Low energy trajectories for the Moon-to-Earth space flight

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The Moon-to-Earth low energy trajectories of 'detour' type are found and studied within the frame of the Moon–Earth–Sun-particle system. These trajectories use a passive flight to the Earth from an initial elliptic selenocentric orbit with a high aposelenium. The Earth perturbation increases the particle selenocentric energy from a negative value first to zero and then to a positive one and therefore leads to a passive escape of the particle motion from the Moon attraction near the translunar libration point L_2. This results in the particle flight to a distance of about 1.5 million km from the Earth where the Sun gravitation decreases the particle orbit perigee distance to a small value that leads to the particle approaching the Earth vicinity in about 100 days of the flight. A set of the Moon-to-Earth 'detour' trajectories is defined numerically. Characteristics of these trajectories are presented. The 'detour' trajectories give essential economy of energy (about 150 m/s in Delta V) relative to the usual ones.

1. Introduction

Investigations of space trajectories for flights from the near-Moon vicinity to the Earth are important for both Celestial Mechanics and Astronautics. Usual trajectories (see, e.g., Egorov and Gusev 1980) for the Moon-to-Earth direct space flights within the Earth's sphere of influence with respect to the Sun are well studied. In this case, perturbations caused by the Sun are small, and the model of the restricted three-body problem (Moon–Earth with a particle of negligible mass) is used. Trajectories of this type were used for space flights from the Moon in both the USA project of the Apollo manned flights and the Soviet project for robotic collection of the lunar material and its delivery to the Earth (see, e.g., Gatland 1982). These trajectories are characterized by a small (several days) flight time and by the fact that the departure of the particle from the Moon occurs along a hyperbola. Recently (see, e.g., Belbruno and Miller 1993; Hiroshi Yamakawa *et al* 1993; Belló Mora *et al* 2000; Biesbroek and Janin 2000; Koon *et al* 2001; Ivashkin 2002, 2003, 2004a), a new class of trajectories with the Earth-to-Moon indirect detour space flight was discovered within the framework of the four-body system (Earth–Moon–Sun-particle). These trajectories use first the space flights towards the Sun (or away from the Sun) beyond the Earth's sphere of influence, and only afterwards, space flights towards the Moon. These space flights seem to be similar to bielliptic ones proposed by Sternfeld (1934, 1937, 1956). But they differ in dynamics. Here, the perigee distance increases due to the Sun gravitation. In addition, now the particle approaches the Moon along an elliptic trajectory, i.e., the capture by the Moon takes place. Thus, for the spacecraft capture to the lunar satellite orbit or for its landing onto the Moon, these detour space flights are more profitable than direct or bielliptic ones. An idea arises to employ this detour scheme for the Moon-to-Earth space flights (Hiroshi Yamakawa *et al* 1993; Ivashkin 2004b, 2004c). The present paper describes the main results of numerical and analytical studies in the problem shown. A family of trajectories for a passive space flight to the Earth from an elliptic orbit of a lunar satellite has been constructed, and

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characteristics of these trajectories are analyzed (Ivashkin 2004b, 2004c). In addition, the effects of gravitational perturbations that resulted in the formation of these trajectories, particularly, in both the particle's gravitational escape from the lunar attraction and passive decreasing the perigee distance of the particle orbit (approximately from the value of the lunar orbit radius to almost zero), thus making the passive flights to the Earth possible, have been analyzed.

2. Moon-to-Earth detour trajectories

2.1 *Algorithm of calculations*

As a result of the analysis and taking into account the experience of the Earth-to-Moon trajectories studies (Ivashkin 2002, 2003, 2004a), a numerical algorithm has been developed that has allowed us to find a family of detour trajectories for space flights to the Earth from elliptic orbits of the lunar satellite. These trajectories correspond to the spacecraft starting from both the Moon's surface and the low orbit of the lunar satellite for several positions of the Moon on its orbit. The spacecraft trajectories have been determined by integration using the method described in the equations for the particle motion (Stepan'yants and L'vov 2000). These equations are written in the Cartesian non-rotating geocentric-equatorial coordinate system OXYZ in the attraction field of the Earth (taking into account its main harmonic c_{20}), the Moon, and the Sun with the high-precision determination of the Moon and Sun coordinates, which is based on the DE403 JPL ephemerides. The particle motion in the selenocentric coordinate system MXYZ is also determined.

2.2 *Some numerical characteristics of the Moon-to-Earth detour flights*

Characteristics of a typical detour trajectory are presented in figures 1–3. The solid curve in figure 1 presents the geocentric motion of a spacecraft, and the dot-and-dash line shows the lunar orbit M. At the point D, the spacecraft flies away from the Moon on May 11th, 2001 (corresponding to the position of the Moon near the apogee), from the perilune of an initial elliptic orbit with the perilune altitude $H_{\pi 0} = 100$ km and semi-major axis $a_0 = 38{,}455$ km. This orbit is close to the final orbit of the Earth-to-Moon space flight (Ivashkin 2002, 2003, 2004a). All the following motions of the particle are passive (without taking into account possible corrections). Under the effect of the Earth's gravitation, evolution of the

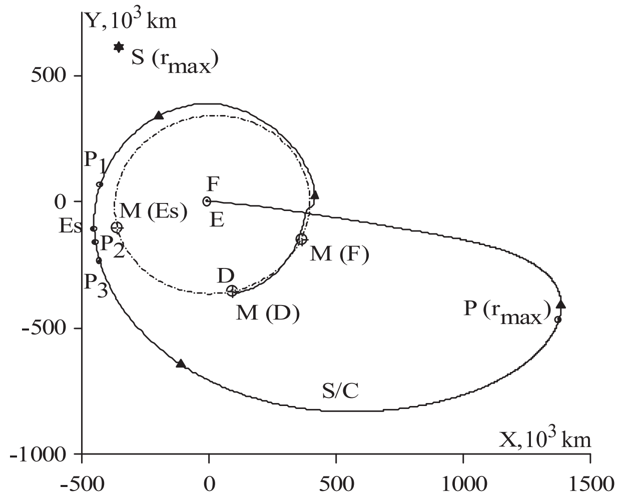

Figure 1. The XY geocentric view for the Moon-to-Earth trajectory of detour flight.

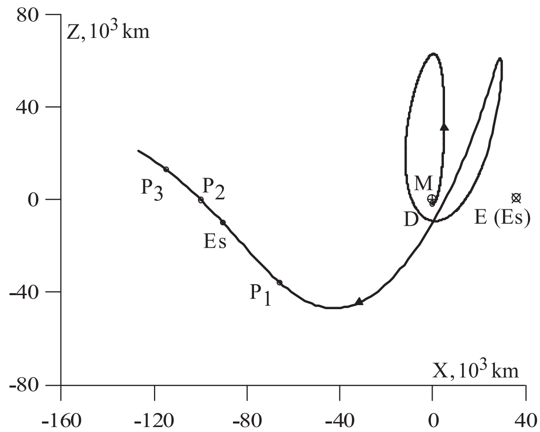

Figure 2. The XZ selenocentric view for the Moon-to-Earth trajectory of the detour type at the initial part of the flight.

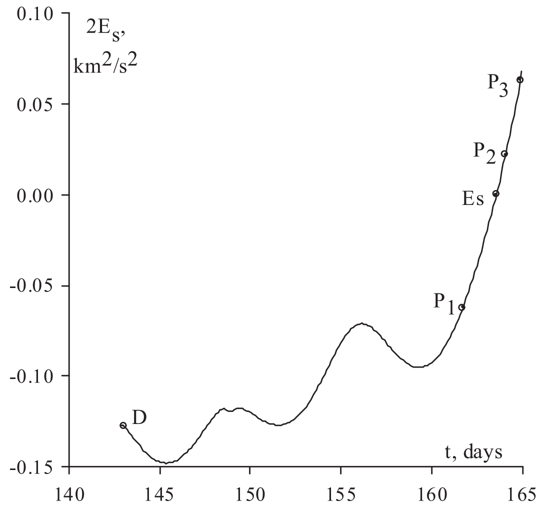

Figure 3. Selenocentric energy versus the time for the initial part of the Moon-to-Earth detour flight with escape from the lunar attraction.

selenocentric orbit and an increase in the selenocentric energy

$$E_s = V^2/2 - \mu_M/\rho = -\mu_M/2a_s \qquad (1)$$

occur. In equation (1) V and ρ are the selenocentric velocity of the particle and its distance from the Moon, respectively, a_s is the semimajor axis of the particle orbit, and $\mu_M(\approx 4903\,\text{km}^3\,\text{s}^{-2})$ is the lunar gravitational parameter. At the point P_1 in the space flight time $\Delta t \approx 19$ days, the energy is $E_s \approx -0.031\,\text{km}^2\,\text{s}^{-2}$, $a_s \approx 79 \cdot 10^3$ km, and $\rho \approx 76 \cdot 10^3$ km. At the point Es for $\Delta t \approx 20.6$ days and $\rho \approx 91.85 \cdot 10^3$ km in the region of the translunar libration point L_2, there is the escape from the lunar attraction, i.e., $E_s = 0$ here, and the orbit is parabolic with zero velocity 'at infinity', $V_\infty = 0$. Further, the particle moves from the Moon along a hyperbola. At the point P_2 for $\Delta t \approx 21.1$ days and $\rho \approx 101 \cdot 10^3$ km, the energy is $E_s \approx 0.011\,\text{km}^2\,\text{s}^{-2}$, $V_\infty = 0.15\,\text{km s}^{-1}$. At the point P_3 for $\Delta t \approx 21.9$ days and $\rho \approx 120.2 \cdot 10^3$ km, the energy becomes equal to $E_s \approx 0.031\,\text{km}^2\,\text{s}^{-2}$, and $V_\infty = 0.25$ km/s. Then, the spacecraft flies away from both the lunar orbit and the Earth and reaches in $\Delta t \approx 70$ days the maximal distance $r_{\max} \approx 1470 \cdot 10^3$ km from the Earth. At that moment, the point $S(r_{\max})$ determines the direction to the Sun. By the effect of the Sun gravitation, the perigee is gradually lowered, and for $\Delta t \approx 113$ days at the point F, the spacecraft approaches the Earth E having the perigee's osculating altitude $H_\pi = 50$ km.

Figures 2 and 3 show the evolution of the spacecraft detour motion with respect to the Moon at the initial part of the space flight where there is the escape from the lunar attraction. Figure 2 gives selenocentric trajectory in the XZ plane. The point E (Es) determines the direction to the Earth at the moment of the escape from the lunar gravitational attraction. Figure 3 gives the selenocentric energy constant $2E_s$ versus the time for the initial part DP_1 Es P_2P_3 of the motion with escape from the lunar attraction. Here and below, on figure 5, the time t is counted off from the Julian date 2451898.5, that is 20.12.2000.0.

The initial velocity of the spacecraft (at the point D, figures 1–3) is $V_0^+ \approx 2282\,\text{m s}^{-1}$. For leaving a circular lunar-satellite orbit with an altitude of 100 km and velocity $V_0^- \approx 1633\,\text{m s}^{-1}$ using a velocity impulse (a high thrust), the velocity increment is $\Delta V_0 \approx 649\,\text{m s}^{-1}$. For the usual direct space flight scheme and the minimal departure energy, $V_\infty \approx 0.8\,\text{km s}^{-1}$, the flight time $T \approx 5.5$ days, we have the initial velocity V_0^+ of about $2443\,\text{m s}^{-1}$, the velocity impulse ΔV_0 of about $810\,\text{m s}^{-1}$, that is at $\text{d}V_0 \approx 161$ m/s more than for the case of detour space flight.

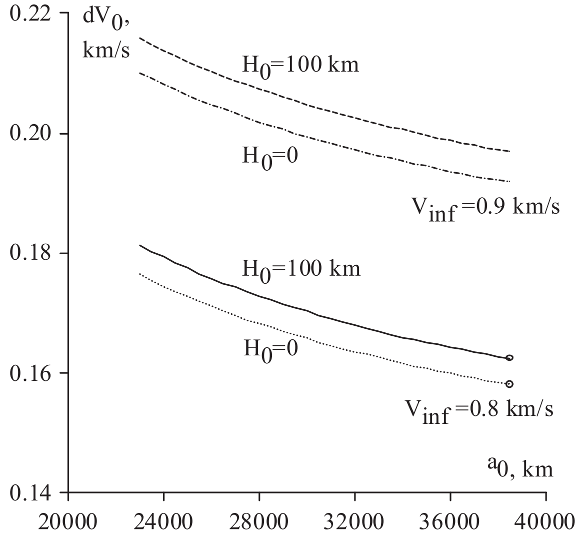

Figure 4. Decrease of the velocity impulse for the Moon–Earth detour flight relative to the direct flight depending on the initial semi-major axis.

For a case when the spacecraft leaves the Moon surface, the detour trajectory (with $a_0 = 38{,}455$ km again) has approximately the same characteristics as for the indicated case of the starting from the lunar satellite orbit. The decrease in the velocity increment is equal to about $\text{d}V_0 \approx 156$ m/s in this case.

If the initial semi-major axis a_0 is less, the decreasing $\text{d}V_0$ of the velocity impulse for the detour flight relative to the direct one is more. Figure 4 gives this decreasing $\text{d}V_0$ versus the initial semi-major axis a_0 of detour flight for two values of the velocity at 'infinity' $V_\infty = V_{\text{inf}}$ for direct flight: $V_\infty = 0.8$ km/s (approximately, for optimal direct flight from the Moon apogee) and $V_\infty = 0.9$ km/s (approximately, for optimal direct flight from the Moon perigee). The lines $H_0 = 100$ km correspond to the spacecraft starting from the satellite orbit perilune with altitude $H_0 = 100$ km. The lines $H_0 = 0$ correspond to the spacecraft starting from the Moon surface. Possible values of the initial semi-major axis a_0 for the detour flight are given below, in section 3.

3. Earth's gravity effect on the particle escape

We now qualitatively analyze the gravitational effects on the formation of the detour trajectory. First, we estimate an increase $\Delta E_s = -E_{s0}$ of selenocentric energy (1) from the negative value E_{s0} for the initial elliptic orbit to the zero energy which can be caused by the Earth gravity during the particle selenocentric motion on the arc D Es from the initial state D to the escape point

Es. On the basis of the orbit evolution theory (Lidov 1961, 1962) and assuming that the particle's selenocentric orbit eccentricity is $e_s \approx 1$, the mean energy is $E_s \approx -\Delta E_s/2$, and taking into account the change in the Moon–Earth direction, we obtain (Ivashkin 2002, 2003, 2004a):

$$\Delta E_s \approx \mathrm{sign}\beta \left(\frac{15}{2} \pi \mu_E \left(\frac{\mu_M}{a_M} \right)^3 n_M \beta \right)^{2/9} > 0. \quad (2)$$

In equation (2) the value μ_E is the Earth's gravitational parameter, value a_M is the semi-major axis of the Moon's orbit, and value n_M is the angular velocity of its orbital motion,

$$\beta = \cos^2 \gamma \sin 2\alpha, \quad (3)$$

here angle γ is the slope of the radius vector r_B for an external body (for the Earth, in this case) to the plane of the particle orbit, and α is the angle between the projection of the radius vector r_B onto this plane and the perilune direction. For $\Delta E_s > 0$, it is necessary to have $\sin 2\alpha > 0, 0 < \alpha < \pi/2$ or $\pi < \alpha < 3\pi/2$. Estimation by equations (2–3) gives $\Delta E_s = -E_{s0} \approx 0.096\,\mathrm{km}^2\,\mathrm{s}^{-2}$, $a_0 \approx 25{,}600$ km for the middle value $\beta = 0.5$. This gives the estimation for a minimal value of semimajor axis a_0 for the initial elliptic selenocentric orbit in the Moon-to-Earth detour trajectory. This theoretical evaluation well fits the results for our numerical calculations of the Moon-to-Earth detour trajectories. For example, for the Moon-to-Earth flight during a month from May 12th, 2001, and for initial inclination $i_0 = 90°$, we obtained for the Moon-to-Earth calculated trajectories the minimal value of semimajor axis $a_{0\,\mathrm{min}} \approx 24{,}500$–$27{,}000$ km for initial ascending node $\Omega_0 = 0$; $a_{0\,\mathrm{min}} \approx 23{,}500$–$28{,}500$ km for $\Omega_0 \approx -63.9°$; $a_{0\,\mathrm{min}} \approx 24{,}000$–$28{,}000$ km for $\Omega_0 = -90°$ (see figure 5).

We can see that, if the orientation of the particle initial orbit relative to the Earth is suitable and its negative energy is large enough, the Earth's gravitation provides a sufficient increase in the particle orbital energy and allows its passive escape from the lunar attraction.

4. Earth's gravity effect on the particle acceleration to hyperbolic selenocentric motion

Now we approximately analyze the acceleration of the particle motion with respect to the Moon from the zero energy to a positive one for a hyperbolic trajectory with velocity at 'infinity' V_∞ which is about 0.15–0.25 km/s on the subsequent short arc Es P_2P_3 (even on the somewhat larger arc

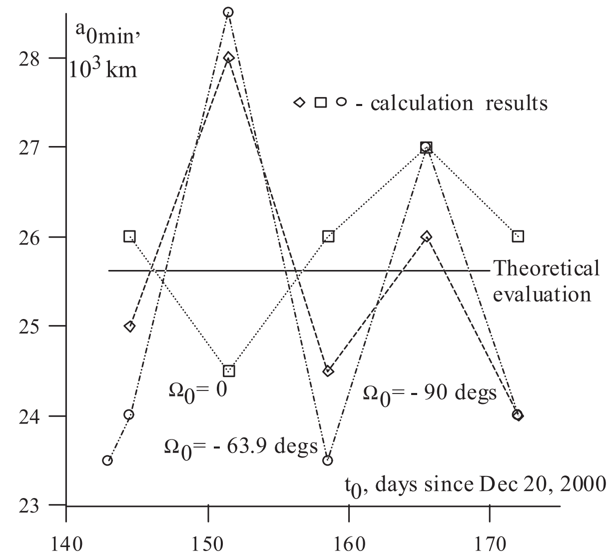

Figure 5. Minimal value of initial semi-major axis depending on the time of start from near-Moon elliptic selenocentric orbit for the Moon-to-Earth detour trajectories.

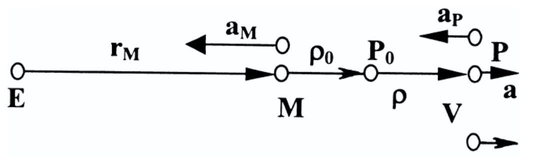

Figure 6. A model for the particle selenocentric motion from the Moon.

P_1 Es P_2P_3 from the energy $E_s < 0$). This acceleration is qualitatively described by an approximate model of the one-dimensional rectilinear particle's motion with the Earth, placed on the same line at a distance r_M beyond the Moon (Ivashkin 2002, 2003, 2004a), see figure 6.

In this case $\mathrm{d}\rho/\mathrm{d}t > 0$, i.e., the particle moves away from the Moon. The Earth's perturbation $\delta a_E = \mu_E/r_M^2 - \mu_E/(r_M + \rho)^2 > 0$, it accelerates the particle motion. For this model, assuming that, approximately, $r_M =$ constant, we can integrate the equations for the perturbed motion of the particle:

$$V^2(\rho) = 2E_s(\rho) + 2\mu_M/\rho; \quad (4)$$

$$t(\rho) - t_0 = \int_{\rho_0}^{\rho} \mathrm{d}\rho/V(\rho); \quad (5)$$

$$E_s(\rho) - E_0 = (\mu_E/r_M^2)(\rho - \rho_0) + \mu_E/(r_M + \rho) - \mu_E/(r_M + \rho_0),$$

$$E_s(\rho_0) = E_0; \quad (6)$$

inversely:

$$\rho(E_s) = B/2 + [B^2/4 + r_M B]^{1/2}; \quad (7)$$

where

$$B = (E_s - E_0)r_M^2/\mu_E + \rho_0^2/(r_M + \rho_0). \quad (8)$$

As an example, we may consider the following case. Let the presented trajectory at the point Es of the gravitational escape the selenocentric energy be $E_s = E_0 = 0, \rho = \rho_0 = 91{,}850\,\mathrm{km}$, $r_M = 376{,}000\,\mathrm{km}$. Then, the model of equations (4–8) gives $\rho \approx 102.5 \cdot 10^3\,\mathrm{km}$ for $V_\infty = 0.15\,\mathrm{km/s}$ (point P_2), $\rho \approx 120.4 \cdot 10^3\,\mathrm{km}$ for $V_\infty = 0.25\,\mathrm{km/s}$ (point P_3), and $\rho \approx 55 \cdot 10^3\,\mathrm{km}$ for $E_s = -0.031\,\mathrm{km}^2/\mathrm{s}^2$ (point P_1). We can see the qualitative correspondence with the numerical results presented above, especially for $E_s > 0$.

Thus, for the given class of the Moon-to-Earth detour space flights, the Earth's gravitation in the region of the translunar libration point L_2 allows increasing the selenocentric energy of the particle motion from the zero value to the positive one for a hyperbolic trajectory. If the selenocentric hyperbolic velocity V_∞ of the particle is directed along the Moon velocity, the geocentric energy and the apogee distance will be increased and the particle runs away from the Moon orbit and from the Earth, at a large geocentric distance.

5. Sun's gravity effect on decrease of the particle orbit perigee distance

Next, we estimate the effect of the Sun gravitation on the variation Δr_π of the particle orbit perigee distance r_π on the final arc P_3F of the space flight. We use the theory (Lidov 1961, 1962) of the orbit evolution for one orbital revolution of a planet's (the Earth's, here) satellite due to an external body's (the Sun's, now) gravity perturbation assuming the Earth–Sun direction to be constant. Since the final geocentric distance $r_{\pi f}$ for the particle orbit perigee is very small ($r_{\pi f} = r_{\pi 0} + \Delta r_\pi \approx 0$), we assume that eccentricity $e \approx 1$ and take for r_π its mean value $r_\pi = (2r_{\pi f} - \Delta r_\pi)/2 \approx -\Delta r_\pi/2$. Thus, we have:

$$\Delta r_\pi \approx \operatorname{sign}\beta \left(\frac{15}{2}\pi\frac{\mu_S}{\mu_E}\beta\right)^2 a^7/a_E^6 < 0. \quad (9)$$

Here, μ_S is the gravitational parameter of the Sun, a_E and a are the semimajor axes for the Earth's orbit and for the particle geocentric orbit, the value β is determined by equation (3) with the Sun as the external body. For $\Delta r_\pi < 0$, it follows from equations (9) and (3) that $\sin 2\alpha < 0, \pi/2 < \alpha < \pi$ or $3\pi/2 < \alpha < 2\pi$. Then, we estimate the desired value of the semimajor axis for the spacecraft orbit as

$$a \approx \left[|\Delta r_\pi|\, a_E^6 \Big/ \left(\frac{15}{2}\pi\frac{\mu_S}{\mu_E}\beta\right)^2\right]^{1/7}. \quad (10)$$

For estimation, we have assumed that $\Delta r_\pi = -500 \cdot 10^3\,\mathrm{km}$ and $\beta = -0.5$. Then, according to equation (10), the semi-major axis of the particle geocentric orbit at the final part of the flight is $a \approx 870 \cdot 10^3\,\mathrm{km}$ and its apogee distance is $r_\alpha \approx 1500 \cdot 10^3\,\mathrm{km}$. Thus, if the orientation of the particle orbit with regard to the Sun is suitable enough and the orbit apogee distance is large enough (of about $1.5 \cdot 10^6\,\mathrm{km}$), the particle perigee distance decreases from about the lunar-orbit radius to almost zero. This allows the particle's passive approach to the Earth.

6. Conclusions

Reviewing the results of our analysis, we can see that gravitational perturbations of the Earth and the Sun make it possible for the particle beginning its motion from the selenocentric elliptic orbit to escape the motion from the lunar attraction, to transfer it to the Moon-to-Earth detour trajectory, and then to approach the Earth. This leads to a noticeable decrease in the energy consumption for the Moon-to-Earth space flights. Such a conclusion is confirmed by both the numerical calculations of relevant trajectories and their theoretical analysis.

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