Tropical systematic and random error energetics based on NCEP (MRF) analysis-forecast system – A barotropic approach *Part II: in wavenumber domain*

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Systematic and random error and their growth rate and different components of growth rate budget in energy/variance form are investigated at wavenumber domain for medium range tropical (30°S–30°N) weather forecast using daily horizontal wind field of 850 hPa up to 5-day forecast for the month of June, 2000 of NCEP (MRF) model. Part I of this paper deals with the study at physical domain.

The following are the major findings in this paper:

- Tropical systematic error is associated with large scale wave of wavenumber 2, unlike the tropical random error, in which case dominant spectra of random error are observed at higher spectral band of wavenumbers 4–7 in comparison to that of systematic error.
- Systematic error growth rate peak is observed at wavenumber 2 up to 4-day forecast then the peak is shifted to wavenumber 1 at 5-day forecast. Random error energy shows maximum growth at wavenumber 4 for 2-day forecast, wavenumber 6 for 3–4 day forecasts and at wavenumber 7 for 1-day and 5-day forecasts.
- In the error growth rate budget, flux of systematic error shows the net increase of error energy at wavenumber 1 through the triad interactions with the pairs of waves of other wavenumbers. Flux and pure generation of random error energy are found to be accumulated at wavenumber 4. Resolving the possible triads in wavenumber 4 associated with these terms, it is shown that the wave receives more energy from the pairs of waves of different wavenumbers than it loses, leading to the error energy peak at wavenumber 4. However, the significant triad interaction occurs among the wavenumber 2 and higher wavenumbers in systematic error energy flux.

1. Introduction

The deterministic prediction of the subsequent evolution of the atmospheric state starting from the observed/analyzed state of the atmosphere loses its accuracy as the forecast time increases, ultimately leading to the total loss of predictability of the atmospheric system. One of the reasons for the loss of predictability is that error is generated and cas-

caded nonlinearly from the small scale to the larger one contaminating all the scales, thereby leaving the atmospheric system totally unpredictable. Two predictions, made from almost identical initial conditions except for some minor differences in initial state, differ from each other. The divergence depends on the growth rate of error generated from the structure of the different initial status, inadequate formulation of the dynamical equations,

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and inefficient parameterization of the physical processes.

Systematic error, identified as a stationary component of total forecast error is generally associated with large-scale error particularly with wavenumber 1 in the tropics (Kanamitsu 1985). Random error, a transient part of total forecast error, is generally associated with higher wavenumbers in comparison to that of systematic error (Dalcher and Kalaney 1987).

There are three different ways to estimate the growth rate of errors in predictability studies. In the dynamical approach the rate of amplification of the differences between two or more solutions, obtained by slightly changing the initial conditions of forecast model is evaluated as the error growth rate. In the empirical approach the difference between two naturally occurring analogues, in which either state may occur separately but similar to each other, is taken as an error and the difference at a future time may be treated as the error growth rate. In the dynamical-empirical approach a system of error equations is derived from the primitive equations and the dependent variables of the error equations are the estimated spectral form of error energy (Lorenz 1969a).

In the first approach, since the estimation of error is only due to the erroneous initial conditions, the error growth rate is underestimated. In the second approach, the main drawback is to find out the truly natural analogues. Van den Dool (1994) computed that it would take probabilistically of the order of 10^{30} years to find out the true natural analogue over the entire northern hemisphere 500 hPa height field. The initial difference between two nearest natural analogues is more than half of the average error. One cannot study the predictability of a system whose initial error is so high. In the third approach, as the error equation is basically derived from the primitive equations, it is well connected with the dynamics. In addition to this, error has propagated nonlinearly from the small scale to the next higher scale and the rate of error growth is inversely proportional to the scale size. In the dynamical-empirical method, the predictability limits of atmospheric phenomena of different scales are well within the observational limit. So, this method is treated as the best approach to study the predictability in terms of error analysis of an operational forecast system (Lorenz 1969a, 1969b).

Inadequate research has so far been performed in the spectral study of error energetics. Lambert and Merilees (1978) studied the 96 hour forecast error characteristics of spectral numerical model associated with planetary scale wave of wavenumber 1. Authors showed largest amplitude of harmonics (1, 1) and (2, 2) in spherical harmonics which

corresponds to zonal wavenumbers 1 and 2 and their two dimensional part respectively are in the tropical regions. Klinker and Capaldo (1986) studied the geographical distribution of analysis and forecast of horizontal and vertical winds, geopotential and temperature and the vertical structure of the prominent baroclinic waves associated with the systematic error of these parameters utilizing 3 years winter data. Dalcher and Kalaney (1987) studied the spectral distribution of systematic and random error in spherical harmonics over the global region and discussed the growth rate of forecast error variance by modifying the empirically formulated error growth rate budget given by Lorenz (1982) using 100 days ECMWF data of 10 days forecasts. Kanamitsu and Saha (1995) performed a budget analysis of short range component of systematic error of the variance on model sigma levels, based on NCEP(MRF) model for precise study to isolate the source of error using spectral coefficients of vorticity, divergence, virtual temperature and specific humidity at every time step during 24 hour model integration taking 150 cases of summer and winter seasons.

Boer (1984, 1993, 1994) first documented dynamical error growth rate equations derived from primitive equations and formulated by the above-mentioned dynamical-empirical approach using spherical harmonics spectra with an application to the extratropical region.

It is the goal of this study to carry out the Fourier spectral analysis of systematic and random error and their growth rate budgets which consist of several factors that govern the error growth in a forecast model using the horizontal wind field exclusively over the global tropics (30°S – 30°N). An attempt is made to investigate the inherent features of the underlying physical and dynamical processes of error growth by resolving the dominant spectral modes into individual triad interactions associated with various terms of growth rate equations. The methodology used here has not previously been applied earlier for error/predictability studies. So, as an initial work in this direction, one-month daily data are used to examine the aspects of error characteristics.

2. Methodology

Computations are performed in spectral domain using wind field based on systematic and random error growth rate equations (Boer 1993). Boer (1984, 1993), Dalcher and Kalaney (1987) have applied spherical harmonics technique in spectral error growth study but authors adopt Fourier decomposition instead of spherical harmonics synthesis for the following reasons.

In spherical harmonics method, motions are spherical in nature whereas in Fourier technique motions are basically planar. Basdevant *et al* (1981) showed that in the presence of rotation, spherical flow which is isotropic in nature seems slightly less predictable compared to the motion on plane which is more anisotropic in nature than the spherical flow. Reason is that the error first contaminates the isotropic part of flow then follows the anisotropic part. So, the more anisotropic the flow, like a flow on plane, the more predictable it would be in comparison to the spherical flow. Therefore authors are intended to use the Fourier synthesis which itself contributes less error compared to the spherical harmonics in spectral error analysis study. Moreover, Dalcher and Kalanay (1987) showed that the summer systematic error has a strong zonal character and is confined to a few low wavenumbers. The random error energy is equipartitioned among the zonal wavenumbers ≤ 20 . So, the influence of summer systematic error and random error variance in the meridional wavenumber is very much reduced for low wavenumbers. In this paper, the wavenumbers are zonal wavenumbers since Fourier spectra are taken in the zonal direction only. Hence, the Fourier technique, if applied in the zonal direction instead of considering spherical harmonics, would not alter the characteristics of the error spectra qualitatively in the study of spectral error energy analysis.

In addition to this, triad interaction technique, first developed by Murakami and Tomatsu (1964), Tomatsu (1979) is a specific as well as unique mathematical tool in the sense that this tool is used only in Fourier method not in spherical harmonics and may be applied to a term of three variables. The advantage of applying this tool is that the dominant spectral mode may be explained in terms of non-linear energy exchange into individual triad interactions, which contributes to the total exchange. When the error energy budgets are examined in spectral domain, one may ask questions on the error growth at a certain wavenumber from its interaction with other spatial scales. This entails the computation of the Fourier series of nonlinearities associated with quadratic and triple product. Here cospectra and coherence of Fourier spectra in the space of nonlinearities constitute the main computations.

3. Systematic and random errors and their growth rate equations

The systematic and random error kinetic energies are the function of forecast time (t) only and are

expressed as

$$Ks(t) = \langle ks(t) \rangle = \left\langle \frac{1}{2} \overline{Ves \bullet Ves} \right\rangle, \quad (1)$$

$$\text{and } Kr(t) = \langle kr(t) \rangle = \left\langle \frac{1}{2} \overline{Ver \bullet Ver} \right\rangle, \quad (2)$$

where $Ves = (ues, ves)$ and $Ver = (uer, ver)$ are the systematic and random error vector of total wind. ues, uer and ves, ver are the systematic, random component of zonal and meridional winds respectively. The over bar represents the spatial average and $\langle \rangle$ represents the ensemble average over all days at a fixed forecast time and the dot represents the dot product. Systematic error (equation 1), which is already time averaged may be written as

$$Ks(t) = \frac{1}{2} \overline{Ves \bullet Ves}. \quad (3)$$

Spatially averaged systematic and random error energy growth rate equations are used following Boer (1993) and are expressed as,

$$\begin{aligned} \frac{\partial ks}{\partial t} = & -\nabla \bullet \left[\left(\frac{Ves \bullet Ves}{2} + Ves \bullet Ver \right) \cdot Vf \right] \\ & + \left[\overline{(uer \cdot Vo + uer \cdot Ver) \bullet \nabla ues} \right. \\ & \left. + \overline{(ver \cdot Vo + ver \cdot Ver) \bullet \nabla ves} \right] \\ & - \left[\overline{(ues \cdot Ves \bullet \nabla uo + ves \cdot Ves \bullet \nabla vo)} \right. \\ & \left. + \overline{(ues \cdot Ver \bullet \nabla uo + ves \cdot Ver \bullet \nabla vo)} \right] \\ & + \overline{Ves \bullet Rs} \end{aligned} \quad (4)$$

and

$$\begin{aligned} \frac{\partial kr}{\partial t} = & -\nabla \bullet \left[\frac{Ver \bullet Ver}{2} \cdot Vf \right] \\ & - \left[\overline{(uer \cdot Vo + uer \cdot Ver) \bullet \nabla ues} \right. \\ & \left. + \overline{(ver \cdot Vo + ver \cdot Ver) \bullet \nabla ves} \right] \end{aligned}$$

$$\begin{aligned}
& - \left[\overline{(uer \cdot Ver \bullet \nabla uo + ver \cdot Ver \bullet \nabla vo)} \right. \\
& \quad \left. + \overline{(uer \cdot ves \bullet \nabla uo + ver \cdot ves \bullet \nabla vo)} \right] \\
& + \overline{Ver \bullet Rr}. \tag{5}
\end{aligned}$$

The details of the variables in equations (4) and (5) are given in part I of this study.

In the theory of harmonic analysis it is assumed that the wind fields are equispaced over a latitudinal circle and periodic having period 2π . The error in the wind fields is also cyclic and periodic in longitude. So, the finite discrete space Fourier series (Köckler 1994) may be applied to error fields as,

$$\begin{aligned}
ues(x_j) &= \frac{uesc_0}{2} + \sum_{n=1}^{N/2} [uesc_n \cos(nx_j) \\
& \quad + uess_n \sin(nx_j)], \\
ves(x_j) &= \frac{vesc_0}{2} + \sum_{r=1}^{N/2} [vesc_r \cos(rx_j) \\
& \quad + vess_r \sin(rx_j)],
\end{aligned}$$

and

$$\begin{aligned}
uer(x_j) &= \frac{uerc_0}{2} + \sum_{s=1}^{N/2} [uerc_s \cos(sx_j) \\
& \quad + uers_s \sin(sx_j)], \\
ver(x_j) &= \frac{verc_0}{2} + \sum_{m=1}^{N/2} [verc_m \cos(mx_j) \\
& \quad + vers_m \sin(mx_j)] \text{ for all } j=1, 2, \dots, N.
\end{aligned}$$

Respective Fourier cos and sin coefficients are

$$\begin{aligned}
uesc_n &= \frac{2}{N} \sum_{j=1}^N ues(x_j) \cos(nx_j); \\
uess_n &= \frac{2}{N} \sum_{j=1}^N ues(x_j) \sin(nx_j), \\
vesc_r &= \frac{2}{N} \sum_{j=1}^N ves(x_j) \cos(rx_j);
\end{aligned}$$

$$vess_r = \frac{2}{N} \sum_{j=1}^N ves(x_j) \sin(rx_j),$$

$$uerc_s = \frac{2}{N} \sum_{j=1}^N uer(x_j) \cos(sx_j);$$

$$uers_s = \frac{2}{N} \sum_{j=1}^N uer(x_j) \sin(sx_j),$$

$$verc_m = \frac{2}{N} \sum_{j=1}^N ver(x_j) \cos(mx_j);$$

$$vers_m = \frac{2}{N} \sum_{j=1}^N ver(x_j) \sin(mx_j).$$

In these expressions, n, r, s, m are zonal wavenumbers, N is the total number of points on a latitudinal circle. Subscript '0' indicates the zonal mean. The present study is not concerned with the zonal mean.

In Fourier spectra, the triad interaction technique (Murakami and Tomatsu 1964) is applied to each term on the r.h.s. of equations (4) and (5) so that those terms may be represented in the form of nonlinear error energy exchange of the parent wave of wavenumber ' n ' arising from the triad interaction with the pair of waves of wavenumbers ' r ' and ' s ' subject to the conditions $n = r + s$ and $n = |r - s|$. The systematic and random error energies in wavenumber domain are formulated by applying the Fourier transform on equations (3) and (2) respectively and are expressed as

$$Ks_n(t) = \overline{\frac{1}{2}(uesc_n^2 + uess_n^2 + vesc_n^2 + vess_n^2)}, \tag{6}$$

and

$$Kr_n(t) = \left\langle \overline{\frac{1}{2}(uerc_n^2 + uers_n^2 + verc_n^2 + vers_n^2)} \right\rangle. \tag{7}$$

Utilizing the Fourier transform in equations (4) and (5) and using the orthogonal property of the trigonometric functions (Chakraborty 1995), the systematic and random error energy growth rate budget equations in wavenumber domain (equations (8) and (9)) are written in the form seen in the Appendix.

4. Data and model

In this study, daily analyzed and five day forecast horizontal wind field of NCEP medium range forecast (MRF) model at 850 hPa available at $2.5^\circ \times 2.5^\circ$ grid size is used for the region of 30°S – 30°N over the global tropical region. The whole data set contains 35 days data of 00GMT from 1st June, 2000 to 5th July, 2000.

The resolution of the NCEP (MRF) model is T126 having 28 number of unevenly spaced sigma levels. The model physics included long- and short-wave radiation with diurnal variation, cloud radiation interaction, planetary boundary layer processes, deep and shallow convection, large scale condensation, gravity wave drag, enhanced orography, simple hydrology and vertical and horizontal diffusion (Kanamitsu and Saha 1995).

5. Results and discussions

Authors discuss the results in two different sections. The first section deals with the analysis of systematic and random error and their different components of growth rate budget in wavenumber domain. In the second section, possible explanations for dominant spectral modes of different component terms of the growth rate budget are given in the form of nonlinear error energy exchange using individual triad interactions.

5.1 Systematic and random error energetics in wavenumber domain

Utilizing the above mentioned data, systematic and random error variances, their growth rates and the related components governing the error growths are computed in wavenumber domain using equations (6), (7), (8) and (9). The Fourier spectral analysis of systematic and random errors and their growth rates and the several components of growth rate budget like flux, conversion, pure generation and mixed generation reveal that the interactions beyond wavenumber 20 are insignificant out of 72 zonal waves obtained from 144 grid points. All the terms are visualized in wavenumber domain at each forecast day where the abscissa shows the wavenumber and the ordinate indicates the value of the respective term in either m^2/sec^2 for systematic and random errors or m^2/sec^3 for the growth rate terms and different terms associated with the growth rate budget.

Figures 1 and 2 represent the spectral distribution of systematic and random errors at each forecast time, respectively, up to the 5-day forecast. The main conclusions from the two figures are as follows:

- (i) For both types of error, error variance decreases with increasing wavenumber at a fixed forecast time. For a fixed wavenumber, error variance, particularly the random error variance, increases with increasing forecast time.
- (ii) Tropical systematic error accumulates primarily at wavenumber 2 with a secondary maximum at wavenumber 5 (figure 1). The maximum random error energy associated with a particular wavenumber at different forecast times is very fluctuating in nature (figure 2). It shows that the random error energy is generally concentrated within the spectral band 4–7.

In the statement (i) above, the first part is in general agreement with previous authors (Lorenz 1969b, Boer 1984, 1993) in spectral error study. Detailed examination of figure 1 reveals that the dominant systematic error variance spectra associated with wavenumber 2 appear at 3–5 day forecasts and the secondary maximum in wavenumber 5 appears on all the forecast times, with its value remaining almost invariant with changing forecast time. Similarly in random error energy (figure 2), the dominant spectrum appears at wavenumber 4 for 3-day forecast, whereas for 4-day and 5-day forecasts it appears at wavenumber 6. There is also large accumulation of random error energy at wavenumber 7 for 5-day forecast. Next, we will examine how the dominant systematic and random error spectral modes are reflected in growth rate terms and explore the factors governing their growth rates.

Figure 3 shows the spectral distribution of systematic error growth rate at each forecast day up to 5-day forecast. It is observed from the figure that the maximum growth rate of systematic error occurs at wavenumber 2 for 1–4 day forecasts. For 5-day forecast, the growth rate maximum shifts to wavenumber 1. There are secondary maxima at wavenumber 5 and 6 in 1–2 day forecasts. In the random error growth rate, shown in figure 4, it is seen that the error energy has maximum growth at wavenumber 6 for 3–4 day forecasts, whereas wavenumber 4 dominates for 2-day forecast and wavenumber 7 dominates for 1-day and 5-day forecasts. By comparing the spectral distribution of systematic and random errors and their growth rate terms over the tropical region it is seen that the random error dominates compared to the systematic error in the total forecast error field and grows in higher wavenumbers (wavenumber band 4–7) in comparison to that in systematic error as was observed by Savijarvi (1984) and Dalcher & Kalanay (1987) in their error analysis study over the global region using 100 days of ECMWF data

Figure 1. Spectral distribution of systematic error variance ($\times 10^3$) in m^2/sec^2 at different forecast time.

Figure 2. Spectral distribution of random error variance ($\times 10^3$) in m^2/sec^2 at different forecast time.

Figure 3. Spectral distribution of systematic error growth rate ($\times 10^8$) in m^2/sec^3 at different forecast time.

Figure 4. Same as figure 3 but for random error growth rate ($\times 10^8$).

of 10 day forecasts and Boer (1993) in extended range forecasting experiment on Canadian Climate Centre GCM over Northern Hemisphere.

In the systematic error growth rate budget, figure 5 shows the spectral distribution of systematic error flux at each forecast day up to the 5-day forecast. It reveals that the tropical systematic error flux converges at wavenumber 1 and 5 for 2–5-day forecasts. The convergence increases with increasing forecast time. In the random error growth rate budget, the spectral distribution of error energy flux at each forecast day (figure 6) shows that the tropical random error flux converges at wavenumber 4 throughout all forecast days though the random error grows at wavenumber band 4–7.

As far as systematic error generation is concerned, figure 7 shows the spectral distribution of pure and mixed generation in m^2/sec^3 for each forecast day. The magnitudes of the systematic error generations are much smaller than that of error flux. Small generation of systematic error occurs at wavenumber 1, whereas mixed generation, i.e., conversion of systematic to random error, shows a small signal at wavenumber 6 for 3-day forecast and at wavenumber 4, 6, 8 for 4–5-day forecasts. Figure 8 displays the spectral distribution of pure and mixed generations of random error up to the 5-day forecast. This reveals that there is a random error generation at wavenumber 4 for 3–5-day forecasts due to the non linear barotropic interaction between the erroneous flow and the forecast flow. As far as mixed generation is concerned (figure 8), there is no such peak observed at any wavenumber for all forecast days. In the conversion term (figures 5 and 6), i.e., conversion of random to systematic error, and vice-versa, no dominant spectral mode is found for any of the forecast days in both figures.

From the overall analysis of systematic and random errors, their growth rates and the different components of growth rate budgets in wavenumber domain, it has been observed that though the dominant spectral energy appears at wavenumber 2 in the systematic error and its growth rate term, the same is observed at wavenumber 1 in systematic error energy flux. The secondary maximum of systematic error at wavenumber 5 is nicely reflected in the flux term. In the random error and its growth rate term, though the dominant peak is shown at wavenumber 6 for most forecast days, the flux and pure generation terms show the spectral peak at wavenumber 4. Only a small signature at wavenumber 6 for 3–5 day forecasts is seen in the systematic error mixed generation (figure 7). In the following section the authors will investigate the possible explanations of the above mentioned discrepancies through computation and

further analysis by resolving the dominant spectral modes into nonlinear individual triad interactions in the different components of the error growth rate budget.

5.2 *Decomposition of spectral error energy exchange into individual triad interactions*

Spectral energy associated with each term of the r.h.s. of equations (8) and (9) may be decomposed into individual triad interaction. As the triad interactions are insignificant beyond wavenumber 20, the first 20 interactions are taken into account in flux and generation terms of systematic and random error energy growth rate equations (equations (8) and (9)). Three waves of wavenumber n, r and s involved with any individual triad interaction (n, r, s) , while interacting among themselves, the parent wave (n) either receives energy from the other two waves r and s or loses energy to those waves according to the positive and negative value of that triad interaction respectively. So, the dominant spectral energy in a particular wavenumber implies larger density of positive value triads in comparison to the density of negative value triads associated with that wavenumber. It may be noted that the parameters associated with r and s in a triad interaction under the condition $n = r - s$ are different from those associated with $n = s - r$. In spectral distribution of triad interaction, ordinates of all the figures indicate the value of individual triad interaction of the corresponding term in m^2/sec^3 . Pairs of r and s , where r and s are separated by comma are expressed as triad interactions with the parent wavenumber n and are shown along abscissa.

Figures 9 and 10 indicate the triad interaction spectra of wavenumber 1 for $n = r - s$ and $n = s - r$ respectively of systematic error flux. Figures 11, 12, 13 and 14 indicate the same but of wavenumber 2 and 5 respectively. Summation of all the triads for $n = r - s$ and $n = s - r$ by combining figures 9 and 10 reveals that the density of positive valued triads is more in wavenumber 1 for 2–5-day forecasts of systematic error flux. In figures 11 and 12, the participating triads in wavenumber 2 for $n = r - s$ and $n = s - r$ are almost opposite in nature. So, the summation of all the triads leads to the less spectral energy associated with wavenumber 2 compared to that associated with wavenumber 1 though the most significant triad interaction (2, 7, 5) of the magnitude of $1.5 \times 10^{-7} \text{ m}^2/\text{sec}^3$ is associated with wavenumber 2 for $n = r - s$ on 2-day forecast in systematic error energy flux. Another significant triad interaction (1, 1, 2) associated with wavenumber 1 for $n = s - r$ is of the magnitude of $1.3 \times 10^{-7} \text{ m}^2/\text{sec}^3$ for systematic error flux on 5-day forecast. In

Figure 5. Spectral distribution of flux ($\times 10^8$) and conversion ($\times 10^8$) in systematic error growth rate budget in m^2/sec^3 at different forecast time.

Figure 6. Same as figure 5 but for random error flux ($\times 10^8$) and conversion ($\times 10^8$) in error energy growth rate budget.

Figure 7. Same as figure 5 but for pure and mixed generations ($\times 10^8$) in systematic error energy growth rate budget.

Figure 8. Same as figure 7 but in random error energy growth rate budget.

wavenumber 5, the triad interaction distribution associated with systematic error flux for $n = r - s$ and $n = s - r$ reveals that the spread of positive contribution is over large number of triad interactions shown in figures 13 and 14 respectively. This leads to the maintenance of the dominant spectral mode in wavenumber 5 though the individual triad is not as strong as that in wavenumber 1 and 2.

Figures 15, 16 show the spectral distribution of random error energy flux into individual triad interactions for $n = r - s, n = s - r$ in wavenumber 4 respectively. Combination of the participating triads for $n = r - s$ and $n = s - r$ in wavenumber 4 shows large density positive value triads whereas the same in wavenumber 6 (figures are not shown) indicates large density negative value triads. This gives the answer of the domination of the spectral energy associated with wavenumber 4 in random error flux even though the random error itself shows the dominant spectral energy in wavenumber 6 (figure 2) for maximum number of forecast days. Spectra of random error pure generation in terms of individual triads for $n = r - s$ (figure 17) and for $n = s - r$ (figure 18) show wavenumber 4 receives more energy than it loses while interacting nonlinearly with the other two waves, causing dominant spectral energy in wavenumber 4 for 3–5-day forecasts. The same feature has been observed in systematic error mixed generation at wavenumber 6 by combining the contributions of all the triads for $n = r - s, n = s - r$ and $n = r + s$ (figures are not shown). This gives a signature of conversion of systematic to random error at wavenumber 6 for 3–5-day forecasts.

Extensive analysis of all possible triad interactions in significant wavenumbers, performed from the spectral distribution of different components of error growth rate budget, shows that though the systematic error and its growth rate indicate dominant spectrum at wavenumber 2, the main reason for less spectral energy associated with wavenumber 2 in systematic error energy flux is that the wave of wavenumber 2 receives as well as loses nearly equal amount of energy through the triad interactions with other pairs of waves. The strongest individual nonlinear error energy exchange occurs between the wavenumber 2 and the pair of wavenumbers 7 and 5 in systematic error flux term. In random error, wavenumber 4 receives more amount of error energy than it loses by the process of triad interactions with other pairs of wave in flux and generation terms whereas wavenumber 6 shows opposite features though the random error and its growth rate show peak at wavenumber 6 for maximum number of forecast days.

6. Conclusions

Utilizing the above mentioned data and on the basis of rigorous computation and analysis of systematic and random error variances, their growth rates and the several factors governing the error growth in wavenumber domain, the whole study can be summarized as follows:

- Spectral distributions of systematic and random error show that error variance decreases with increasing forecast time. For a fixed wavenumber, error variance, particularly the random error variance, increases with increasing forecast time.
- Largest spectrum of tropical systematic error variance is observed at wavenumber 2 with a secondary maximum at wavenumber 5. The dominant wavenumber for tropical random error variance changes at different forecast times. Random error shows up in the spectral band 4–7, with a maximum at wavenumber 6. This represents higher wavenumbers compared to that of systematic error.
- Spectral distribution of systematic error growth rate up to 5-day forecast shows maximum at wave number 2 for 1–4 day forecasts and at wavenumber 1 for 5-day forecast. There are secondary maxima at wavenumber 5 and 6 for 1–2 day forecasts. Random error energy has maximum growth at wavenumber 6 for 3–4 day forecasts whereas wavenumber 4 dominates for 2-day forecast and wavenumber 7 dominates for 1-day and 5-day forecast.
- In error growth rate budget, systematic error energy flux shows its dominance at wavenumbers 1 and 5 though the error itself has maximum at wavenumber 2. This may be explained as there is a net increase of error energy in flux at these waves through their triad interactions with other pairs of waves. The most significant triad interaction (2, 7, 5) is associated with wavenumber 2 at 2-day forecast. Similarly, the random error energy flux and pure generation show their maximum spectral energy at wavenumber 4. Resolving the dominant spectra of these terms into individual triad interactions, it is observed that wavenumber 4 receives more energy than it loses into the interaction with the pairs of other waves whereas wavenumber 6, showing peak in random error and its growth rate, loses more error energy than it receives. Only a small amount of systematic error in its mixed generation term is converted into the random error at wavenumber 6.

It is to be noted here that in different components of the random error energy growth rate budget, the error energy associated with wavenum-

Figure 9. Individual triad interaction spectra of systematic error flux in wavenumber $n = 1$ for $n = r - s$ on 2-day, 3-day, 4-day and 5-day forecast.

Figure 10. Same as figure 9 but for $n = s - r$.

Figure 11. Same as figure 9 but in wavenumber $n = 2$ for $n = r - s$. Figure 12. Same as figure 9 but in wavenumber $n = 2$ for $n = s - r$.

Figure 13. Same as figure 9 but in wavenumber $n = 5$ for $n = r - s$. Figure 14. Same as figure 9 but in wavenumber $n = 5$ for $n = s - r$.

Figure 15. Individual triad interaction spectra of random error flux in wavenumber $n = 4$ for $n = r - s$ on 3-day, 4-day and 5-day forecast.

Figure 16. Same as figure 15 but for $n = s - r$.

Figure 17. Same as figure 15 but in wavenumber $n = 4$ for $n = r - s$ of random error pure generation.

Figure 18. Same as figure 17 but for $n = s - r$.

ber 6 appears to be underestimated, though the spectral distribution of random error energy (figure 2) shows maximum accumulation of error at wavenumber 6 for most of the forecast days. In majority too, individual triad interactions in wavenumber 6 show negative values. The possible reasons may be that waves of respective wavenumber bands 1–4 and 5–8 are basically barotropic and baroclinic in nature (Savijarvi 1984; Klinker and Capaldo 1986). But here, the authors examined the barotropic processes of systematic and random error growth. So naturally, the random error growth rate equation considered here is not sufficient to project the required amount of spectral energy associated with wavenumber 6 which is baroclinic in nature. Therefore, random error growth due to the baroclinic effect must be incorporated in the corresponding error growth rate equation.

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Appendix

$$\begin{aligned} & \sum_n \left[\frac{\partial}{\partial t} \left(\frac{1}{2} uesc_n^2 \right) + \frac{\partial}{\partial t} \left(\frac{1}{2} uess_n^2 \right) \right. \\ & \quad \left. + \frac{\partial}{\partial t} \left(\frac{1}{2} vesc_n^2 \right) + \frac{\partial}{\partial t} \left(\frac{1}{2} vess_n^2 \right) \right] \\ & = -\frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right) \end{aligned}$$

$$\left\{ \begin{aligned} & ufc_n \cdot uesc_r \frac{\partial uesc_s}{\partial x} \\ & + ufc_n \cdot vesc_r \frac{\partial vesc_s}{\partial x} \\ & + \frac{1}{2} \frac{\partial ufc_n}{\partial x} uesc_r \cdot uesc_s \\ & + \frac{1}{2} \frac{\partial ufc_n}{\partial x} vesc_r \cdot vesc_s \\ & + vfc_n \cdot uesc_r \frac{\partial uesc_s}{\partial y} \\ & + vfc_n \cdot vesc_r \frac{\partial vesc_s}{\partial y} \\ & + \frac{1}{2} \frac{\partial vfc_n}{\partial y} uesc_r \cdot uesc_s \\ & + \frac{1}{2} \frac{\partial vfc_n}{\partial y} vesc_r \cdot vesc_s \\ & + \frac{\partial ufc_n}{\partial x} uesc_r \cdot uerc_s \\ & + ufc_n \frac{\partial uesc_r}{\partial x} uerc_s \\ & + ufc_n \cdot uesc_r \frac{\partial uerc_s}{\partial x} \\ & + \frac{\partial ufc_n}{\partial x} vesc_r \cdot verc_s \\ & + ufc_n \frac{\partial vesc_r}{\partial x} verc_s \\ & + ufc_n \cdot vesc_r \frac{\partial verc_s}{\partial x} \\ & + \frac{\partial vfc_n}{\partial y} uesc_r \cdot uerc_s \\ & + vfc_n \frac{\partial uesc_r}{\partial y} uerc_s \\ & + vfc_n \cdot uesc_r \frac{\partial uerc_s}{\partial y} \\ & + \frac{\partial vfc_n}{\partial y} vesc_r \cdot verc_s \\ & + vfc_n \frac{\partial vesc_r}{\partial y} verc_s \\ & + vfc_n \cdot vesc_r \frac{\partial verc_s}{\partial y} \end{aligned} \right\} F$$

$$\begin{aligned}
& -\frac{1}{4} \left(\begin{array}{c} -\sum_{n=r+s} \\ +\sum_{n=r-s} \\ +\sum_{n=s-r} \end{array} \right) \left\{ \begin{array}{l} ufc_n \cdot uess_r \frac{\partial uess_s}{\partial x} \\ + ufc_n \cdot vess_r \frac{\partial vess_s}{\partial x} \\ + \frac{1}{2} \frac{\partial ufc_n}{\partial x} uess_r \cdot uess_s \\ + \frac{1}{2} \frac{\partial ufc_n}{\partial x} vess_r \cdot vess_s \\ + vfc_n \cdot uess_r \frac{\partial uess_s}{\partial y} \\ + vfc_n \cdot vess_r \frac{\partial vess_s}{\partial y} \\ + \frac{1}{2} \frac{\partial vfc_n}{\partial y} uess_r \cdot uess_s \\ + \frac{1}{2} \frac{\partial vfc_n}{\partial y} vess_r \cdot vess_s \\ + \frac{\partial ufc_n}{\partial x} uess_r \cdot uers_s \\ + ufc_n \frac{\partial uess_r}{\partial x} uers_s \\ + ufc_n \cdot uess_r \frac{\partial uers_s}{\partial x} \\ + \frac{\partial ufc_n}{\partial x} vess_r \cdot vers_s \\ + ufc_n \frac{\partial vess_r}{\partial x} vers_s \\ + ufc_n \cdot vess_r \frac{\partial vers_s}{\partial x} \\ + \frac{\partial vfc_n}{\partial y} uess_r \cdot uers_s \\ + vfc_n \frac{\partial uess_r}{\partial y} uers_s \\ + vfc_n \cdot uess_r \frac{\partial uers_s}{\partial y} \\ + \frac{\partial vfc_n}{\partial y} vess_r \cdot vers_s \\ + vfc_n \frac{\partial vess_r}{\partial y} vers_s \\ + vfc_n \cdot vess_r \frac{\partial vers_s}{\partial y} \end{array} \right\} L \\
& -\frac{1}{4} \left(\begin{array}{c} \sum_{n=r+s} \\ -\sum_{n=r-s} \\ +\sum_{n=s-r} \end{array} \right) \left\{ \begin{array}{l} ufs_n \cdot uesc_r \frac{\partial uess_s}{\partial x} \\ + ufs_n \cdot vesc_r \frac{\partial vess_s}{\partial x} \\ + \frac{1}{2} \frac{\partial ufs_n}{\partial x} uesc_r \cdot uess_s \\ + \frac{1}{2} \frac{\partial ufs_n}{\partial x} vesc_r \cdot vess_s \\ + vfs_n \cdot uesc_r \frac{\partial uess_s}{\partial y} \\ + vfs_n \cdot vesc_r \frac{\partial vess_s}{\partial y} \\ + \frac{1}{2} \frac{\partial vfs_n}{\partial y} uesc_r \cdot uess_s \\ + \frac{1}{2} \frac{\partial vfs_n}{\partial y} vesc_r \cdot vess_s \\ + \frac{\partial ufs_n}{\partial x} uesc_r \cdot uers_s \\ + ufs_n \frac{\partial uesc_r}{\partial x} uers_s \\ + ufs_n \cdot uesc_r \frac{\partial uers_s}{\partial x} \\ + \frac{\partial ufs_n}{\partial x} vesc_r \cdot vers_s \\ + ufs_n \frac{\partial vesc_r}{\partial x} vers_s \\ + ufs_n \cdot vesc_r \frac{\partial vers_s}{\partial x} \\ + \frac{\partial vfs_n}{\partial y} uesc_r \cdot uers_s \\ + vfs_n \frac{\partial uesc_r}{\partial y} uers_s \\ + vfs_n \cdot uesc_r \frac{\partial uers_s}{\partial y} \\ + \frac{\partial vfs_n}{\partial y} vesc_r \cdot vers_s \\ + vfs_n \frac{\partial vesc_r}{\partial y} vers_s \\ + vfs_n \cdot vesc_r \frac{\partial vers_s}{\partial y} \end{array} \right\} U
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} - \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned}
& ufs_n \cdot uess_r \frac{\partial uesc_s}{\partial x} \\
& + ufs_n \cdot vess_r \frac{\partial vesc_s}{\partial x} \\
& + \frac{1}{2} \frac{\partial ufs_n}{\partial x} uess_r \cdot uesc_s \\
& + \frac{1}{2} \frac{\partial ufs_n}{\partial x} vess_r \cdot vesc_s \\
& + vfs_n \cdot uess_r \frac{\partial uesc_s}{\partial y} \\
& + vfs_n \cdot vess_r \frac{\partial vesc_s}{\partial y} \\
& + \frac{1}{2} \frac{\partial vfs_n}{\partial y} uess_r \cdot uesc_s \\
& + \frac{1}{2} \frac{\partial vfs_n}{\partial y} vess_r \cdot vesc_s \\
& + \frac{\partial ufs_n}{\partial x} uess_r \cdot uerc_s \\
& + \frac{\partial ufs_n}{\partial x} vess_r \cdot verc_s \\
& + ufs_n \frac{\partial uess_r}{\partial x} uerc_s \\
& + ufs_n \cdot uess_r \frac{\partial uerc_s}{\partial x} \\
& + ufs_n \frac{\partial vess_r}{\partial x} verc_s \\
& + ufs_n \cdot vess_r \frac{\partial verc_s}{\partial x} \\
& + \frac{\partial vfs_n}{\partial y} uess_r \cdot uerc_s \\
& + vfs_n \frac{\partial uess_r}{\partial y} uerc_s \\
& + vfs_n \cdot uess_r \frac{\partial uerc_s}{\partial y} \\
& + \frac{\partial vfs_n}{\partial y} vess_r \cdot verc_s \\
& + vfs_n \frac{\partial vess_r}{\partial y} verc_s \\
& + vfs_n \cdot vess_r \frac{\partial verc_s}{\partial y}
\end{aligned} \right\} X \\
& + \frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned}
& \frac{\partial uesc_n}{\partial x} uoc_r \cdot uerc_s \\
& + \frac{\partial uesc_n}{\partial y} voc_r \cdot uerc_s \\
& + \frac{\partial uesc_n}{\partial x} uerc_r \cdot uerc_s \\
& + \frac{\partial uesc_n}{\partial y} uerc_r \cdot uerc_s \\
& + \frac{\partial vesc_n}{\partial x} uoc_r \cdot verc_s \\
& + \frac{\partial vesc_n}{\partial y} voc_r \cdot verc_s \\
& + \frac{\partial vesc_n}{\partial x} uerc_r \cdot verc_s \\
& + \frac{\partial vesc_n}{\partial y} verc_r \cdot verc_s
\end{aligned} \right\} CON \\
& + \frac{1}{4} \left(- \sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned}
& \frac{\partial uesc_n}{\partial x} uos_r \cdot uers_s \\
& + \frac{\partial uesc_n}{\partial y} vos_r \cdot uers_s \\
& + \frac{\partial uesc_n}{\partial x} uers_r \cdot uers_s \\
& + \frac{\partial uesc_n}{\partial y} uers_r \cdot vers_s \\
& + \frac{\partial vesc_n}{\partial x} uos_r \cdot vers_s \\
& + \frac{\partial vesc_n}{\partial y} vos_r \cdot vers_s \\
& + \frac{\partial vesc_n}{\partial x} uers_r \cdot vers_s \\
& + \frac{\partial vesc_n}{\partial y} vers_r \cdot vers_s
\end{aligned} \right\} VER
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{4} \left(\sum_{n=r+s} - \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uess_n}{\partial x} uoc_r \cdot uers_s \\ & + \frac{\partial uess_n}{\partial y} voc_r \cdot uers_s \\ & + \frac{\partial uess_n}{\partial x} uerc_r \cdot uers_s \\ & + \frac{\partial uess_n}{\partial y} uerc_r \cdot vers_s \\ & + \frac{\partial vess_n}{\partial x} uoc_r \cdot vers_s \\ & + \frac{\partial vess_n}{\partial y} voc_r \cdot vers_s \\ & + \frac{\partial vess_n}{\partial x} uerc_r \cdot vers_s \\ & + \frac{\partial vess_n}{\partial y} uerc_r \cdot vers_s \end{aligned} \right\} SI \\
& + \frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} - \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uess_n}{\partial x} uos_r \cdot uerc_s \\ & + \frac{\partial uess_n}{\partial y} vos_r \cdot uerc_s \\ & + \frac{\partial uess_n}{\partial x} uers_r \cdot uerc_s \\ & + \frac{\partial uess_n}{\partial y} uers_r \cdot verc_s \\ & + \frac{\partial vess_n}{\partial x} uos_r \cdot verc_s \\ & + \frac{\partial vess_n}{\partial y} vos_r \cdot verc_s \\ & + \frac{\partial vess_n}{\partial x} uers_r \cdot verc_s \\ & + \frac{\partial vess_n}{\partial y} vers_r \cdot verc_s \end{aligned} \right\} ON \\
& - \frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uoc_n}{\partial x} uesc_r \cdot uesc_s \\ & + \frac{\partial uoc_n}{\partial y} uesc_r \cdot vesc_s \\ & + \frac{\partial voc_n}{\partial x} uesc_r \cdot vesc_s \\ & + \frac{\partial voc_n}{\partial y} vesc_r \cdot vesc_s \end{aligned} \right\} PU \\
& - \frac{1}{4} \left(- \sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uoc_n}{\partial x} uess_r \cdot uess_s \\ & + \frac{\partial uoc_n}{\partial y} uess_r \cdot vess_s \\ & + \frac{\partial voc_n}{\partial x} uess_r \cdot vess_s \\ & + \frac{\partial voc_n}{\partial y} vess_r \cdot vess_s \end{aligned} \right\} RE \\
& - \frac{1}{4} \left(\sum_{n=r+s} - \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uos_n}{\partial x} uesc_r \cdot uess_s \\ & + \frac{\partial uos_n}{\partial y} uesc_r \cdot vess_s \\ & + \frac{\partial vos_n}{\partial x} uesc_r \cdot vess_s \\ & + \frac{\partial vos_n}{\partial y} vesc_r \cdot vess_s \end{aligned} \right\} GENE
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} - \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uos_n}{\partial x} uess_r \cdot uesc_s \\ & + \frac{\partial uos_n}{\partial y} uess_r \cdot vesc_s \\ & + \frac{\partial vos_n}{\partial x} uess_r \cdot vesc_s \\ & + \frac{\partial vos_n}{\partial y} vess_r \cdot vesc_s \end{aligned} \right\} RATION \\
& -\frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uoc_n}{\partial x} uesc_r \cdot uerc_s \\ & + \frac{\partial uoc_n}{\partial y} uesc_r \cdot verc_s \\ & + \frac{\partial voc_n}{\partial x} vesc_r \cdot uerc_s \\ & + \frac{\partial voc_n}{\partial y} vesc_r \cdot verc_s \end{aligned} \right\} MIXED \\
& -\frac{1}{4} \left(-\sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uoc_n}{\partial x} uess_r \cdot uers_s \\ & + \frac{\partial uoc_n}{\partial y} uess_r \cdot vers_s \\ & + \frac{\partial voc_n}{\partial x} vess_r \cdot uers_s \\ & + \frac{\partial voc_n}{\partial y} vess_r \cdot vers_s \end{aligned} \right\} GE \\
& -\frac{1}{4} \left(\sum_{n=r+s} - \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uos_n}{\partial x} uesc_r \cdot uers_s \\ & + \frac{\partial uos_n}{\partial y} uesc_r \cdot vers_s \\ & + \frac{\partial vos_n}{\partial x} vesc_r \cdot uers_s \\ & + \frac{\partial vos_n}{\partial y} vesc_r \cdot vers_s \end{aligned} \right\} NE \\
& -\frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} - \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uos_n}{\partial x} uess_r \cdot uerc_s \\ & + \frac{\partial uos_n}{\partial y} uess_r \cdot verc_s \\ & + \frac{\partial vos_n}{\partial x} vess_r \cdot uerc_s \\ & + \frac{\partial vos_n}{\partial y} vess_r \cdot verc_s \end{aligned} \right\} RATION \\
& + Ves \bullet Rs(RESIDUAL) \tag{8}
\end{aligned}$$

and

$$\begin{aligned}
& \sum_n \left[\frac{\partial}{\partial t} \left(\frac{1}{2} uerc_n^2 \right) + \frac{\partial}{\partial t} \left(\frac{1}{2} uers_n^2 \right) \right. \\
& \quad \left. + \frac{\partial}{\partial t} \left(\frac{1}{2} verc_n^2 \right) + \frac{\partial}{\partial t} \left(\frac{1}{2} vers_n^2 \right) \right] \\
& = -\frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right)
\end{aligned}$$

$$\begin{aligned}
& \left\{ \begin{aligned} & \frac{1}{2} \frac{\partial u f c_n}{\partial x} u e r c_r \cdot u e r c_s \\ & + \frac{1}{2} \frac{\partial u f c_n}{\partial x} v e r c_r \cdot v e r c_s \\ & + u f c_n \cdot u e r c_r \frac{\partial u e r c_s}{\partial x} \\ & + u f c_n \cdot v e r c_r \frac{\partial v e r c_s}{\partial x} \\ & + \frac{1}{2} \frac{\partial v f c_n}{\partial y} u e r c_r \cdot u e r c_s \\ & + \frac{1}{2} \frac{\partial v f c_n}{\partial y} v e r c_r \cdot v e r c_s \\ & + v f c_n \cdot v e r c_r \frac{\partial v e r c_s}{\partial y} \\ & + v f c_n \cdot u e r c_r \frac{\partial u e r c_s}{\partial y} \end{aligned} \right\} F \\
& - \frac{1}{4} \left(- \sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{1}{2} \frac{\partial u f c_n}{\partial x} u e r s_r \cdot u e r s_s \\ & + \frac{1}{2} \frac{\partial u f c_n}{\partial x} v e r s_r \cdot v e r s_s \\ & + u f c_n \cdot u e r s_r \frac{\partial u e r s_s}{\partial x} \\ & + u f c_n \cdot v e r s_r \frac{\partial v e r s_s}{\partial x} \\ & + \frac{1}{2} \frac{\partial v f c_n}{\partial y} u e r s_r \cdot u e r s_s \\ & + \frac{1}{2} \frac{\partial v f c_n}{\partial y} v e r s_r \cdot v e r s_s \\ & + v f c_n \cdot v e r s_r \frac{\partial v e r s_s}{\partial y} \\ & + v f c_n \cdot u e r s_r \frac{\partial u e r c_s}{\partial y} \end{aligned} \right\} L \\
& - \frac{1}{4} \left(\sum_{n=r+s} - \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{1}{2} \frac{\partial u f s_n}{\partial x} u e r c_r \cdot u e r s_s \\ & + \frac{1}{2} \frac{\partial u f s_n}{\partial x} v e r c_r \cdot v e r s_s \\ & + u f s_n \cdot u e r c_r \frac{\partial u e r s_s}{\partial x} \\ & + u f s_n \cdot v e r c_r \frac{\partial v e r s_s}{\partial x} \\ & + \frac{1}{2} \frac{\partial v f s_n}{\partial y} u e r c_r \cdot u e r s_s \\ & + \frac{1}{2} \frac{\partial v f s_n}{\partial y} v e r c_r \cdot v e r s_s \\ & + v f s_n \cdot v e r c_r \frac{\partial v e r s_s}{\partial y} \\ & + v f s_n \cdot u e r c_r \frac{\partial u e r s_s}{\partial y} \end{aligned} \right\} U \\
& - \frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} - \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{1}{2} \frac{\partial u f s_n}{\partial x} u e r s_r \cdot u e r c_s \\ & + \frac{1}{2} \frac{\partial u f s_n}{\partial x} v e r s_r \cdot v e r c_s \\ & + u f s_n \cdot u e r s_r \frac{\partial u e r c_s}{\partial x} \\ & + u f s_n \cdot v e r s_r \frac{\partial v e r c_s}{\partial x} \\ & + \frac{1}{2} \frac{\partial v f s_n}{\partial y} u e r c_r \cdot u e r c_s \\ & + \frac{1}{2} \frac{\partial v f s_n}{\partial y} v e r s_r \cdot v e r c_s \\ & + v f s_n \cdot v e r s_r \frac{\partial v e r c_s}{\partial y} \\ & + v f s_n \cdot u e r s_r \frac{\partial u e r c_s}{\partial y} \end{aligned} \right\} X
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uesc_n}{\partial x} uoc_r \cdot uerc_s \\ & + \frac{\partial uesc_n}{\partial y} voc_r \cdot uerc_s \\ & + \frac{\partial uesc_n}{\partial x} uerc_r \cdot uerc_s \\ & + \frac{\partial uesc_n}{\partial y} uerc_r \cdot verc_s \\ & + \frac{\partial vesc_n}{\partial x} uoc_r \cdot verc_s \\ & + \frac{\partial vesc_n}{\partial y} voc_r \cdot verc_s \\ & + \frac{\partial vesc_n}{\partial x} uerc_r \cdot verc_s \\ & + \frac{\partial vesc_n}{\partial y} verc_r \cdot verc_s \end{aligned} \right\} CON \\
& -\frac{1}{4} \left(-\sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uesc_n}{\partial x} uos_r \cdot uers_s \\ & + \frac{\partial uesc_n}{\partial y} vos_r \cdot uers_s \\ & + \frac{\partial uesc_n}{\partial x} uers_r \cdot uers_s \\ & + \frac{\partial uesc_n}{\partial y} uers_r \cdot vers_s \\ & + \frac{\partial vesc_n}{\partial x} uos_r \cdot vers_s \\ & + \frac{\partial vesc_n}{\partial y} vos_r \cdot vers_s \\ & + \frac{\partial vesc_n}{\partial x} uers_r \cdot vers_s \\ & + \frac{\partial vesc_n}{\partial y} vers_r \cdot vers_s \end{aligned} \right\} VER \\
& -\frac{1}{4} \left(\sum_{n=r+s} - \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uess_n}{\partial x} uoc_r \cdot uers_s \\ & + \frac{\partial uess_n}{\partial y} voc_r \cdot uers_s \\ & + \frac{\partial uess_n}{\partial x} uerc_r \cdot uers_s \\ & + \frac{\partial uess_n}{\partial y} uerc_r \cdot vers_s \\ & + \frac{\partial vess_n}{\partial x} uoc_r \cdot vers_s \\ & + \frac{\partial vess_n}{\partial y} voc_r \cdot vers_s \\ & + \frac{\partial vess_n}{\partial x} uerc_r \cdot vers_s \\ & + \frac{\partial vess_n}{\partial y} verc_r \cdot vers_s \end{aligned} \right\} SI \\
& -\frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} - \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uess_n}{\partial x} uos_r \cdot uerc_s \\ & + \frac{\partial uess_n}{\partial y} vos_r \cdot uerc_s \\ & + \frac{\partial uess_n}{\partial x} uers_r \cdot uerc_s \\ & + \frac{\partial uess_n}{\partial y} uers_r \cdot verc_s \\ & + \frac{\partial vess_n}{\partial x} uos_r \cdot verc_s \\ & + \frac{\partial vess_n}{\partial y} vos_r \cdot verc_s \\ & + \frac{\partial vess_n}{\partial x} uers_r \cdot verc_s \\ & + \frac{\partial vess_n}{\partial y} vers_r \cdot verc_s \end{aligned} \right\} ON
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uoc_n}{\partial x} uerc_r \cdot uerc_s \\ & + \frac{\partial uoc_n}{\partial y} uerc_r \cdot verc_s \\ & + \frac{\partial voc_n}{\partial x} uerc_r \cdot verc_s \\ & + \frac{\partial voc_n}{\partial y} verc_r \cdot verc_s \end{aligned} \right\} \text{PURE} \\
& -\frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} - \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uos_n}{\partial x} uers_r \cdot uerc_s \\ & + \frac{\partial uos_n}{\partial y} uers_r \cdot verc_s \\ & + \frac{\partial vos_n}{\partial x} uers_r \cdot verc_s \\ & + \frac{\partial vos_n}{\partial y} vers_r \cdot verc_s \end{aligned} \right\} \text{RATION} \\
& -\frac{1}{4} \left(-\sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uoc_n}{\partial x} uers_r \cdot uers_s \\ & + \frac{\partial uoc_n}{\partial y} uers_r \cdot vers_s \\ & + \frac{\partial voc_n}{\partial x} uers_r \cdot vers_s \\ & + \frac{\partial voc_n}{\partial y} vers_r \cdot vers_s \end{aligned} \right\} \text{GE} \\
& -\frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uoc_n}{\partial x} uerc_r \cdot uesc_s \\ & + \frac{\partial uoc_n}{\partial y} uerc_r \cdot vesc_s \\ & + \frac{\partial voc_n}{\partial x} uesc_r \cdot verc_s \\ & + \frac{\partial voc_n}{\partial y} vesc_r \cdot verc_s \end{aligned} \right\} \text{MIXED} \\
& -\frac{1}{4} \left(\sum_{n=r+s} - \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uos_n}{\partial x} uerc_r \cdot uers_s \\ & + \frac{\partial uos_n}{\partial y} uerc_r \cdot vers_s \\ & + \frac{\partial vos_n}{\partial x} uerc_r \cdot vers_s \\ & + \frac{\partial vos_n}{\partial y} verc_r \cdot vers_s \end{aligned} \right\} \text{NE} \\
& -\frac{1}{4} \left(-\sum_{n=r+s} + \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uoc_n}{\partial x} uers_r \cdot uess_s \\ & + \frac{\partial uoc_n}{\partial y} uers_r \cdot vess_s \\ & + \frac{\partial voc_n}{\partial x} uess_r \cdot vers_s \\ & + \frac{\partial voc_n}{\partial y} vess_r \cdot vers_s \end{aligned} \right\} \text{GE}
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{4} \left(\sum_{n=r+s} - \sum_{n=r-s} + \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uos_n}{\partial x} uerc_r \cdot uess_s \\ & + \frac{\partial uos_n}{\partial y} uerc_r \cdot vess_s \\ & + \frac{\partial vos_n}{\partial x} uesc_r \cdot vers_s \\ & + \frac{\partial vos_n}{\partial y} vesc_r \cdot vers_s \end{aligned} \right\} NE \\
& -\frac{1}{4} \left(\sum_{n=r+s} + \sum_{n=r-s} - \sum_{n=s-r} \right) \\
& \left\{ \begin{aligned} & \frac{\partial uos_n}{\partial x} uers_r \cdot uesc_s \\ & + \frac{\partial uos_n}{\partial y} uers_r \cdot vesc_s \\ & + \frac{\partial vos_n}{\partial x} uess_r \cdot vercs_s \\ & + \frac{\partial vos_n}{\partial y} vess_r \cdot vercs_s \end{aligned} \right\} RATION \\
& + Ver \bullet Rr(RESIDUAL). \quad (9)
\end{aligned}$$

Ensemble averages of equations (8) and (9) are taken for all days at each forecast time. In the r.h.s. of both equations, the extent of flux, conversion, pure and mixed generation terms is indicated by writing the term outside the second bracket. The left hand side indicates the growth rate term for systematic and random error respectively.

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