Generalized normal	0.162	204.825	4.3	Gamma	0.324	325.726
45°–50°E	4.1	Generalized normal	0.163	249.109	4.4	Generalized normal	0.141	560.203
	4.2	Generalized normal	0.155	370.077	4.5	Generalized normal	0.150	715.759
30°–35°N	4.0	Gamma	0.473	182.551	4.3	Gamma	0.441	375.183
50°–55°E	4.1	Gamma	0.456	243.019	4.4	Gamma	0.438	432.037
	4.2	Gamma	0.453	291.213	4.5	Gamma	0.490	479.048
30°–35°N	4.0	Gamma	0.281	245.211	4.3	Generalized normal	0.143	670.823
55°–60°E	4.1	Generalized normal	0.124	444.335	4.4	Generalized normal	0.338	493.965
	4.2	Generalized normal	0.127	543.196	4.5	Gamma	0.387	517.498
35°–40°N	4.0	Gamma	0.286	307.385	4.3	Generalized normal	0.125	931.319
45°–50°E	4.1	Generalized normal	0.134	576.598	4.4	Generalized normal	0.122	981.188
	4.2	Generalized normal	0.123	760.362		Generalized normal		
35°–40°N	4.0	Generalized normal	0.199	429.896	4.2	Generalized normal	0.200	527.128
50°–55°E	4.1	Generalized normal	0.199	429.896	4.3	Generalized normal	0.203	692.167
35°–40°N	4.0	Generalized normal	0.226	439.540	4.2	Generalized normal	0.213	634.615
55°–60°E	4.1	Gamma	0.519	289.539				

provinces, the most suitable distribution was the generalized normal at all threshold magnitudes. The gamma distribution was the most suitable for the interoccurrence time data at a variety of cut-off magnitudes in the Makran seismotectonic province.

Acknowledgement

This work is partially supported by the Department of Geophysics, Science and Research branch, Islamic Azad University.

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MS received 20 February 2011; revised 23 September 2011; accepted 20 October 2011