Reflection of P and SV waves from free surface of an elastic solid with generalized thermodiffusion

Baljeet Singh

Department of Mathematics, Government College, Sector-11, Chandigarh 160 011, India.
e-mail: baljeet@networkindia.net

The governing equations for generalized thermodiffusion in an elastic solid are solved. There exists three kinds of dilatational waves and a Shear Vertical (SV) wave in a two-dimensional model of the solid. The reflection phenomena of P and SV waves from free surface of an elastic solid with thermodiffusion is considered. The boundary conditions are solved to obtain a system of four non-homogeneous equations for reflection coefficients. These reflection coefficients are found to depend upon the angle of incidence of P and SV waves, thermodiffusion parameters and other material constants. The numerical values of modulus of the reflection coefficients are presented graphically for different values of thermodiffusion parameters. The dimensional velocities of various plane waves are also computed for different material constants.

1. Introduction

Duhamel (1837) and Neumann (1885) introduced the theory of uncoupled thermoelasticity. There are two shortcomings with this theory. First, the fact that the mechanical state of the elastic body has no effect on the temperature is not in accordance with the true physical experiments. Second, the heat equation being parabolic predicts an infinite speed of propagation for the temperature, which is not physically admissible.

Biot (1956) developed the coupled theory of thermoelasticity which deals with the first defect of uncoupled theory, but shares the second defect of uncoupled theory. In the classical theory of thermoelasticity, when a homogeneous isotropic elastic solid is subjected to a thermal disturbance, the effect is felt at a location far from the source, instantaneously. This implies that the thermal wave propagates with infinite speed, a physically impossible result. In contrast to the conventional thermoelasticity, nonclassical theories came into existence during the last three decades. These theories, referred to as generalized thermoelasticity, were introduced into the literature in an attempt to eliminate the shortcomings of the classical dynamical thermoelasticity. For example, Lord and Shulman (1967), by incorporating a flux-rate term into Fourier's law of heat conduction, formulated a generalized theory which involves a hyperbolic heat transport equation admitting finite speed for thermal signals. Green and Lindsay (1972), by including temperature rate among the constitutive variables, developed a temperature-rate-dependent thermoelasticity that does not violate the classical Fourier law of heat conduction, when the body under consideration has a centre of symmetry and this theory also predicts a finite speed for heat propagation. Chandrasekharaiah (1986) referred to this wavelike thermal disturbance as 'second sound'. The Lord and Shulman theory of generalized thermoelasticity was further extended by Sherief (1980) and Dhaliwal and Sherief (1980) to include the anisotropic case. A survey article of representative theories in the range of generalized thermoelasticity is due to Hetnarski and Ignaczak (1999).

Sinha and Sinha (1974) and Sinha and Elsibai (1996a, b) studied the reflection of thermoelastic waves from the free surface of a solid half-space and

Keywords. Thermoelasticity; thermodiffusion; mass diffusion; reflection; reflection coefficients.

J. Earth Syst. Sci. **114**, No. 2, April 2005, pp. 159–168

at the interface of two semi-infinite media in welded contact, in the context of generalized thermoelasticity. Abd-Alla and Al-Dawy (2000) studied the reflection phenomena of SV waves in a generalized thermoelastic medium. Recently, Sharma *et al* (2003) investigated the problem of thermoelastic wave reflection from the insulated and isothermal stress-free as well as rigidly fixed boundaries of a solid half-space in the context of different theories of generalized thermoelasticity.

Diffusion may be defined as the random walk, of an ensemble of particles, from regions of high concentration to regions of lower concentration. The study of this phenomenon is of great concern due to its many geophysical and industrial applications. In integrated circuit fabrication, diffusion is used to introduce 'dopants' in controlled amounts into the semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors, form integrated resistors, form the source/drain regions in Metal Oxide Semiconductor (MOS) transistors and dope poly-silicon gates in MOS transistors. Study of the phenomenon of diffusion is used to improve the conditions of oil extractions (seeking ways of more efficiently recovering oil from oil deposits). These days, oil companies are interested in the process of thermodiffusion for more efficient extraction of oil from oil deposits.

Thermodiffusion in an elastic solid is due to coupling of the fields of temperature, mass diffusion and that of strain. Heat and mass exchange with the environment during thermodiffusion in an elastic solid. Using the coupled thermoelastic model, Nowacki (1974a, b, c, 1976) developed the theory of thermoelastic diffusion and discussed dynamical problems of diffusion in solids. Dudziak and Kowalski (1989) also discussed the theory of thermodiffusion for solids. Olesiak and Pyryev (1995) discussed a coupled quasi-stationary problem of thermodiffusion for an elastic cylinder. They studied the influences of cross effects arising from the coupling of the fields of temperature, mass diffusion and strain. Due to these cross effects, the thermal excitation results in an additional mass concentration and the mass concentration generates the additional field of temperature. Recently, Sherief *et al* (2004) generalized the theory of thermoelastic diffusion, which allows the finite speeds of propagation of waves. The development of generalized theory of thermoelastic diffusion by Sherief *et al* (2004) provides a chance to study the wave propagation in such an interesting media. The paper is organized as follows: In section 2, the wave propagation in an isotropic, homogeneous model of elastic solid with generalized thermoelastic diffusion is studied. The governing equations are solved in x–z plane to show the existence of three kinds of dilatational waves and an SV wave. In section 3, the expressions for reflection coefficients are obtained for the incidence of P and SV wave at a thermally insulated free surface. In the last section, a numerical example is given to discuss the dependence of reflection coefficients upon the angle of incidence of P and SV waves as well as on thermodiffusion parameters. This dependence is also shown graphically.

2. Governing equations and solution

Following, Sherief *et al* (2004), the governing equations for an isotropic, homogeneous elastic solid with generalized thermodiffusion at constant temperature T_0 in the absence of body forces are:

(i) the equation of motion

$$\mu u_{i,jj} + (\lambda+\mu)u_{j,ij} - \beta_1\Theta_{,i} - \beta_2 C_{,i} = \rho\ddot{u}_i, \quad (1)$$

(ii) the equation of heat conduction

$$\rho c_E(\dot{\Theta} + \tau_0\ddot{\Theta}) + \beta_1 T_0(\dot{e} + \tau_0\ddot{e}) + aT_0(\dot{C} + \tau_0\ddot{C}) = K\Theta_{,ii}, \quad (2)$$

(iii) the equation of mass diffusion

$$D\beta_2 e_{,ii} + Da\Theta_{,ii} + \dot{C} + \tau\ddot{C} - DbC_{,ii} = 0, \quad (3)$$

(iv) the constitutive equations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}(\lambda e_{kk} - \beta_1\Theta - \beta_2 C), \quad (4)$$

$$\rho T_0 S = \rho c_E\Theta + \beta_1 T_0 e_{kk} + aT_0 C, \quad (5)$$

$$P = -\beta_2 e_{kk} + bC - a\Theta, \quad (6)$$

where $\beta_1 = (3\lambda+2\mu)\alpha_t$ and $\beta_2 = (3\lambda+2\mu)\alpha_c$, λ, μ are Lame's constants, α_t is the coefficient of linear thermal expansion and α_c is the coefficient of linear diffusion expansion. $\Theta = T - T_0$, T_0 is the temperature of the medium in its natural state assumed to be such that $|\Theta/T_0| \ll 1$. σ_{ij} are the components of the stress tensor, u_i are the components of the displacement vector, ρ is the density assumed independent of time, e_{ij} are the components of the strain tensor, T is the absolute temperature, S is the entropy per unit mass, P is the chemical potential per unit mass, C is the mass concentration, c_E is the specific heat at constant strain, K is the coefficient of thermal conductivity, D is thermodiffusion constant. τ_0 is the thermal relaxation time, which will ensure that the heat conduction equation, satisfied by the temperature Θ will predict finite speeds of heat propagation. τ is the diffusion relaxation time, which will ensure that the equation, satisfied by the concentration C will also predict finite speeds of propagation of matter from one medium to the other. The constants a and b are the measures of thermodiffusion effects and diffusive effects, respectively. The superposed dots denote derivative with respect to time.

For two-dimensional motion in x–z plane, the equations (1)–(3) are written as

$$(\lambda+2\mu)u_{1,11}+(\mu+\lambda)u_{3,13}+\mu u_{1,33} - \beta_1\Theta_{,1}-\beta_2 C_{,1}=\rho\ddot{u}_1, \tag{7}$$

$$\mu u_{3,11}+(\mu+\lambda)u_{1,13}+(\lambda+2\mu)u_{3,33} - \beta_1\Theta_{,3}-\beta_2 C_{,3}=\rho\ddot{u}_3, \tag{8}$$

$$K(\Theta_{,11}+\Theta_{,33}) = \rho c_E\tau_m\dot{\Theta}+\beta_1 T_0\tau_m\dot{e}+aT_0\tau_m\dot{C}, \tag{9}$$

$$D\beta_2(e_{,11}+e_{,33})+Da(\Theta_{,11}+\Theta_{,33}) - Db(C_{,11}+C_{,33})+\tau_n\dot{C}=0, \tag{10}$$

where $\tau_m = 1+\tau_0\frac{\partial}{\partial t}$ and $\tau_n = 1+\tau\frac{\partial}{\partial t}$. The displacement components u_1 and u_3 may be written in terms of potential functions ϕ and ψ as

$$u_1=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial z}, \qquad u_3=\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x}. \tag{11}$$

Using (11) into equations (7)–(10), we obtain

$$\mu\nabla^2\psi=\rho\ddot{\psi}, \tag{12}$$

$$(\lambda+2\mu)\nabla^2\phi-\beta_1\Theta-\beta_2 C=\rho\ddot{\phi}, \tag{13}$$

$$K(\Theta_{,11}+\Theta_{,33})=\rho c_E\tau_m\dot{\Theta}+\beta_1 T_0\tau_m\frac{\partial}{\partial t}\nabla^2\phi + aT_0\tau_m\dot{C}, \tag{14}$$

$$D\beta_2\nabla^4\phi+Da(\Theta_{,11}+\Theta_{,33}) - Db(C_{,11}+C_{,33})+\tau_n\dot{C}=0. \tag{15}$$

Equation (12) is uncoupled, whereas the equations (13)–(15) are coupled in ϕ, Θ and C. From equations (12)–(15), we see that while the P wave is affected due to the presence of thermal and mass diffusion waves, the SV remains unaffected. The solution of equation (12) corresponds to the propagation of SV wave with velocity $v_4=\sqrt{\mu/\rho}$.

Solutions of the equations (13)–(15) are now sought in the form of the harmonic travelling wave

$$\{\phi,\Theta,C\}=\{\phi_0,\Theta_0,C_0\}e^{\iota k(x\sin\theta+z\cos\theta-vt)}, \tag{16}$$

in which v is the phase speed, k is the wave number, $(\sin\theta,\cos\theta)$ denotes the projection of the wave normal onto the x–z plane. The homogeneous system of equations in ϕ_0, Θ_0, and C_0, obtained by inserting (16) into equations (13)–(15), admits non-trivial solutions and enables one to conclude that ξ satisfies the cubic equation

$$\xi^3+L\xi^2+M\xi+N=0, \tag{17}$$

where

$$\xi=\rho v^2,$$

$$L=-(\epsilon+\epsilon\epsilon_2{\epsilon_1}^2+d_1+d_2+\lambda+2\mu),$$

$$M=(\lambda+2\mu)(d_1+d_2+\epsilon\epsilon_2{\epsilon_1}^2) + d_1d_2+d_2\epsilon-2\epsilon\epsilon_1\epsilon_2-\epsilon_2,$$

$$N=-d_1d_2(\lambda+2\mu)+\epsilon_2 d_1,$$

$$d_1=K/c_E{\tau_m}^1, \quad d_2=\rho Db/{\tau_n}^1,$$

$$\epsilon={\beta_1}^2T_0/\rho c_E, \quad \epsilon_1=-a/\beta_1\beta_2,$$

$$\epsilon_2=\rho D{\beta_2}^2/{\tau_n}^1,$$

$${\tau_m}^1=\tau_0+(\iota/\omega), \quad {\tau_n}^1=\tau+(\iota/\omega).$$

Equation (17) is cubic in ξ. The roots of this equation give three values of ξ. Each value of ξ corresponds to a wave if v^2 is real and positive. Hence, three positive values of v will be the velocities of propagation of three possible waves. Using Cardan's method, equation (17) is written as:

$$\Lambda^3+3H\Lambda+G=0, \tag{18}$$

where

$$\Lambda = \xi + \frac{L}{3}, \qquad H = \frac{3M - L^2}{9},$$

$$G = \frac{27N - 9LM + 2L^3}{27}. \tag{19}$$

For all the three roots of equation (18) to be real, Δ_0 $(= G^2 + 4H^3)$ should be negative. Assuming the Δ_0 to be negative, we obtain the three roots of equation (18) as

$$\Lambda_n = 2\sqrt{-H}\cos\left(\frac{\phi + 2\pi(n-1)}{3}\right), \quad (n{=}1,\ 2,\ 3), \tag{20}$$

where

$$\phi = \tan^{-1}\left(\frac{\sqrt{|\Delta_0|}}{-G}\right). \tag{21}$$

Hence,

$$v_n = \sqrt{\left(\Lambda_n - \frac{L}{3}\right)\Big/\rho}, \quad (n = 1,\ 2,\ 3), \tag{22}$$

are velocities of propagation of the three kinds of possible dilatational waves. The waves with velocities v_1, v_2, v_3 $(v_1 > v_3 > v_2)$ correspond to P wave, Mass Diffusion (MD) wave and Thermal (T) wave, respectively. This fact may be verified, when we solve the equation (17), using a computer program of Cardan's method. If we neglect thermal effects, *i.e.*, for $\epsilon = 0$, $d_1 = 0$, the cubic equation (17) reduces to a quadratic equation whose roots are as

$$2\rho v^2 = [d_2 + (\lambda + 2\mu)] \pm \sqrt{[d_2 - (\lambda + 2\mu)]^2 + 4\epsilon_2}, \tag{23}$$

where the positive and negative signs correspond to P wave and MD wave, respectively. Moreover, the MD wave exists if ${\beta_2}^2 < b(\lambda + 2\mu)$. Similarly, if we neglect the diffusion effects, *i.e.*, for $\epsilon_1 = \epsilon_2 = d_2 = 0$, the equation (17) reduces to a quadratic equation whose roots are as

$$2\rho v^2 = [\{d_1 + (\lambda + 2\mu)\} + \epsilon] \pm \sqrt{[\{d_1 - (\lambda + 2\mu) - \epsilon]^2 + 4\epsilon d_1}, \tag{24}$$

where the positive and negative signs correspond to P wave and T waves, respectively. The T wave exists, if $d_1 > 0$, which is true. These two-dimensional roots are in agreement with the non-dimensional roots obtained by Abd-alla and Al-Dawy (2000) for Lord and Shulman theory. In absence of thermodiffusion effects, the equation (17) corresponds to P wave with velocity $v_1 = \sqrt{(\lambda + 2\mu)/\rho}$.

It may be pointed here that an analogy may exist between the present three dilatational waves and the three kinds of P waves obtained by Lu and Hangya (2004), as an analogy exists between thermoelastic propagation and poroelastic propagation (Biot 1956; Norris 1992). P, T and MD waves may correspond to the P1, P2 and P3 waves obtained by Lu and Hangya (2004). Here P and P1 waves correspond to P1 wave in Biot's theory. Also, T and P2 waves correspond to diffusive P2 wave in Biot's theory. MD wave is related to mass diffusion, where as P3 wave is related to capillary pressure effects.

3. Reflection coefficients

In the previous section, it has been discussed that there exists three kinds of dilatational waves and an SV wave in an isotropic elastic solid with generalized thermodiffusion. Any incident wave at the interface of two elastic solid bodies, in general, produce dilatational and rotational waves in both media (see for example, Ewing *et al* 1957; Ben-Menahem and Singh 1981). Let us now consider an incident P or SV wave (figure 1). The boundary conditions at the free surface $z = 0$ are satisfied, if the incident P or SV wave gives rise to a reflected SV and three reflected dilatational waves (*i.e.*, P, MD and T waves). The surface $z = 0$ is free from surface tractions and is assumed thermally insulated so that there is no variation of temperature

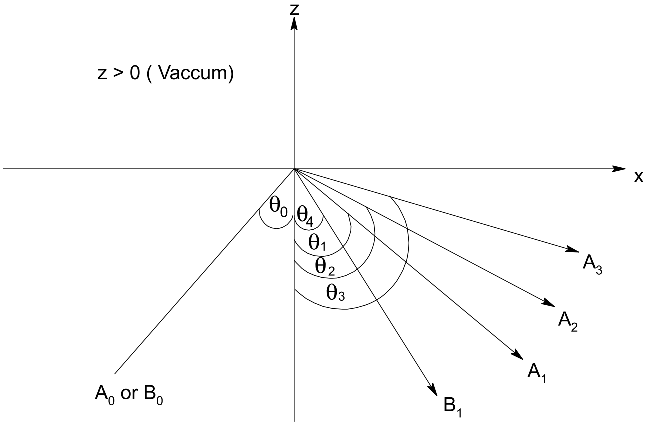

Figure 1. Schematic diagram for the problem (for incident P wave, $\theta_0 = \theta_1$, for incident SV wave, $\theta_0 = \theta_4$).

and concentration on it. Therefore, the boundary conditions on $z = 0$ may be written as

$$\sigma_{zz} = 0, \qquad \sigma_{zx} = 0, \qquad \frac{\partial \Theta}{\partial z} = 0,$$

$$\frac{\partial C}{\partial z} = 0, \quad \text{on} \quad z = 0. \qquad (25)$$

The appropriate displacement potentials ϕ and ψ, temperature Θ and concentration C are taken in the form

$$\psi = B_0 \exp[\iota k_4(x \sin\theta_0 + z\cos\theta_0) - \iota\omega t] + B_1 \exp[\iota k_4(x \sin\theta_4 - z\cos\theta_4) - \iota\omega t], \qquad (26)$$

$$\phi = A_0 \exp[\iota k_1(x \sin\theta_0 + z\cos\theta_0) - \iota\omega t] + A_1 \exp[\iota k_1(x \sin\theta_1 - z\cos\theta_1) - \iota\omega t] + A_2 \exp[\iota k_2(x \sin\theta_2 - z\cos\theta_2) - \iota\omega t] + A_3 \exp[\iota k_3(x \sin\theta_3 - z\cos\theta_3) - \iota\omega t], \qquad (27)$$

$$\Theta = \zeta_1 A_0 \exp[\iota k_1(x \sin\theta_0 + z\cos\theta_0) - \iota\omega t] + \zeta_1 A_1 \exp[\iota k_1(x \sin\theta_1 - z\cos\theta_1) - \iota\omega t] + \zeta_2 A_2 \exp[\iota k_2(x \sin\theta_2 - z\cos\theta_2) - \iota\omega t] + \zeta_3 A_3 \exp[\iota k_3(x \sin\theta_3 - z\cos\theta_3) - \iota\omega t], \qquad (28)$$

$$C = \eta_1 A_0 \exp[\iota k_1(x \sin\theta_0 + z\cos\theta_0) - \iota\omega t] + \eta_1 A_1 \exp[\iota k_1(x \sin\theta_1 - z\cos\theta_1) - \iota\omega t] + \eta_2 A_2 \exp[\iota k_2(x \sin\theta_2 - z\cos\theta_2) - \iota\omega t] + \eta_3 A_3 \exp[\iota k_3(x \sin\theta_3 - z\cos\theta_3) - \iota\omega t], \qquad (29)$$

where the wave normal of the incident P or SV wave makes angle θ_0 with the positive direction of the z-axis, and those of reflected P, MD, T and SV waves make $\theta_1, \theta_2, \theta_3$ and θ_4 with the same direction, and

$$\zeta_i = {k_i}^2 G_i(\rho {v_i}^2 - \lambda - 2\mu), \qquad (i = 1, 2, 3) \qquad (30)$$

$$\eta_i = {k_i}^2 H_i(\rho {v_i}^2 - \lambda - 2\mu),$$

and

$$G_i = \frac{\epsilon\rho {v_i}^2(\epsilon_1\epsilon_2 - d_2 + \rho {v_i}^2)}{d_1\epsilon_2 + \rho {v_i}^2[\epsilon(d_2 - \rho {v_i}^2) - \epsilon_2 - 2\epsilon\epsilon_1\epsilon_2]}, \quad (31)$$

$$H_i = \frac{\epsilon_2[\rho {v_i}^2(\epsilon\epsilon_1 + 1) - d_1]}{d_1\epsilon_2 + \rho {v_i}^2[\epsilon(d_2 - \rho {v_i}^2) - \epsilon_2 - 2\epsilon\epsilon_1\epsilon_2]}. \quad (32)$$

The ratios of the amplitudes of the reflected waves to the amplitude of the incident P wave, namely B_1/A_0, A_1/A_0, A_2/A_0 and A_3/A_0 give the reflection coefficients for reflected SV, reflected P, reflected MD and reflected T waves, respectively. Similarly for incident SV wave, $B_1/B_0, A_1/B_0, A_2/B_0$ and A_3/B_0 are the reflection coefficients for reflected SV, reflected P, reflected MD and reflected T waves, respectively. The wave number k_1, k_2, k_3, k_4 and the angles $\theta_1, \theta_2, \theta_3, \theta_4$ are connected by the relation

$$k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3 = k_4 \sin\theta_4, \quad (33)$$

at surface $z = 0$. The relation (33) may also be written in order to satisfy the boundary conditions (25) as

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta_3}{v_3} = \frac{\sin\theta_4}{v_4}, \qquad (34)$$

where $v_4 = \sqrt{\mu/\rho}$ is the velocity of SV wave and v_i, $(i = 1, 2, 3)$ are the velocities of three kinds of reflected dilatational waves. Using the potentials given by (26)–(29) in boundary conditions (25), we obtain a system of four non-homogeneous equations

$$\Sigma a_{ij} Z_j = b_i, \quad (i, j = 1, 2, \ldots, 4), \qquad (35)$$

where

$$a_{11} = -\left[\lambda + 2\mu\cos^2\theta_1 + \beta_1\frac{\zeta_1}{{k_1}^2} + \beta_2\frac{\eta_1}{{k_1}^2}\right]\left(\frac{k_1}{l}\right)^2,$$

$$a_{12} = -\left[\lambda + 2\mu\cos^2\theta_2 + \beta_1\frac{\zeta_2}{{k_2}^2} + \beta_2\frac{\eta_2}{{k_2}^2}\right]\left(\frac{k_2}{l}\right)^2,$$

$$a_{13} = -\left[\lambda + 2\mu\cos^2\theta_3 + \beta_1\frac{\zeta_3}{{k_3}^2} + \beta_2\frac{\eta_3}{{k_3}^2}\right]\left(\frac{k_3}{l}\right)^2,$$

$$a_{14} = \mu \sin 2\theta_4\left(\frac{k_4}{l}\right)^2,$$

$$a_{21} = \sin 2\theta_1 \left(\frac{k_1}{l}\right)^2,$$

$$a_{22} = \sin 2\theta_2 \left(\frac{k_2}{l}\right)^2,$$

$$a_{23} = \sin 2\theta_3 \left(\frac{k_3}{l}\right)^2,$$

$$a_{24} = \cos 2\theta_4 \left(\frac{k_4}{l}\right)^2,$$

$$a_{31} = \cos\theta_1 \frac{\zeta_1}{{k_1}^2} \left(\frac{k_1}{l}\right)^3,$$

$$a_{32} = \cos\theta_2 \frac{\zeta_2}{{k_2}^2} \left(\frac{k_2}{l}\right)^3,$$

$$a_{33} = \cos\theta_3 \frac{\zeta_3}{{k_3}^2} \left(\frac{k_3}{l}\right)^3,$$

$$a_{34} = 0,$$

$$a_{41} = \cos\theta_1 \frac{\eta_1}{{k_1}^2} \left(\frac{k_1}{l}\right)^3,$$

$$a_{42} = \cos\theta_2 \frac{\eta_2}{{k_2}^2} \left(\frac{k_2}{l}\right)^3,$$

$$a_{43} = \cos\theta_3 \frac{\eta_3}{{k_3}^2} \left(\frac{k_3}{l}\right)^3,$$

$$a_{44} = 0.$$

For incident P wave

$$b_1 = -a_{11}, \quad b_2 = a_{21}, \quad b_3 = a_{31},$$

$$b_4 = a_{41}, \quad l = k_1.$$

For incident SV wave

$$b_1 = a_{14}, \quad b_2 = -a_{24}, \quad b_3 = a_{34},$$

$$b_4 = a_{44}, \quad l = k_4,$$

and Z_1, Z_2, Z_3, Z_4 are reflection coefficients of reflected P, MD, T and SV waves, respectively.

In the absence of thermodiffusion, these reflection coefficients reduce to:

For incident P wave

$$\frac{A_1}{A_0} = \frac{\sin 2\theta_1 \sin 2\theta_4 - (v_1/v_4)^2 \cos^2 2\theta_4}{\sin 2\theta_1 \sin 2\theta_4 + (v_1/v_4)^2 \cos^2 2\theta_4}, \tag{36}$$

$$\frac{B_1}{A_0} = \frac{-2(v_1/v_4) \sin 2\theta_1 \cos 2\theta_4}{\sin 2\theta_1 \sin 2\theta_4 + (v_1/v_4)^2 \cos^2 2\theta_4}. \tag{37}$$

For incident SV wave

$$\frac{A_1}{B_0} = \frac{(v_1/v_4) \sin 4\theta_4}{\sin 2\theta_1 \sin 2\theta_4 + (v_1/v_4)^2 \cos^2 2\theta_4}, \tag{38}$$

$$\frac{B_1}{B_0} = \frac{\sin 2\theta_1 \sin 2\theta_4 - (v_1/v_4)^2 \cos^2 2\theta_4}{\sin 2\theta_1 \sin 2\theta_4 + (v_1/v_4)^2 \cos^2 2\theta_4}, \tag{39}$$

which are the same as those given by Ben-Menahem and Singh (1981), if θ_1, θ_4, v_1 and v_4 are replaced by e, f, α and β, respectively. It may also be mentioned that the MD and T waves will disappear in the absence of thermodiffusion.

4. Numerical results

For computational work, the following material constants at $T_0 = 27°\mathrm{C}$ are considered for an elastic solid with generalized thermodiffusion

$$\lambda = 5.775 \times 10^{11}\,\mathrm{dyne/cm}^2,$$

$$\mu = 2.646 \times 10^{11}\,\mathrm{dyne/cm}^2,$$

$$\rho = 2.7\,\mathrm{gm/cm}^3, \quad c_E = 2.361\,\mathrm{cal/gm}^\circ\mathrm{C},$$

$$K = 0.492\,\mathrm{cal/cm\ s}^\circ\mathrm{C},$$

$$\tau_0 = 0.05\,\mathrm{s}, \quad \tau = 0.04\,\mathrm{s}, \quad \alpha_t = 0.05,$$

$$\alpha_c = 0.06, \quad \omega = 2\,\mathrm{s}^{-1}, \quad a = 0.005,$$

$$b = 0.05, \quad D = 0.5.$$

The cubic equation (17) is solved by the computer program of Cardan method to obtain the numerical values of velocities of three dilatational waves *viz.*, P, MD and T waves. The variations of velocities of these waves are shown graphically in figures 2–6 for various material parameters. The P wave is

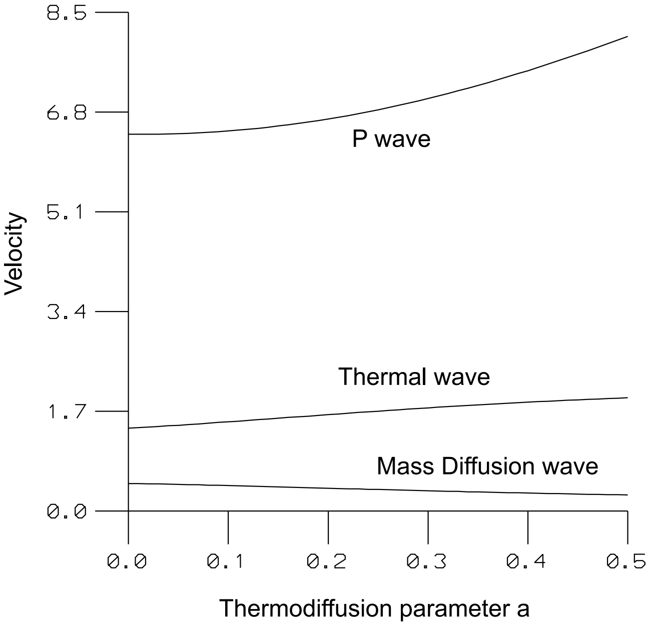

Figure 2. Variations of velocities of dilatational waves with thermodiffusion parameter 'a'.

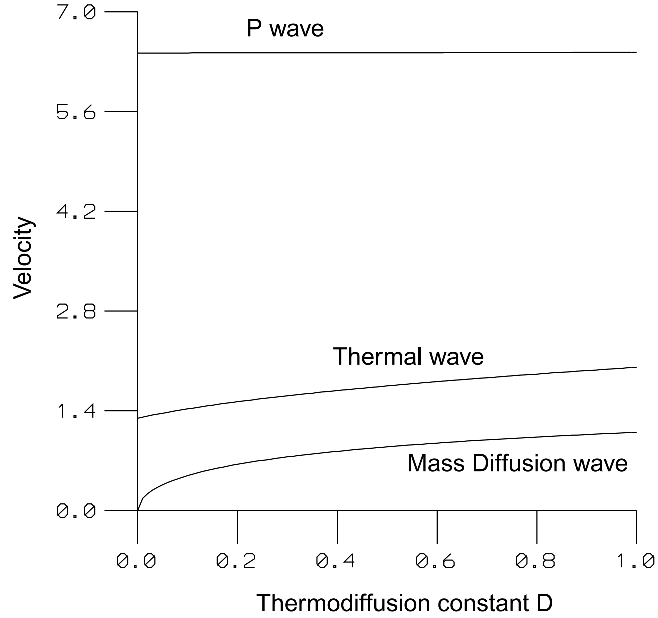

Figure 4. Variations of velocities of dilatational waves with thermodiffusion constant D.

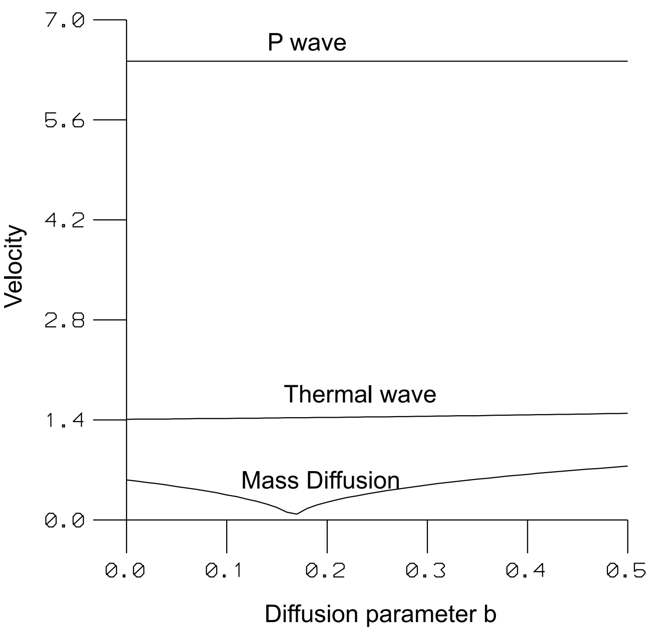

Figure 3. Variations of velocities of dilatational waves with diffusion parameter 'b'.

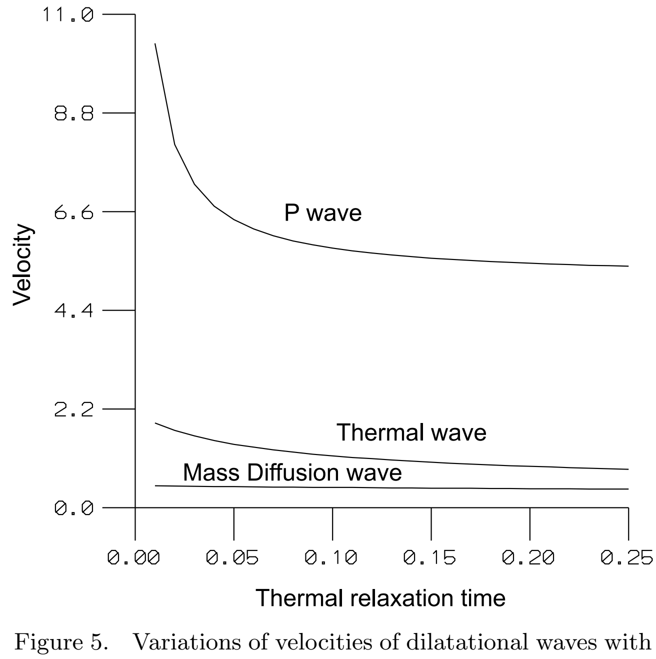

Figure 5. Variations of velocities of dilatational waves with thermal relaxation time.

found to be the fastest wave, whereas MD wave is found to be the slowest wave. All the three dilatational waves are found to depend on thermodiffusion parameters 'a'. The velocities of P and T waves increase with the increase in the value of 'a', whereas the velocity of MD wave decreases. This dependence is shown graphically in figure 2. Similarly, the dependence of velocities on diffusion parameter 'b', thermodiffusion constant 'D', thermal relaxation time and diffusion relaxation time are shown in figures 3–6, respectively. The system (35) of four non-homogeneous equations is solved to obtain the numerical values of reflection coefficients of various reflected waves for incidence of P as well as SV waves.

4.1 *Incident P wave*

The reflection coefficients are computed for the angle of incidence varying from 1° to 90° for thermodiffusion constant $D = 0.1$, 0.5 and 0.9. These

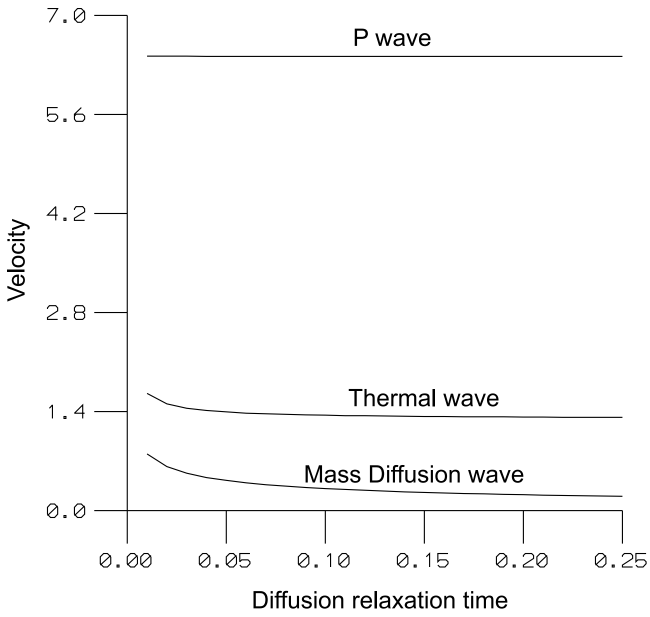

Figure 6. Variations of velocities of dilatational waves with diffusion relaxation time.

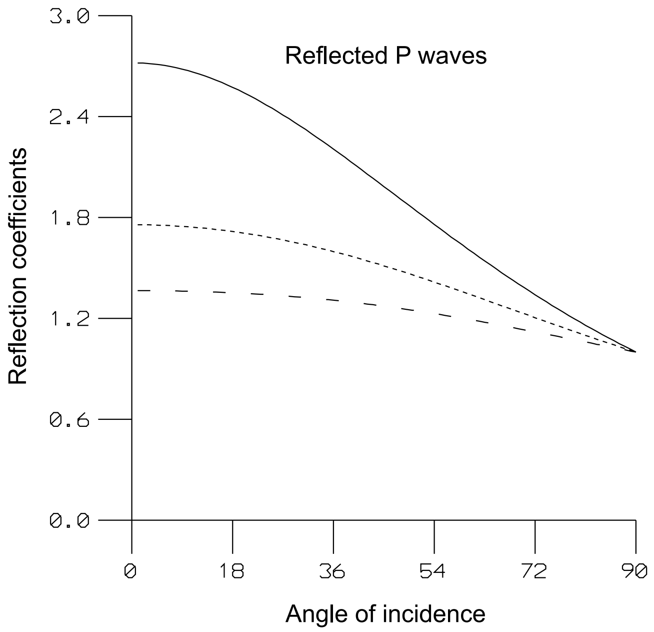

Figure 8. Variations of reflection coefficients of P waves with the angle of incidence of P wave for $D = 0.1$, 0.5 and 0.9.

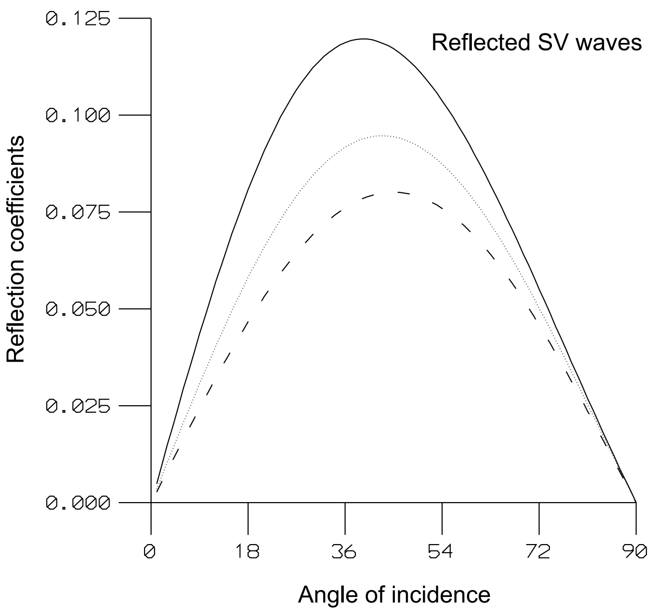

Figure 7. Variations of reflection coefficients of SV waves with the angle of incidence of P wave for $D = 0.1$, 0.5 and 0.9.

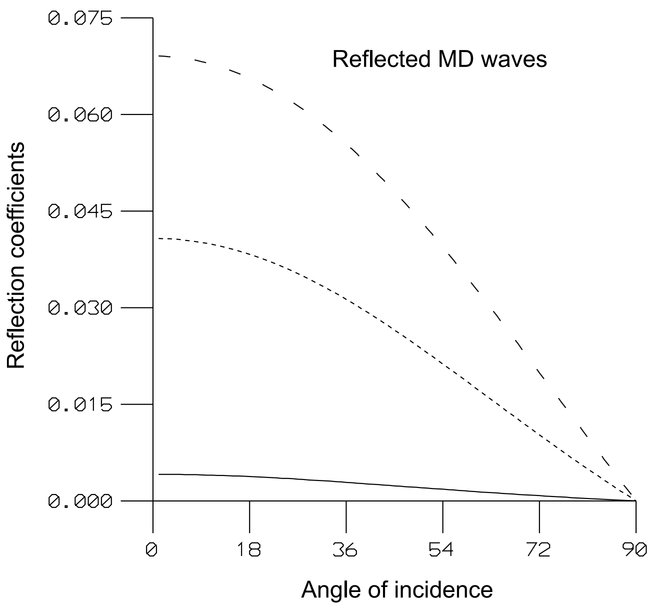

Figure 9. Variations of reflection coefficients of MD waves with the angle of incidence of P wave for $D = 0.1$, 0.5 and 0.9.

numerical values of reflection coefficients are shown graphically in figures 7–10 where solid lines, small dashed lines and long dashed lines correspond to $D = 0.1$, $D = 0.5$ and $D = 0.9$, respectively.

Figure 7 shows the reflection coefficients for reflected SV waves for $D = 0.1$, $D = 0.5$ and $D = 0.9$. The reflection coefficient for each value of D, first increases from its minima to maxima and then decreases to its minima. Figures 8–10 show the variations of reflection coefficients for reflected P, MD and T, respectively. These coefficients decrease with the increase in the angle of incidence. The comparison among solid, small dashed and long dashed lines in figures 7–10, shows the changes in reflection coefficients due to thermodiffusion constant 'D'.

4.2 *Incident SV wave*

The reflection coefficients are computed for the angle of incidence varying from 1° to 35° for

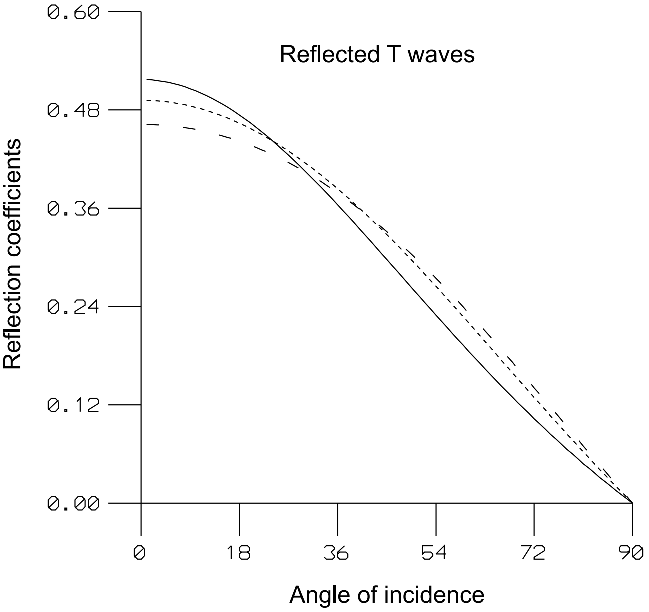

Figure 10. Variations of reflection coefficients of T waves with the angle of incidence of P wave for $D = 0.1$, 0.5 and 0.9.

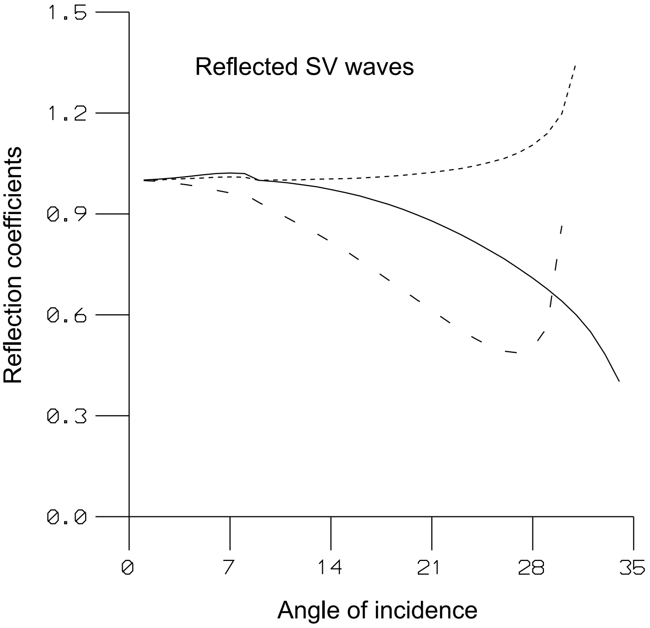

Figure 11. Variations of reflection coefficients of SV waves with the angle of incidence of SV wave for $a = 0.005$, 0.05 and 0.10.

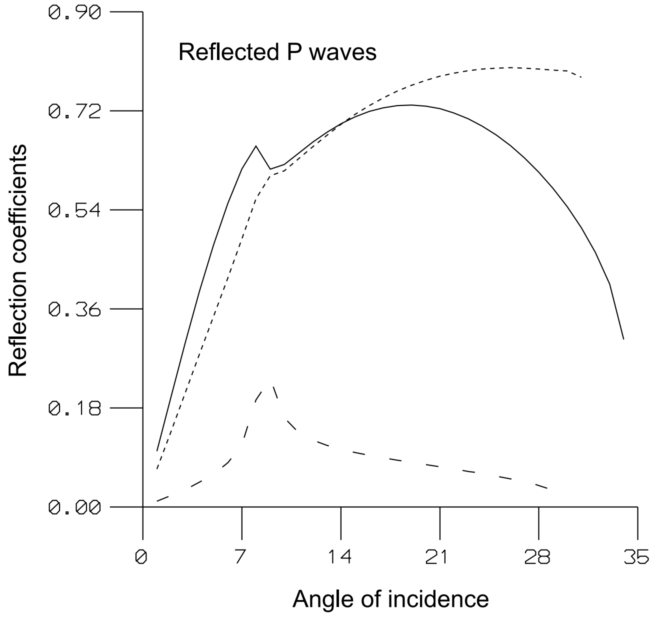

Figure 12. Variations of reflection coefficients of P waves with the angle of incidence of SV wave for $a = 0.005$, 0.05 and 0.1.

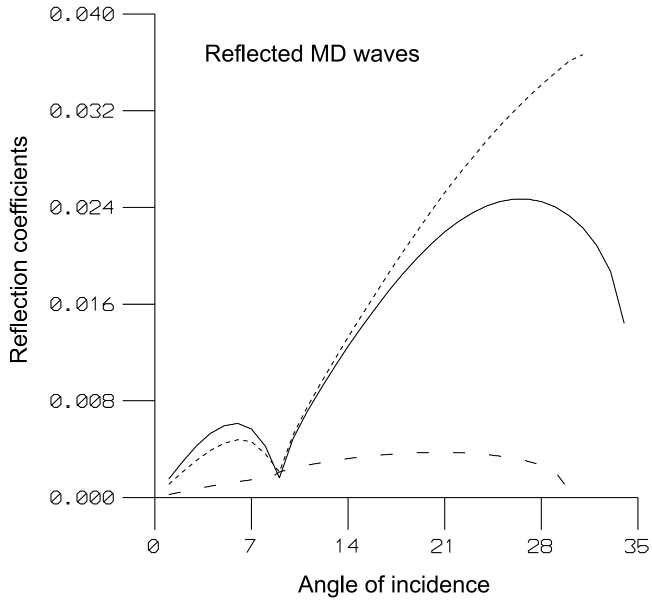

Figure 13. Variations of reflection coefficients of MD waves with the angle of incidence of SV wave for $a = 0.005$, 0.05 and 0.1.

thermodiffusion parameter $a = 0.005$, 0.05 and 0.1. These numerical values of reflection coefficients are shown graphically in figures 11–14 where solid lines, small dashed lines and long dashed lines correspond to $a = 0.005$, $a = 0.05$ and $a = 0.1$ respectively.

Figure 11 shows the reflection coefficients for reflected SV waves for $a = 0.005$, $a = 0.05$ and $a = 0.1$. For $a = 0.005$, the value of reflection coefficient of reflected wave is one beyond 34°. Similarly, it is one beyond 31° and 30° for $a = 0.05$ and $a = 0.1$, respectively. Figures 12–14 show the variations for reflected dilatational waves *viz.*, P, MD and T, respectively. From these figures, it may be noticed that the reflection coefficients at each angle of incidence are affected by thermodiffusion parameter a.

Finally, it may be concluded that there exist three kinds of dilatational waves, *viz.*, P wave, Mass

Figure 14. Variations of reflection coefficients of T waves with the angle of incidence of SV wave for $a = 0.005$, 0.05 and 0.1.

Diffusion wave, Thermal wave alongwith a rotational wave travelling with distinct velocities for two-dimensional motion in an elastic solid with thermodiffusion. The reflection coefficients for the incidence of P and SV waves are computed for a certain range of angle of incidence and for various thermodiffusion parameters. These coefficients depend on the angle of incidence, thermodiffusion parameters, and other material constants. For example, the change in the value of thermodiffusion parameters D and 'a' have considerable impact on the reflection coefficients at certain angles of incidence. The present model of elastic solid with thermodiffusion becomes more interesting due to the existence of three dilatational waves. The present theoretical results may provide some useful information for experimental scientists/researchers/seismologists working in the area of wave propagation in elastic solids.

Acknowledgements

This work was partially completed while the author was visiting the University of Salford under Bilateral Exchange Programme of Indian National Science Academy and Royal Society in the year 2003. The award of Visiting Fellowship from Royal Society, London is very gratefully acknowledged.

References

Abd-Alla A N and Al-Dawy A A S 2000 The reflection phenomena of SV waves in a generalized thermoelastic medium; *Int. J. Math. Sci.* **23** 529–546.

Ben-Menahem A and Singh S J 1981 *Seismic Waves and Sources* (New York: Springer-Verlag) Pp. 89–95.

Biot M 1956 Thermoelasticity and irreversible thermodynamics; *J. Appl. Phys.* **27** 249–253.

Chandrasekharaiah D S 1986 Thermoelasticity with second sound: A Review; *Appl. Mech. Rev.* **39** 355–376.

Dhaliwal R S and Sherief H H 1980 Generalized thermoelasticity for anisotropic media; *Quart. Appl. Math.* **33** 1–8.

Dudziak W and Kowalski S J 1989 Theory of thermodiffusion for solids; *Int. J. Heat and Mass Transfer* **32** 2005–2013.

Duhamel J 1837 Some memoire sur les phenomenes thermomechanique; *J. de L Ecole polytechnique* **15** 1–31.

Ewing W B, Jardetzky W S and Press F 1957 *Elastic waves in Layered Media* (New York: McGraw-Hill) Pp. 76.

Green A E and Lindsay K A 1972 Thermoelasticity; *J. Elasticity* **2** 1–7.

Hetnarski R B and Ignaczak J 1999 Generalized Thermoelasticity; *J. Thermal Stresses* **22** 451–476.

Lord H and Shulman Y 1967 A generalized dynamical theory of thermoelasticity; *J. Mech. Phys. Solids* **15** 299–309.

Lu J F and Hanyga A 2004 Linear dynamic model for porous media saturated by two immiscible fluids; *Int. J. Solids Structure* (in Press, Online).

Neumann F 1885 *Vorlesungen Uber die Theorie der Elasticitat* (Meyer: Brestau).

Norris A 1992 *J. Appl. Phys.* **71** 1138.

Nowacki W 1974a Dynamical problems of thermodiffusion in solids I; *Bull. Acad. Pol. Sci. Ser. Sci. Tech.* **22** 55–64.

Nowacki W 1974b Dynamical problems of thermodiffusion in solids II; *Bull. Acad. Pol. Sci. Ser. Sci. Tech.* **22** 129–135.

Nowacki W 1974c Dynamical problems of thermodiffusion in solids III; *Bull. Acad. Pol. Sci. Ser. Sci. Tech.* **22** 275–266.

Nowacki W 1976 Dynamical problems of diffusion in solids; *Eng. Frac. Mech.* **8** 261–266.

Olesiak Z S and Pyryev Y A 1995 A coupled quasi-stationary problem of thermodiffusion for an elastic cylinder; *Int. J. Engg. Sci.* **33** 773–780.

Sharma J N, Kumar V and Chand D 2003 Reflection of generalized thermoelastic waves from the boundary of a half-space; *J. Thermal Stresses* **26** 925–942.

Sherief H H 1980 *On Generalized Thermoelasticity*, Ph.D. Thesis, University of Calgary, Canada.

Sherief H H, Hamza F and Saleh H 2004 The theory of generalized thermoelastic diffusion; *Int. J. Engg. Sci.* **42** 591–608.

Sinha A N and Sinha S B 1974 Reflection of thermoelastic waves at a solid half-space with thermal relaxation; *J. Phys. Earth* **22** 237–244.

Sinha S B and Elsibai K A 1996a Reflection of thermoelastic waves at a solid half-space with two relaxation times; *J. Thermal Stresses* **19** 763–777.

Sinha S B and Elsibai K A 1996b Reflection and refraction of thermoelastic waves at an interface of two semi-infinite media with two relaxation times; *J. Thermal Stresses* **20** 129–146.

MS received 19 July 2004; accepted 22 November 2004