2-D Deformation analysis of a half-space due to a long dip-slip fault at finite depth

Sushil K Tomar[1] and Naresh K Dhiman[2]

[1]*Department of Mathematics, Panjab University, Chandigarh 160 014, India.*
e-mail: sktomar@yahoo.com
[2]*S. U. S. College of Engineering and Technology, Tangori (Mohali) 140 306, Punjab, India.*

Closed form analytical expressions of stresses and displacements at any field point due to a very long dip-slip fault of finite width buried in a homogeneous, isotropic elastic half-space, are presented. Airy stress function is used to derive the expressions of stresses and displacements which depend on the dip angle and depth of the upper edge of the fault. The effect of dip angle and depth of the upper edge of the fault on stresses and displacements is studied numerically and the results obtained are presented graphically. Contour maps for stresses and displacements are also presented. The results of Rani and Singh (1992b) and Freund and Barnett (1976) have been reproduced.

1. Introduction

The problems relating to the application of the elastic dislocation theory to earth faulting have been investigated by many researchers and have appeared in the open literature. Savage (1974) among others is an important reference in the area of dislocation theory. 2-D dip-slip dislocation models of faulting have been used extensively to model the lithospheric deformation associated with faulting; see, Cohen (1992) and the references listed therein. Freund and Barnett (1976) developed a 2-D model of dip-slip faulting in a uniform half-space and obtained the relationship between fault slip and surface deformation. The problem of static deformation in a uniform half-space due to long faults has been attempted by a number of researchers. Singh and Rani (1996) presented step-by-step progress made in the direction of crustal deformation modeling associated with strike-slip and dip-slip faulting in the earth. Cohen (1996) gave convenient formulas for determining dip-slip parameters for geophysical observables. Singh *et al* (1997) investigated the problem of deformation of a layered half-space due to a very long dip-slip fault. Okada (1985, 1992) investigated 3-D problems of surface deformation and internal deformation due to shear and tensile faults in a half-space. Rani and Singh (1992b) attempted the static problem of elastic deformation of a half-space due to a long dip-slip fault of finite width. They obtained closed form expressions of the displacements and stresses at an arbitrary point of a uniform half-space. Other notable references in this field of study are Maruyama (1964, 1966); Savage (1980); Rybicki (1986); Rani *et al* (1991); Rani and Singh (1992a); Singh *et al* (1992); Singh and Rani (1993); and Singh *et al* (1994). Recently, Singh and Singh (2000) and Singh *et al* (2002) studied similar problems due to tensile fault.

In the present analysis, a 2-D faulting problem is investigated. We have obtained the expressions for displacements and stresses at an arbitrary field point of a uniform half-space due to a long dip-slip inclined fault. It is believed that all the faults are not surface breaking and blind faults are also there beneath the ground surface, therefore, we have considered the upper edge of the inclined dip-slip fault at some finite depth h below the earth surface. By adopting simple and straightforward techniques we have been able to obtain the analytical results for 2-D case, which have not been derived by Okada

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(1992) and do not form the special case derived from the results obtained by him.

2. Stresses and displacements

We consider a long dip-slip inclined fault, whose upper edge is at a depth h below the earth surface. We take the origin, x and y-axis on the surface of the earth and z-axis pointing downward (see figure 1). We shall use the notations $x = x_1$, $y = x_2$, $z = x_3$. The problem is 2-dimensional in $x_2 - x_3$ plane, in which the displacement components u_i in the direction of x_i $(i = 1, 2, 3)$ are independent of x_1 co-ordinate so that $u_1 = 0$ and $\partial/\partial x_1 \equiv 0$. As given in Rani and Singh (1992b), the Airy stress function U for dip-slip line source locating at (y_2, y_3) is given by

$$U = \frac{\mu\ b\ ds}{2\pi(1-\sigma)} \Bigg\{\cos 2\delta \left[(x_2 - y_2)(x_3 - y_3)\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right) + 4x_3y_3(x_2 - y_2)(x_3 + y_3)\frac{1}{R_2^4}\right] + \sin 2\delta \left[(x_3 - y_3)^2 \frac{1}{R_1^2} + (x_3^2 - y_3^2 + 2x_3y_3)\frac{1}{R_2^2} - 4x_3y_3(x_3 + y_3)^2 \frac{1}{R_2^4}\right]\Bigg\}, \tag{1}$$

where

$$R_1^2 = (x_2 - y_2)^2 + (x_3 - y_3)^2,$$
$$R_2^2 = (x_2 - y_2)^2 + (x_3 + y_3)^2.$$

Substituting $y_2 = s\cos\delta$ and $y_3 = h + s\sin\delta$ into equation (1) and integrating over s between the limits 0 to L, where $L = s_2 - s_1$ $(s_1 \le s \le s_2)$ is the width of the fault, we obtain

$$U = \frac{\mu\ b}{2\pi(1-\sigma)} \Bigg[(x_2\sin\delta - x_3\cos\delta + h\cos\delta)\ln\frac{R_1}{R_2} + 2x_3\{\sin\delta(x_2\sin\delta + x_3\cos\delta)s + h[(x_3 + h)\cos\delta + (x_2 + s\cos\delta)\sin\delta]\}\frac{1}{R_2^2}\Bigg]\Bigg|_0^L, \tag{2}$$

and the above expressions of R_1^2 and R_2^2 now take the following form

$$R_1^2 = (x_2 - s\cos\delta)^2 + (x_3 - h - s\sin\delta)^2,$$
$$R_2^2 = (x_2 - s\cos\delta)^2 + (x_3 + h + s\sin\delta)^2.$$

Using the well known relations between the Airy stress function U and the stresses p_{ij}, we obtain

$$p_{22} = \frac{\mu\ b}{2\pi(1-\sigma)}\Bigg\{(x_2\sin\delta - 3x_3\cos\delta + h\cos\delta)\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right) + (s\sin 2\delta + 2h\cos\delta)\left(\frac{1}{R_1^2} + \frac{3}{R_2^2}\right) - 2(x_2\sin\delta - x_3\cos\delta + h\cos\delta)\left[(x_3 - h - s\sin\delta)^2\frac{1}{R_1^4} - (x_3 + h + s\sin\delta)^2\frac{1}{R_2^4}\right] - 4[s\sin\delta(3x_2x_3\sin\delta + 5x_3^2\cos\delta + 2s\sin\delta[2x_3\cos\delta + x_2\sin\delta]) + 2h(x_3 + h + s\sin\delta)[(2x_3 + h)\cos\delta + (x_2 + s\cos\delta)\sin\delta] + 2h(x_2\sin\delta + 2x_3\cos\delta)s\sin\delta + hx_3[(x_3 + h)\cos\delta + (x_2 + s\cos\delta)\sin\delta]]\frac{1}{R_2^4} + 16(x_3 + h + s\sin\delta)^2 x_3[\sin\delta(x_2\sin\delta + x_3\cos\delta)s + h\{(x_3 + h)\cos\delta + (x_2 + s\cos\delta)\sin\delta\}]\frac{1}{R_2^6}\Bigg\}\Bigg|_0^L, \tag{3}$$

$$p_{23} = \frac{\mu b}{2\pi(1-\sigma)}\Bigg\{(x_2\cos\delta - x_3\sin\delta + h\sin\delta)\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right) + s\cos 2\delta\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right) + 2(x_2\sin\delta - x_3\cos\delta + h\cos\delta)(x_2 - s\cos\delta)\left[(x_3 - h - s\sin\delta)\frac{1}{R_1^4} - (x_3 + h + s\sin\delta)\frac{1}{R_2^4}\right] + 4(x_2 - s\cos\delta)[s\sin\delta(x_2\sin\delta + 2x_3\cos\delta) + h\{(2x_3 + h)\cos\delta + (x_2 + s\cos\delta)\sin\delta\}]\frac{1}{R_2^4} + 4(x_3 + h + s\sin\delta)x_3(s\sin^2\delta + h\sin\delta)\frac{1}{R_2^4} - 16(x_2 - s\cos\delta)(x_3 + h + s\sin\delta) x_3\{s\sin\delta(x_2\sin\delta + x_3\cos\delta) + h[(x_3 + h)\cos\delta + (x_2 + s\cos\delta)\sin\delta]\}\frac{1}{R_2^6}\Bigg\}\Bigg|_0^L, \tag{4}$$

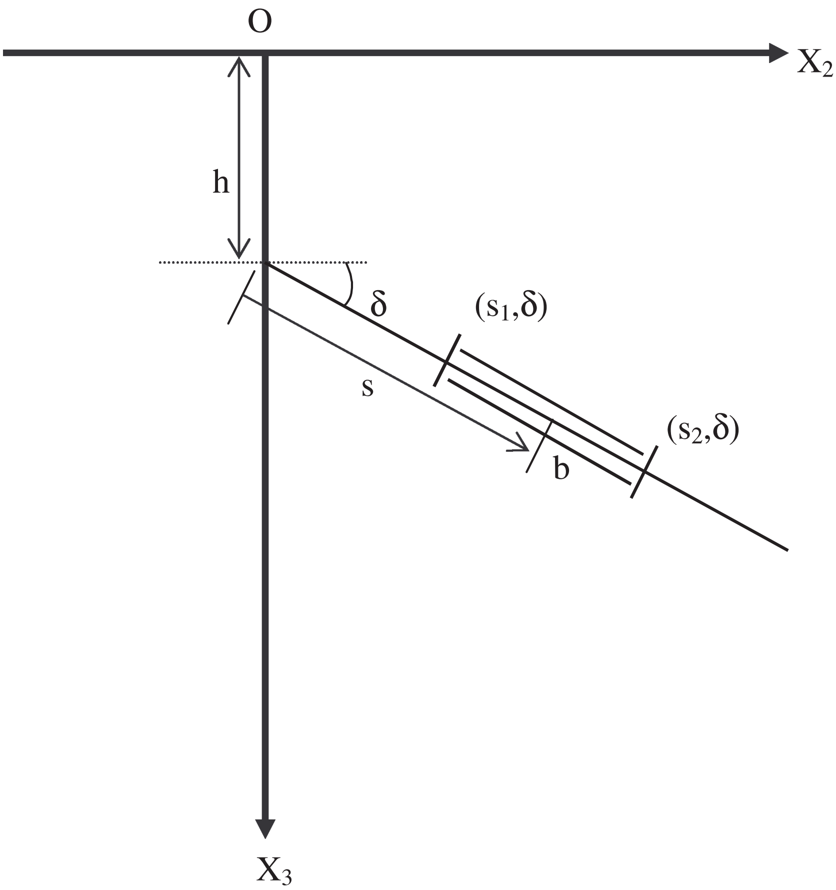

Figure 1. Geometry of the problem.

$$p_{33} = \frac{\mu b}{2\pi(1-\sigma)}\Bigg\{(x_2\sin\delta + x_3\cos\delta - h\cos\delta)$$
$$\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right) - s\sin 2\delta\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right)$$
$$+ 2(x_2\sin\delta - x_3\cos\delta + h\cos\delta)$$
$$\left[(x_3 - h - s\sin\delta)^2\frac{1}{R_1^4} - (x_3 + h + s\sin\delta)^2\frac{1}{R_2^4}\right]$$
$$+ 4x_3\{s\sin\delta(x_2\sin\delta + 3x_3\cos\delta + s\sin 2\delta)$$
$$- 2h\sin\delta(x_2 - s\cos\delta) + 3h[\cos\delta(x_3 + h)$$
$$+ \sin\delta(x_2 + s\cos\delta)]\}\frac{1}{R_2^4} - 16(x_3 + h + s\sin\delta)^2$$
$$x_3\{s\sin\delta(x_2\sin\delta + x_3\cos\delta) + h[(x_3 + h)\cos\delta$$
$$+(x_2 + s\cos\delta)\sin\delta]\}\frac{1}{R_2^6}\Bigg\}\Bigg|_0^L. \tag{5}$$

Next, using the equations (3) and (5) into the relations between displacements and stresses (see Sokolnikoff 1956; pp: 265), we obtain

$$u_2 = \frac{b}{2\pi}\Bigg[\frac{(1-2\sigma)}{2(1-\sigma)}\sin\delta\ln\frac{R_1}{R_2}$$
$$- \cos\delta\tan^{-1}\left(\frac{x_2 - s\cos\delta}{x_3 - h - s\sin\delta}\right)$$
$$+ \cos\delta\tan^{-1}\left(\frac{x_2 - s\cos\delta}{x_3 + h + s\sin\delta}\right)$$
$$- \frac{1}{2(1-\sigma)}(x_2\sin\delta - x_3\cos\delta + h\cos\delta)$$
$$\times (x_2 - s\cos\delta)\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right)$$
$$- \frac{1}{(1-\sigma)}x_3\sin\delta(s\sin\delta + h)\frac{1}{R_2^2}$$
$$+ \frac{2}{(x_3 + h + s\sin\delta)^2}[s\sin\delta(x_3 + s\sin\delta)$$
$$\{(x_3 + h + s\sin\delta)^2\sin\delta - (x_3 + s\sin\delta)$$
$$(x_2 - s\cos\delta)\cos\delta\} - h\{\cos\delta(x_2 - s\cos\delta)$$
$$[(x_3 + h + s\sin\delta)^2 + x_3 s\sin\delta]$$

$$+\sin\delta(x_3+2s\sin\delta+h)[s\cos\delta(x_2-s\cos\delta)$$

$$-(x_3+s\sin\delta+h)^2]\}]\frac{1}{R_2^2}$$

$$+\frac{2}{(1-\sigma)}(x_2-s\cos\delta)x_3$$

$$[s\sin\delta(x_2\sin\delta+x_3\cos\delta)+h\{(x_3+h)\cos\delta$$

$$+(x_2+s\cos\delta)\sin\delta\}]\frac{1}{R_2^4}\Bigg]\Bigg|_0^L, \qquad (6)$$

$$u_3=\frac{b}{2\pi}\left[-\frac{(1-2\sigma)}{2(1-\sigma)}\cos\delta\ln\frac{R_1}{R_2}\right.$$

$$-\sin\delta\tan^{-1}\left(\frac{x_3+s\sin\delta+h}{x_2-s\cos\delta}\right)$$

$$+\sin\delta\tan^{-1}\left(\frac{x_3-s\sin\delta-h}{x_2-s\cos\delta}\right)$$

$$+2(s\sin\delta+h)(x_2\sin\delta+x_3\cos\delta+h\cos\delta)\frac{1}{R_2^2}$$

$$-\frac{1}{(1-\sigma)}[s\sin\delta(x_2\sin\delta+2x_3\cos\delta)$$

$$+h\{(2x_3+h)\cos\delta+(x_2+s\cos\delta)\sin\delta\}]\frac{1}{R_2^2}$$

$$-\frac{1}{2(1-\sigma)}(x_2\sin\delta-x_3\cos\delta+h\cos\delta)$$

$$\left[(x_3-h-s\sin\delta)\frac{1}{R_1^2}-(x_3+h+s\sin\delta)\frac{1}{R_2^2}\right]$$

$$+\frac{2}{(1-\sigma)}(x_3+h+s\sin\delta)x_3$$

$$[s\sin\delta(x_2\sin\delta+x_3\cos\delta)+h\{(x_3+h)\cos\delta$$

$$+(x_2+s\cos\delta)\sin\delta\}]\frac{1}{R_2^4}\Bigg]\Bigg|_0^L. \qquad (7)$$

It is clear from the formulae (3)–(7) that the stresses and displacements depend on the depth of the upper edge of the fault h and the dip angle δ.

3. Particular cases

If we take $h = 0$, then the problem reduces to that considered by Rani and Singh (1992b). In this case, putting $h = 0$ into equations (3)–(7), we obtain the expressions of stresses and displacements given by (11)–(15) of Rani and Singh (1992b) for the relevant problem. It can be verified that by putting both $x_3 = 0$ and $h = 0$ into equations (6) and (7), one can obtain the same expressions of displacements as given by Freund and Barnett (1976) (cf. Rani and Singh (1992b) for corrections in their results).

4. Numerical results and discussion

We shall compute the stresses and the displacements numerically on the surface of the elastic half-space containing a long dip-slip inclined fault of width L. We therefore take $s_1 = 0$ and $s_2 = L$. We also assume that $\sigma = 0.25$ and adopt the notations for non-dimensional quantities given in (13) of Rani and Singh (1992) alongwith $H = h/L$. When $\delta = 90^0$, the fault is vertical. Figure 2 shows the variation of horizontal displacement U_2 with the distance from the fault, Y for three positions of the fault, namely $H = 0, 1$ and 2. We notice from this figure that the maximum value of U_2 is attained at $Y = 0$. The value of U_2 decreases monotonically as Y increases. We also notice that the maximum value of U_2 is equal to one at $Y = 0$ on the surface, irrespective of the value of depth parameter H, while for all other values of $Y > 0$, the value of U_2 depends on H. Figure 3 shows the variation of vertical displacement U_3 with the distance from the fault. We notice that the value of U_3 on the half-space is different for different value of H for $Y > 0$. Its value is maximum in magnitude at $Y = 0$, irrespective of the position of the fault and it decreases as Y increases and ultimately dies out as Y approaches to infinity. Figure 4 shows the variation of U_2 with Y, when fault is inclined at an angle $\delta = 45^0$. We notice from this figure that U_2 is discontinuous at origin for inclined surface breaking fault ($H = 0$), while it is continuous for all values of Y for inclined buried faults ($H = 1$ and 2). Figure 5 shows the variation of U_2 with Y, when fault is horizontal i.e., when the angle $\delta = 0^0$. We notice from this figure that the displacement U_2 is continuous for all values of Y and is symmetrical about $Y = 0.5$. Here, we would like to mention that the displacement U_2 could not be calculated at $Z = 0$ for $H = 0$. The reason being that in this case, since $\delta = 0^0$, so physically the fault is along the surface of the elastic half-space, which is meaningless. Figures 6–8 show the contour maps for stresses around a long vertical dip-slip fault whose upper edge is at $h = L$. We observed on comparing the contour maps for stresses with that of Rani and Singh (1992b) that these maps are influenced greatly by the depth parameter H. The contour maps for displacements are shown through figures 9 and 10 for $H = 1$. These maps exhibit

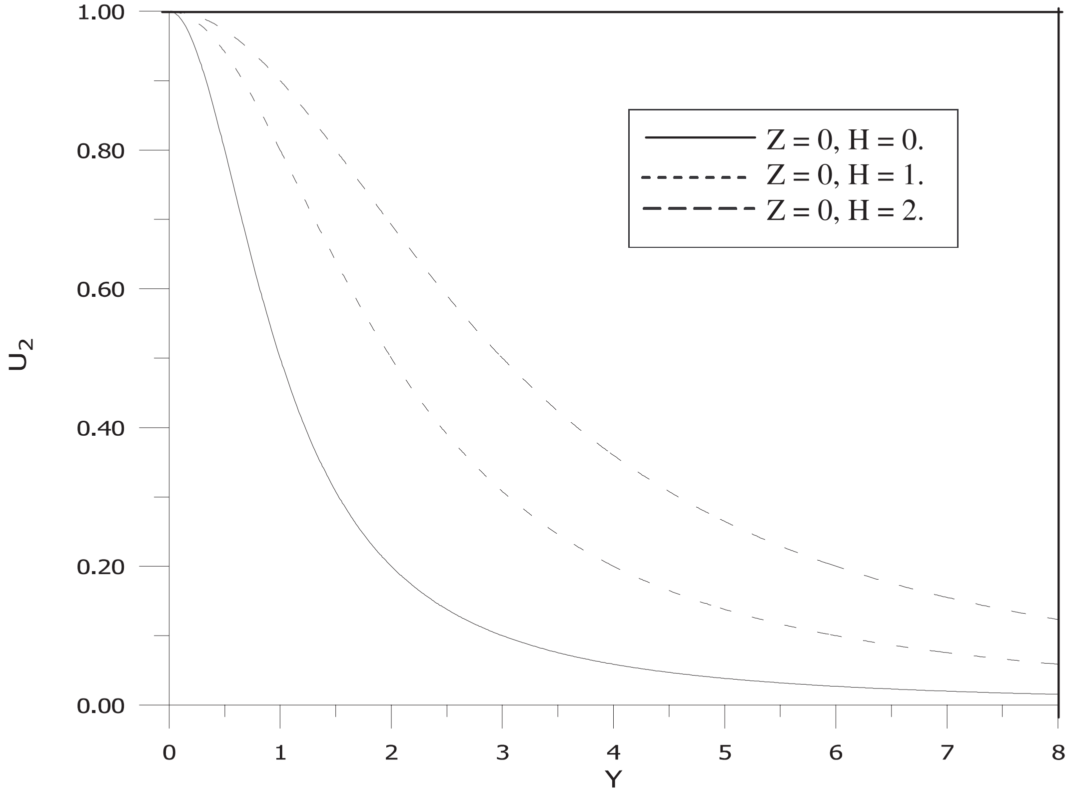

Figure 2. Variation of U_2 versus Y at dip angle $\delta = 90^0$.

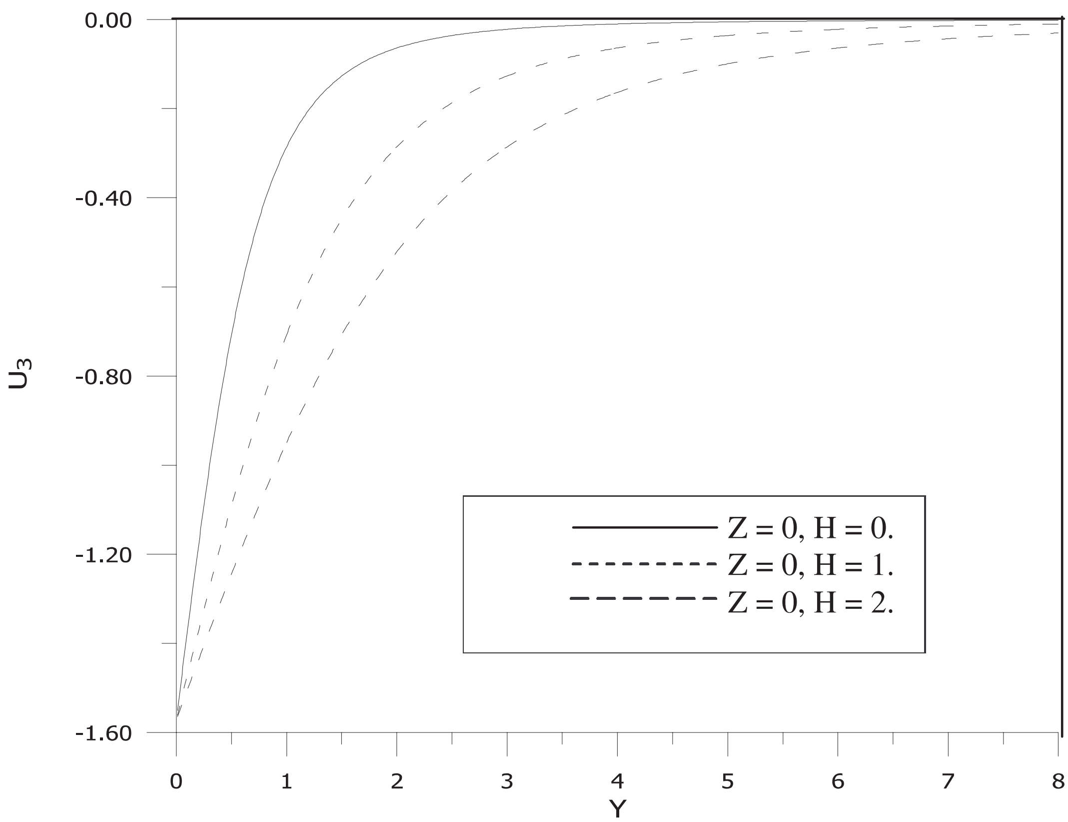

Figure 3. Variation of U_3 versus Y at dip angle $\delta = 90^0$.

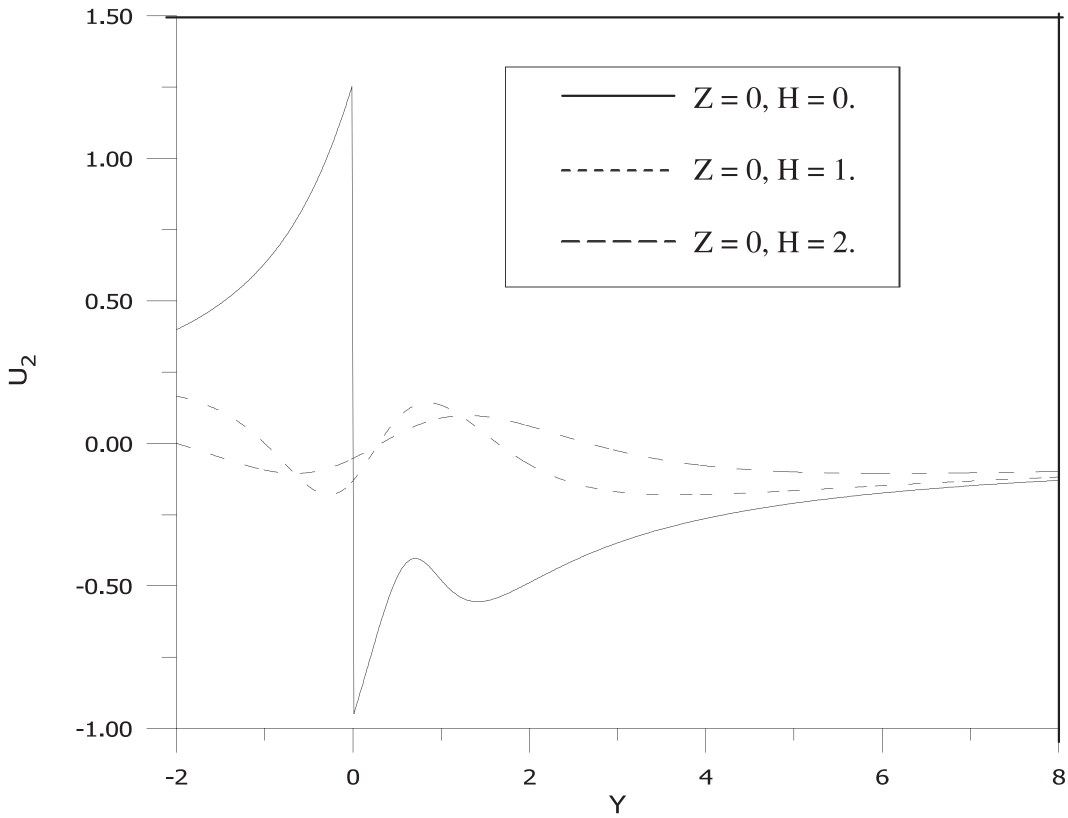

Figure 4. Variation of U_2 versus Y at dip angle $\delta = 45^0$.

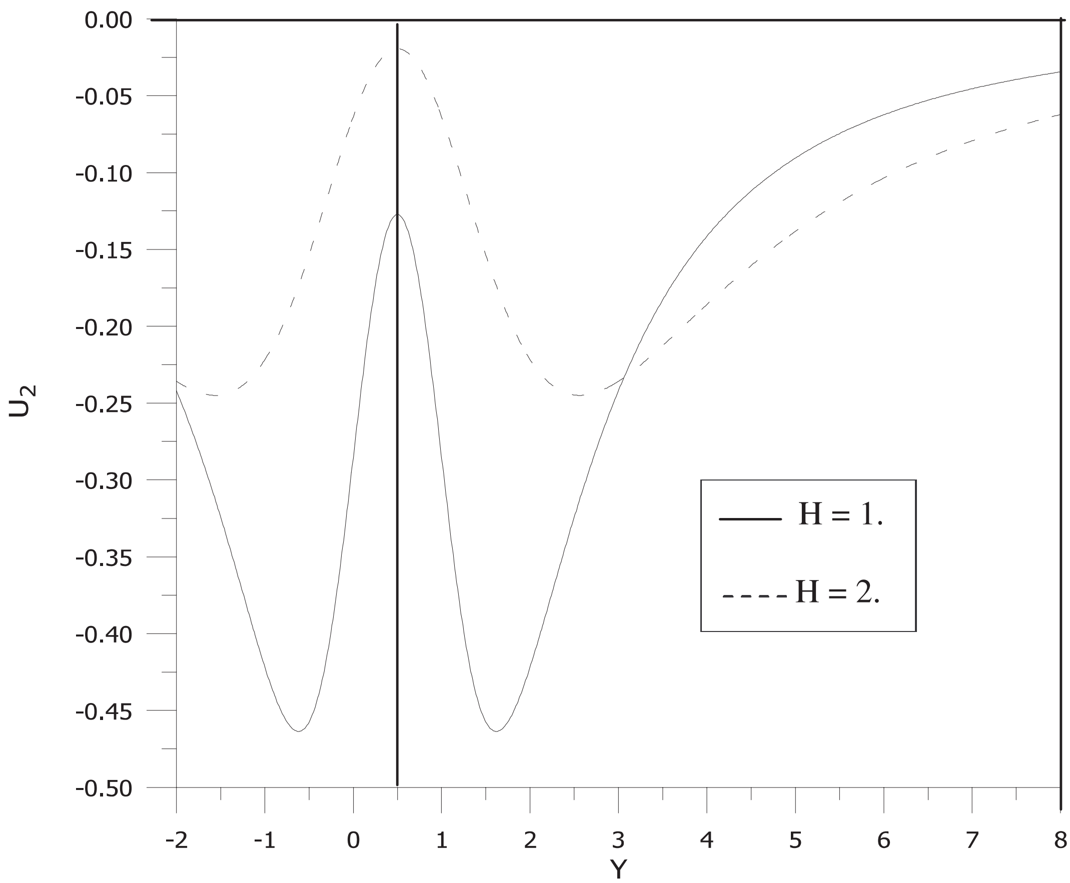

Figure 5. Variation of U_2 versus Y at dip angle $\delta = 0^0$.

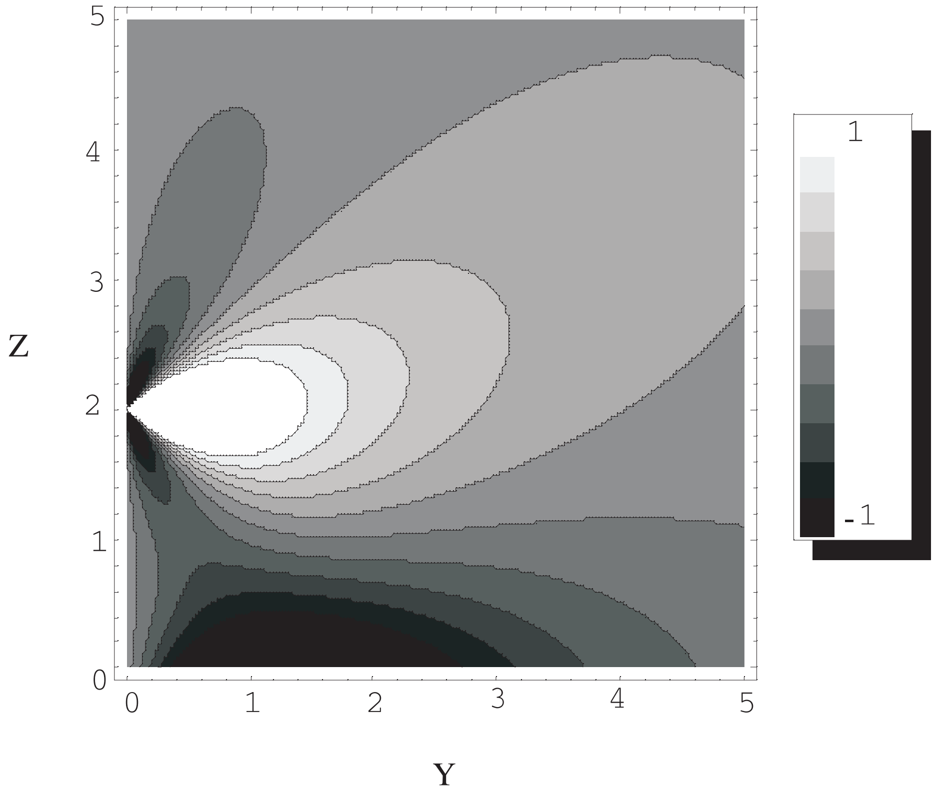

Figure 6. Contour map for P_{22} when $\delta = 90^0$ and $H = 1$.

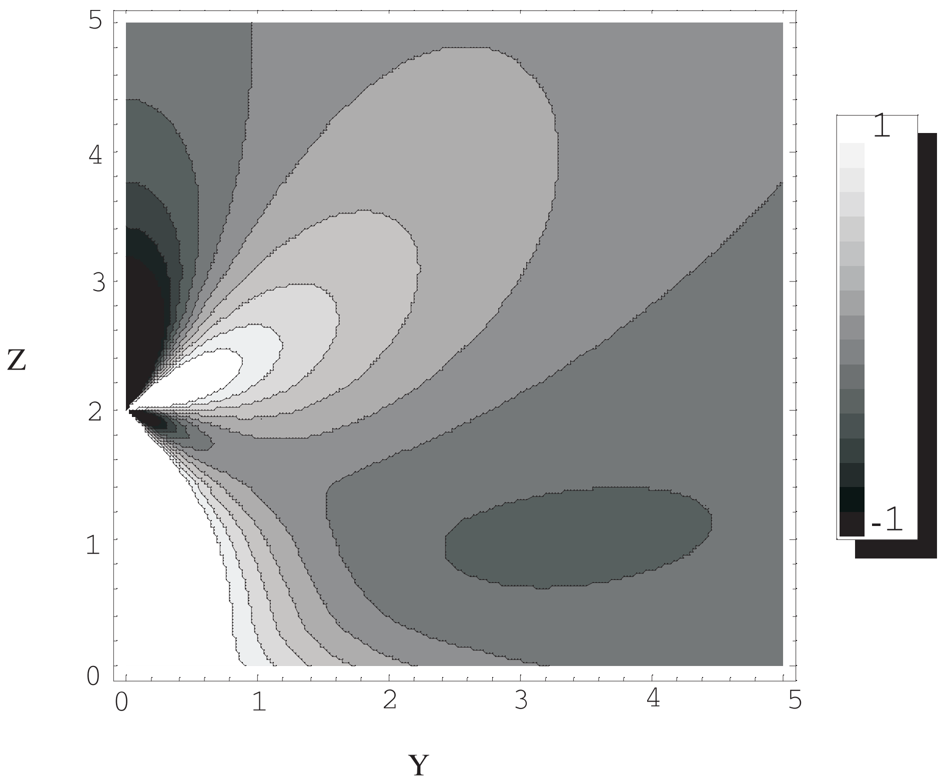

Figure 7. Contour map for P_{23} when $\delta = 90^0$ and $H = 1$.

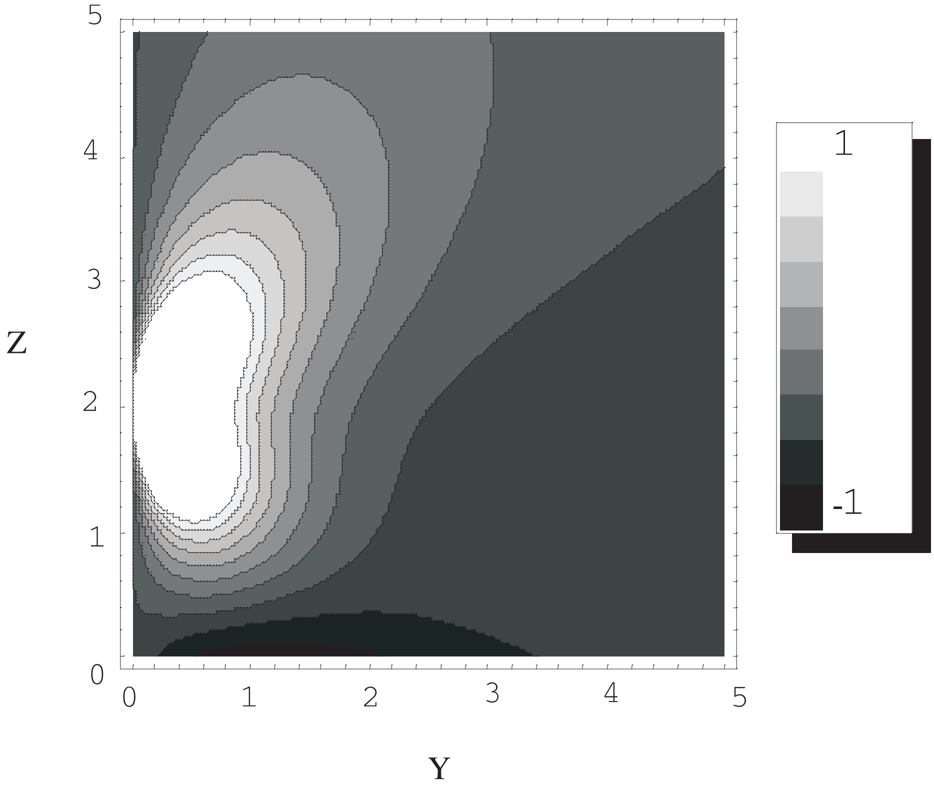

Figure 8. Contour map for P_{33} when $\delta = 90^0$ and $H = 1$.

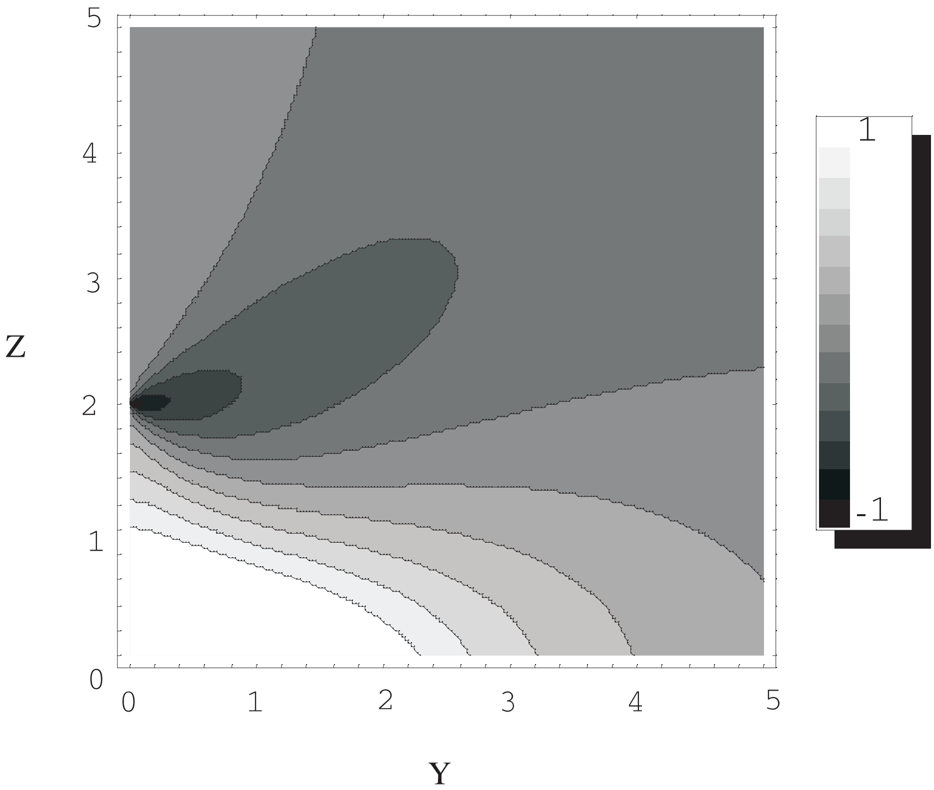

Figure 9. Contour map for U_2 when $\delta = 90^0$ and $H = 1$.

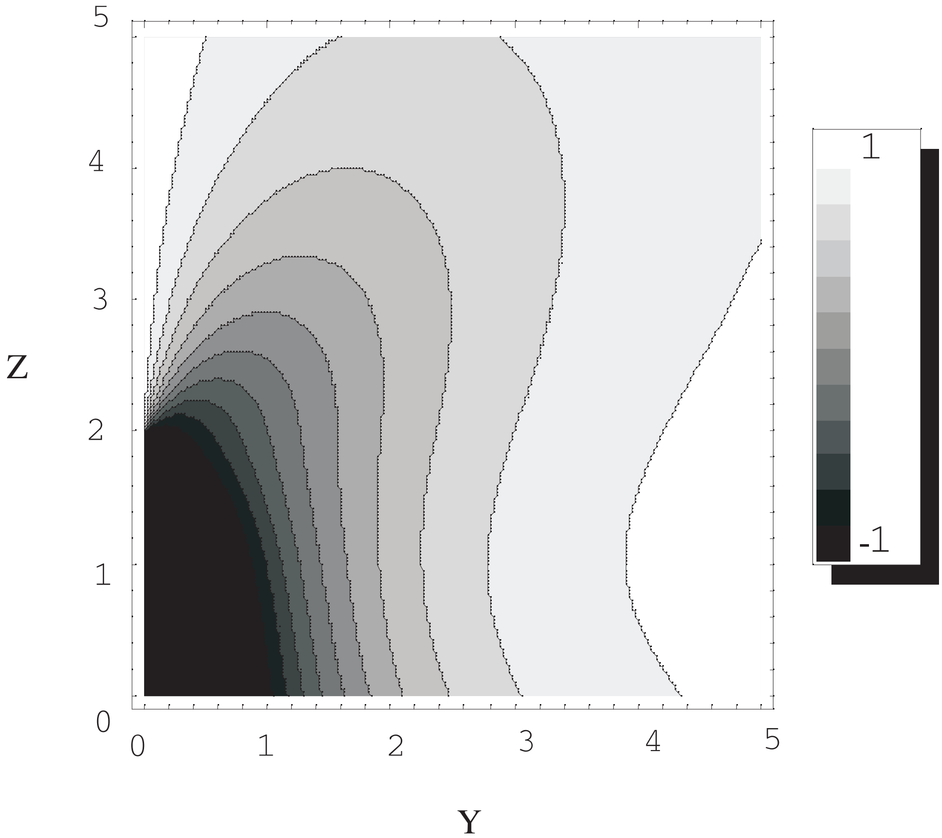

Figure 10. Contour map for U_3 when $\delta = 90^0$ and $H = 1$.

the variation of displacement field around the vertical dip-slip fault. In the legend of contour maps, symbol "1" stands for maximum value and "-1" stands for minimum value.

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