Study of the relationship between non-dimensional roughness length and wave age, effected by wave directionality

Naoya Suzuki[1], Naoto Ebuchi[2], Chaofang Zhao[1,3], Isao Watabe[4] and Yasuhiro Sugimori[1]

[1] *Center for Environmental Remote Sensing (CEReS), Chiba University, 1-33 Yayoi Inage Chiba, 263-8522 Japan.*
[2] *Center for Atmospheric and Oceanic Studies, Graduate School of Science, Tohoku University, Aoba, Sendai 980-8578, Japan.*
[3] *Ocean Remote Sensing Laboratory, Ocean University of Qingdao, China.*
[4] *National Research Institute for Earth Science and Disaster Prevention, Science and Technology Agency, 9-2 Nijigahama, Hiratsuka, Kanagawa, 254-0823 Japan.*

Relationship between the non-dimensional roughness length and inverse of wave age has been discussed without consideration of wave directions, though wind wave field consists of various directional component waves. In this study we observe wave heights by an array of four wave gauges at the Hiratsuka Tower of (Independent Administrative Institution) National Research Institute for Earth Science and Disaster Prevention (NIED), Japan, and discuss the effect of wave directionality. As a result, the data sets were classified into two different groups according to the directional wave spectrum distribution. In case 1 only swell and wind waves exist and in case 2 there exist wave components from several directions. It is shown that the case of multiple-directional component waves (case 2) may affect the non-dimensional roughness length and friction velocity.

1. Introduction

The sea surface wind stress has generally been estimated using the drag coefficient C_D. Many efforts have recently been made to elucidate the wave dependence of the sea-surface roughness parameter z_0, which has one-to-one correspondence to C_D under near-neutral conditions. However, there still exists considerable disagreement among investigators for the parameterization of the coefficient, especially for its dependence on wave growth.

In order to estimate the wind stress and roughness length, Charnock's (1955) well known formula,

$$\frac{gz_0}{u_*^2} = \beta, \qquad (1)$$

has widely been utilized, where g is the acceleration of gravity, and β is a constant. For the value of β, there seems to be considerable disagreement among authors. For example, Charnock (1955) proposed 0.0068, Smith and Banke (1975) gave 0.0130, Garrat (1977) suggested 0.0144, and Wu (1980) proposed 0.0185. A possible reason for the disagreement might be poor quality and quantity of wind stress measurements. Also effects of waves on the wind stress should be considered explicitly.

Stewart (1974) presented a general form for the wave dependence of the wind stress as a function of wave age,

$$\frac{gz_0}{u_*^2} = f\left[\frac{C_P}{u_*}\right], \qquad (2)$$

where C_P is the phase speed of wind waves of the spectral peak frequency. By assuming the lin-

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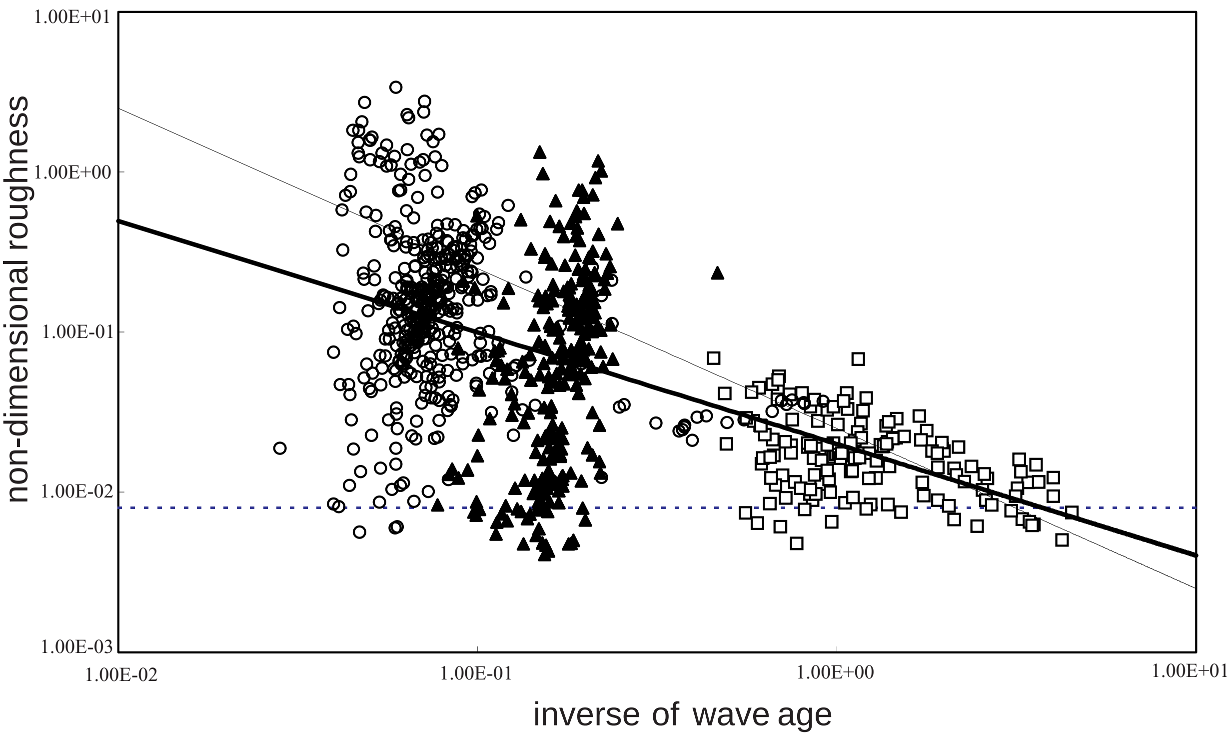


Figure 1. The relation between non-dimensional roughness length and inverse wave age. **Solid line (bold):** result of Suzuki *et al* (1998), **solid line (thin):** Toba & Koga (1986) $z_0\sigma_P/u_* = 0.025$, **dashed line:** Charnock (1955) $gz_0/u_*^2 = 0.008$ (cited from Suzuki *et al* 1998).
□: Former laboratory data set: Hamada (1963), Kunishi (1963), Kunishi and Imasato (1966), Toba (1961, 1972), Hsu *et al* (1982), Masuda and Kusaba (1987).
◯: Former field data set: Kawai *et al* (1977), Donelan (1979), Merzi and Graf (1985), Bass Strait, Geernaert et al (1987).
▲: Data obtained at the Hiratsuka Tower (Suzuki *et al* 1998).

ear dispersion relationship of deep water waves, the wave age C_P/u_* can be expressed as $C_P/u_* = (\sigma_P u_*/g)^{-1}$, where σ_P is the angular frequency of the wind-wave spectral peak. Toba and Koga (1986) proposed a formula involving the peak frequency of the wind wave as,

$$\frac{z_0\sigma_P}{u_*} = \gamma, \tag{3}$$

Masuda and Kusaba (1987) proposed a functional form of equation (2) with non-dimensional roughness length and inverse wave age,

$$\frac{gz_0}{u_*^2} = a\left(\frac{\sigma_P u_*}{g}\right)^b, \tag{4}$$

where a and b are constants. The Charnock's (1955) formula in equation (1) corresponds to $b = 0$ and $a = \beta$ in equation (4). Constants a and b differ in various investigations. Toba *et al* (1990) took $b = -0.5$. Masuda and Kusaba (1987) took $b = 1.10$. Donelan *et al* (1993) took $b = 1.0$. A negative value of the exponent b is suggested from the data set ranging from laboratories to open oceans, though scatter of the data points is very large. The negative exponent of b indicates positive dependence of the wind stress on the wave age or wave growth. In the composite data set collected from previous studies, there is a lack of data in a range of wave age between laboratory and open ocean waves. As shown in figure 1, Suzuki *et al* (1998) tried to fill this lack of data using data from the Hiratsuka Tower of National Research Institute for Earth Science and Disaster Prevention (NRIESDP). They proposed,

$$\frac{gz_0}{u_*^2} = 0.020\left(\frac{\sigma_P u_*}{g}\right)^{-0.697}. \tag{5}$$

As seen in figure 1, scattering of the non-dimensional roughness length is very large. Because of the large variation of non-dimensional roughness length in the relationship between non-dimensional roughness length and inverse wave age, such relation is still to be determined. We think that it is difficult to always express the influence of waves by a single parameter of wave age. The relationship between the non-dimensional roughness length and inverse of wave age has been discussed without consideration of wave directions, though wind wave field consists of various directional component waves. In this study, we observe the directional spectrum, which is the parameter representing an essential structure of an ocean wave, to survey the state of waves for the roughness, and discuss the effect of wave directionality.

2. Method and observation

The EMEP (extended maximum entropy principle method, Hashimoto 1997) was used to estimate the directional wave spectrum. The directional spectrum $S(f, \theta)$ is commonly expressed as the product of the frequency spectrum $S(f)$ and the directional spreading function $G(\theta|f)$,

$$S(f, \theta) = S(f)G(\theta|f). \tag{6}$$

In EMEP, directional spreading function $G(\theta|f)$ is expressed as

$$G(\theta|f) = \frac{\exp\left[\sum_{n=1}^{N}\{a_n(f)\cos n\theta + b_n(f)\sin n\theta\}\right]}{\int_0^{2\pi}\exp\left[\sum_{n=1}^{N}\{a_n(f)\cos n\theta + b_n(f)\sin n\theta\}\right]\mathrm{d}\theta}, \tag{7}$$

where $a_n(f)$ and $b_n(f)(n = 1, \ldots, N)$ are unknown parameters and N is the order of the model, and the cross-power spectrum is expressed as

$$\phi_i(f) = \int_0^{2\pi} H_i(f, \theta)G(\theta|f)\mathrm{d}\theta \quad (i = 1, \ldots, N), \tag{8}$$

where N is the number of equations. After substituting equation (7) into (8), the obtained equation must be modified by considering the existence of errors ε_i defined as

$$\varepsilon_i = \frac{\int_0^{2\pi}\{\phi_i - H_i(\theta)\}\exp\left\{\sum_{n=1}^{N}(a_n\cos n\theta + b_n\sin n\theta)\right\}\mathrm{d}\theta}{\int_0^{2\pi}\exp\left\{\sum_{n=1}^{N}(a_n\cos n\theta + b_n\sin n\theta)\right\}\mathrm{d}\theta} \quad (i = 1, \ldots, N), \tag{9}$$

where N is the number of remaining independent equations following the elimination of meaningless equations such as those determined from the zero co-spectrum and zero quadrature-spectrum, etc. However, as equation (9) is nonlinear with respect to $a_n, b_n(n = 1, \ldots, N)$, it is difficult to solve. Consequently, if the approximate solution $\tilde{a}_n, \tilde{b}_n(n = 1, \ldots, N)$ is assumed, the solution $a_n, b_n(n = 1, \ldots, N)$ can be rewritten as

$$\left.\begin{aligned} a_n &= \tilde{a}_n + a'_b \\ b_n &= \tilde{b}_n + b'_n \end{aligned}\right\}, \tag{10}$$

where a'_n and b'_n are the residuals between the solution a_n and b_n and approximate solution $\tilde{a}_n$ and $\tilde{b}_n$. Minimizing $\sum \varepsilon_i^2$ for a particular data set also introduces an additional problem of choosing the optimal finite model order N (N-th order model) in equation (7); here the following Akaike's Information Criterion (AIC, (Akaike 1973)) is used.

$$\mathrm{AIC} = M(\ln 2\pi\hat{\sigma}^2 + 1) + 2(2N + 1), \tag{11}$$

where M is the number of equation (equation (9)) and $\hat{\sigma}^2$ is the estimated variance of $\varepsilon_i(i = 1, \ldots, M)$. The directional spreading function $G(\theta|f)$ is estimated by calculating a_n and b_n to the minimum AIC, and the directional spectrum can be derived from equation (6).

The observation was conducted at the coastal marine observation tower of Hiratsuka Experiment Station (HES) of (Independent Administrative Institution) the National Research Institute for Earth Science and Disaster Prevention (NIED) in Japan (figure 2). The distance of the tower from the shore is about 1000 m, and the depth is about 20 m. Three steel piles, with diameter of 80 cm, support the observation tower. The tower above the ocean is some 21 m in height, and the observation room is located at the highest part, where several measuring devices are installed.

The wind speed was observed by the aerovane-type and sonic anemometers situated on the roof of the observation room, about 21.5 m height from the mean sea surface. Two horizontal and one vertical component of the wind velocity are recorded with a sampling interval of 0.25 s.

The wave gauges were capacitance type, and installed on poke poles from the operation platform. Measurement of the directional spectra is needed to correlate water surface elevation that can minimize four wave gauges. Frequency is calculated using measure water surface elevation by using a Fast Fourier Transform (FFT), which is used for calculating (equations (6–11)) the EMEP (Hashimoto 1997), which gives the directional spectra. In this study, an array of four capacitance type wave gauges were used to measure the directional spectrum. A schematic diagram of the wave gauge array is shown in figure 3. Sampling interval was 0.1 sec, and the number of data for one series of record was 6000 (10 min.). The observation conducted period was from 18th to 19th October 2000 and 19th to 23rd February 2001.

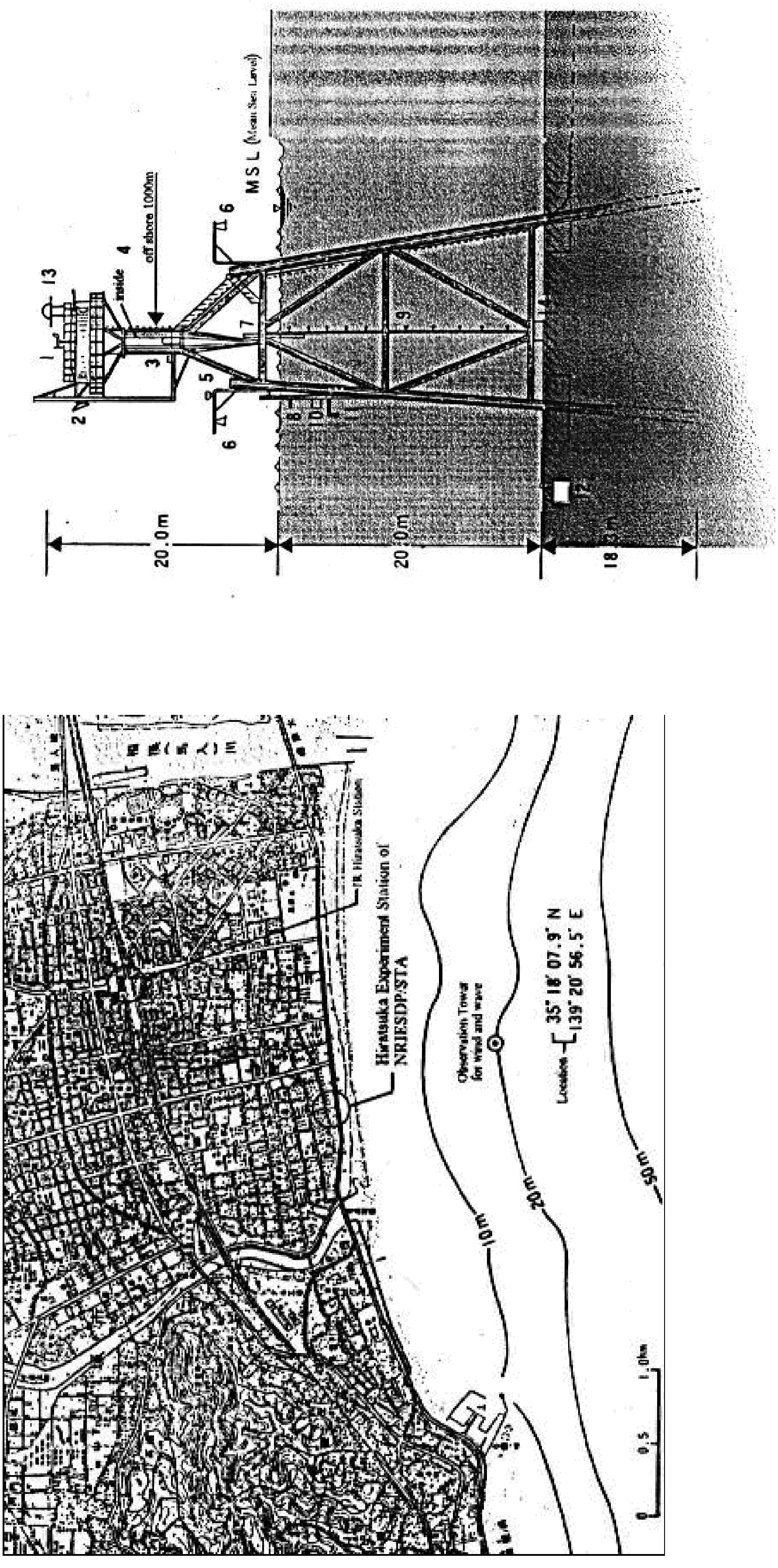


Figure 2. Location of the Hiratsuka Experiment Station and a schematic picture of the tower.

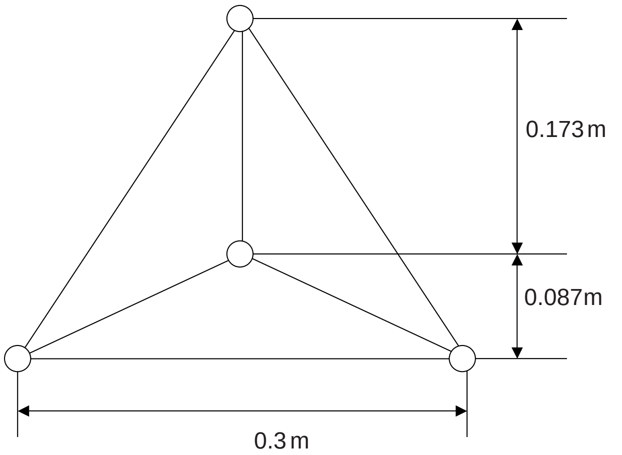


Figure 3. Wave measurement array (Star Array). This triangle is a regular triangle in which one side of a triangle is 0.3 m.

3. Data Processing

The friction velocity u_* is obtained every 10 min. using the eddy correlation method. The roughness length z_0 is given by the log +linear law of wind profile under near-neutral conditions (Toba and Ebuchi 1991) as,

$$\frac{U(z)}{u_*} = \left[\ln\left(\frac{z}{z_0}\right) + cRi\right]/k, \qquad (12)$$

where Ri is the bulk Richardson number and c is a constant of 3. The second term is to compensate for the effect of stratification. The bulk Richardson number is estimated by,

$$Ri = zg(Ta - Tw)/[(273.0 + Ta)U(z)^2] \qquad (13)$$

where z is the anemometer height, Ta and Tw are the temperature of air and water in °C, respectively. In order to select data obtained under near-neutral conditions, only the data of $|Ri| < 0.02$ are used in this analysis according to Toba *et al* (1990).

Since the discussion of wind stress reviewed in the previous section and the wave dependence to be examined in this study are based on aerodynamically rough flow, we also applied a criterion for the roughness Reynolds number proposed by Toba *et al* (1990) as,

$$Re\,(\equiv u_* z_0/\nu) > 2.3 \qquad (14)$$

where ν is the kinematic viscosity of air.

As mentioned in the previous section, this study is focused on data of young wave ages under short fetches. Thus, only data for periods when the wind was blowing off-shore are selected. Also we assumed quasi-static conditions where wind and waves satisfy the local equilibrium and discarded data if wind speed or direction showed large variations in the observation period of 10 min.

The directional wave spectrum and power spectrum are calculated at an interval of 10 min. by using EMEP (Hashimoto 1997) as described in section 2. Data that showed swell-dominant features in the power spectrum were discarded. The angular frequency of the spectral peak σ_P is determined from the power spectrum.

In order to confirm the local equilibrium between wind and waves, we examined the value of a constant B in Toba's (1972) 3/2-power law defined by,

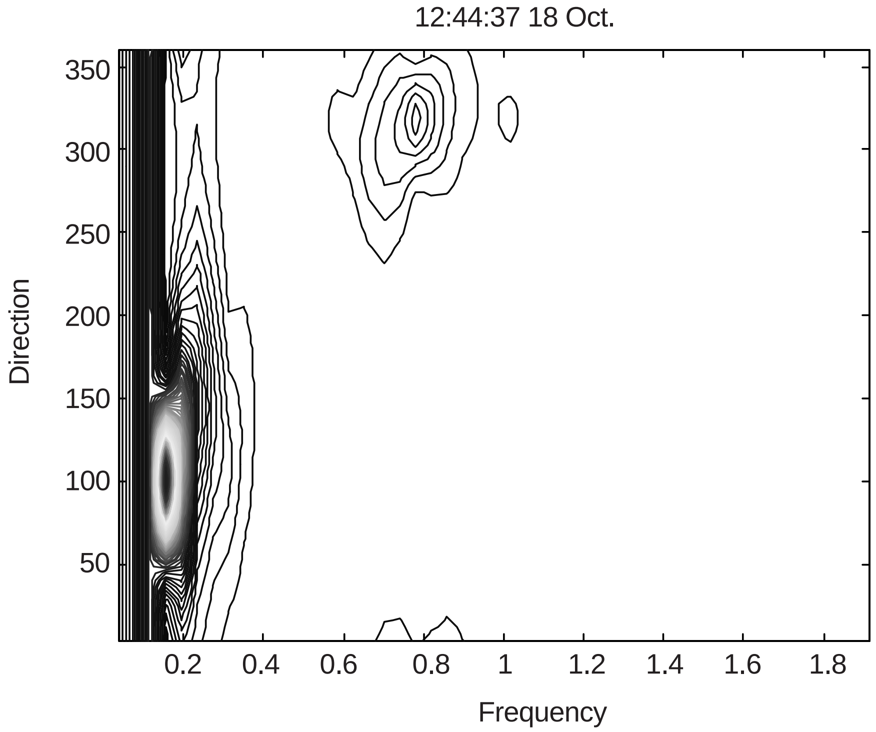
$$B = H(gu_*)^{1/2}T^{3/2} \qquad (15)$$

where H and T are the significant wave height and period. A criterion proposed by Toba *et al* (1990) as $0.050 < B < 0.074$ is used to eliminate data obtained under non-equilibrium conditions.

4. Results and discussion

The directional spectra were separated into two different groups in this observation. In case 1, only swell and wind waves exist. In case 2, there exist multi-directional component waves. From the power spectrum, case 1 has two clear peaks corresponding to wind wave and swell respectively. There exist several peaks in case 2 (see figures 4 and 5). In figure 4, peak frequency of swell is 0.18 Hz and that of wind wave is 0.78 Hz. The direction of swell is West-South-West (100°) and North-North-East (320°) in the case of wind wave. During this period wind direction was NNE (310°). Figure 5 shows peak frequency of swell 0.16 Hz and 120° direction (WSW). Due to the presence of multi-directional component waves, the wind wave is not known. The peak in directional wave spectra in case 1 and case 2 is identified in frequency spectra with approximately 0.05 Hz frequency error. Therefore, we used peak frequency spectra in calculating FFT. Wind wave data are selected by Toba's (1972) 3/2-power law for case 1 and case 2. The selection of wind waves in case 2 has been made by assigning the wind and wave direction less than 60°. In this case, the resultant wind and wave direction has been found to be 30°.

We estimated the relationship between the non-dimensional roughness length gz_0/u_*^2 and inverse of wave age $\sigma_P u_*/g$ for case 1 and case 2 using the friction velocity, roughness and frequency of spectral peak on the wind wave. Using case 1 and case 2 data in figure 6, we plotted the relation between non-dimensional roughness length and inverse wave age. This shows clearly the separation in non-dimensional roughness length.



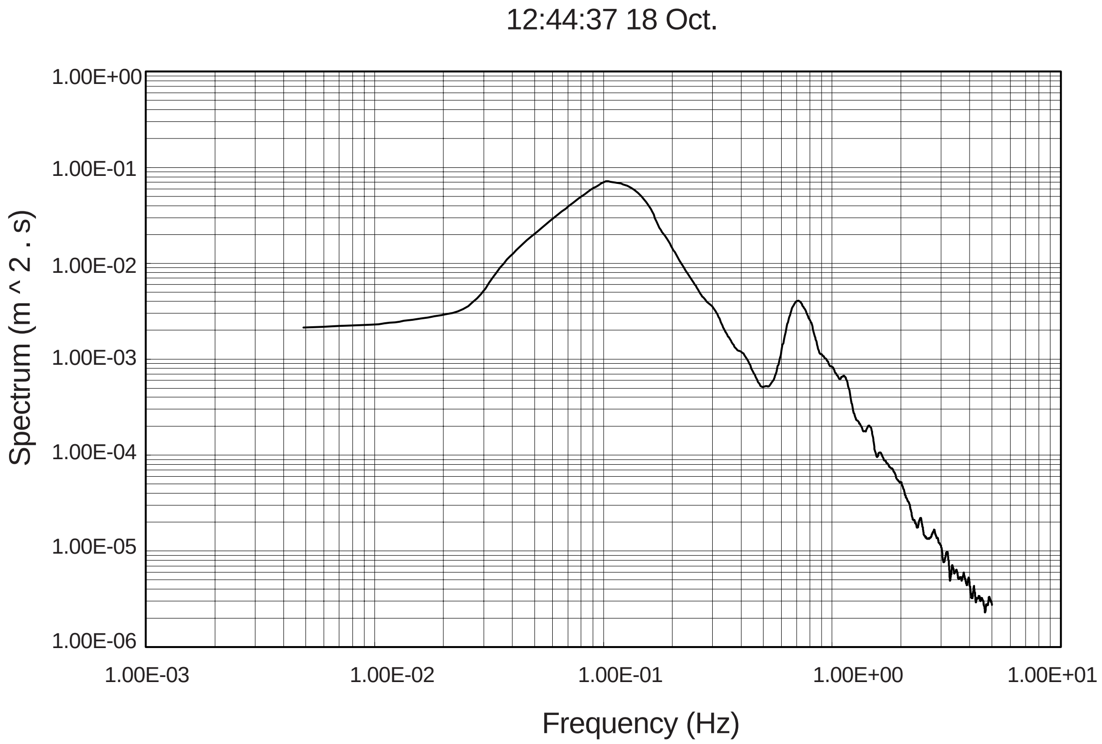


Figure 4. Directional wave spectra and frequency spectra with two peaks (swell and wind wave) in spectrum (case 1). 0° is North, 90° is West, 180° is South, and 270° is East.

For case 1 data, the similar relationship between non-dimensional roughness length and inverse of wave age can be obtained close to the Toba and Koga (1986) formula. On the other hand, the relationship will be near to the Charnock (1955) formula for case 2. Case 1 and case 2 are divided about 10^{-1} line in non-dimensional roughness length.

The relationship between friction velocity u_* and wind speed above 10 m sea surface height U_{10} (figure 7) with case 1 and case 2 are plotted. This shows low values in case 2 as shown in figure 6. Since the friction velocity is calculated directly by the eddy correlation method, it may reflect the sea surface state of multi-directional component waves.

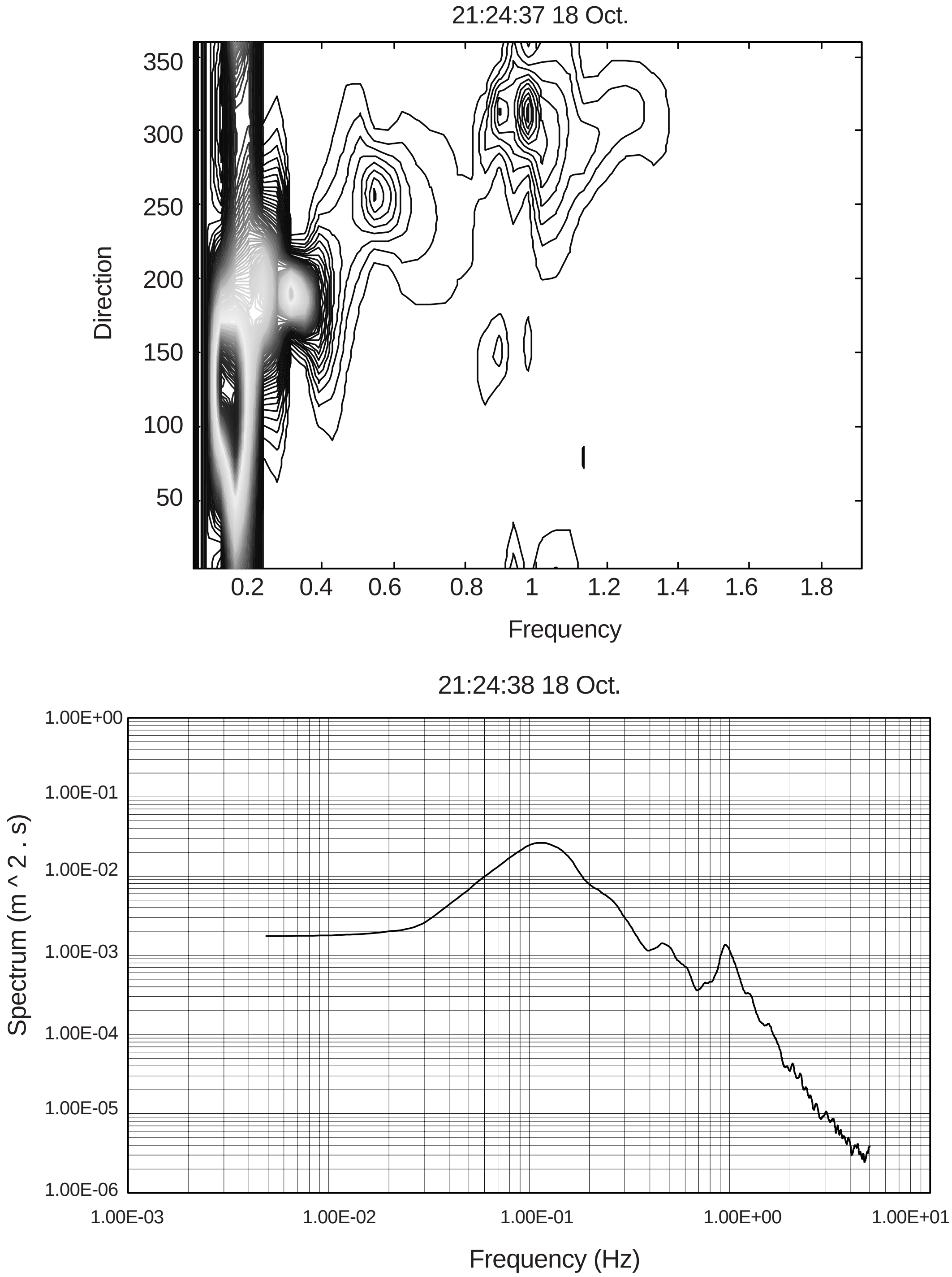


Figure 5. Directional wave spectra and frequency spectra with more than 2 peaks in spectrum (case 2). 0° is North, 90° is West, 180° is South, and 270° is East.

Also the non-dimensional roughness length shows low values estimated based on the logarithmic wind profile.

Consequently, though wave age can be solved by using the frequency of the spectral peak of the wind wave as identified by Toba's (1972) 3/2-power law, the non-dimensional roughness length may or may not be processed using case 1 data and case 2 data methods. Therefore we think that it is difficult to express the influence of waves by a single parameter of wave age.

5. Summary

In the present study, we analyzed wind and wave data observed for a period of 7 days at a coastal tower of the Hiratsuka Experiment Station to investigate wave dependence of the sea surface

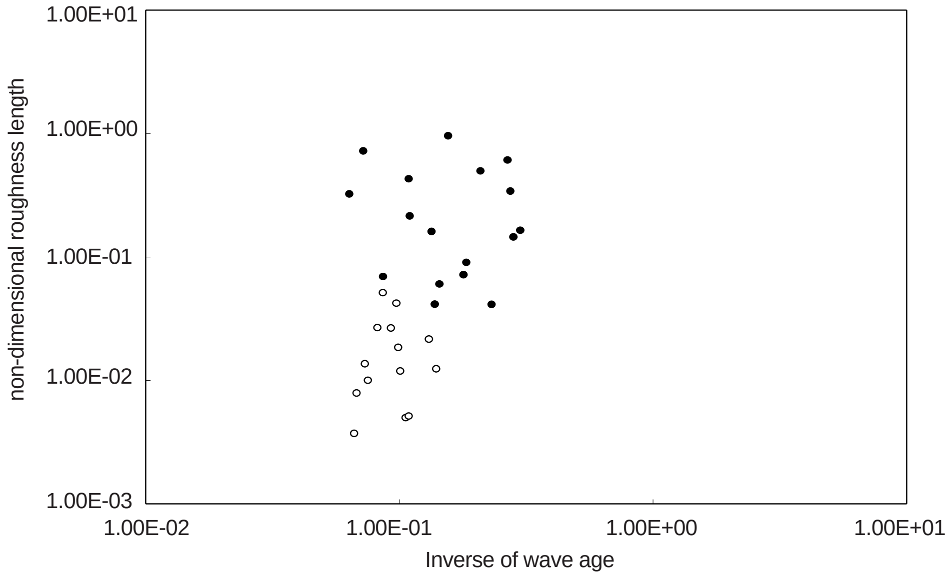


Figure 6. Relation between non-dimensional roughness length and wave age.
●: Case 1 (only two peaks in spectrum (swell and wind wave)).
○: Case 2 (more than two peaks in spectrum).

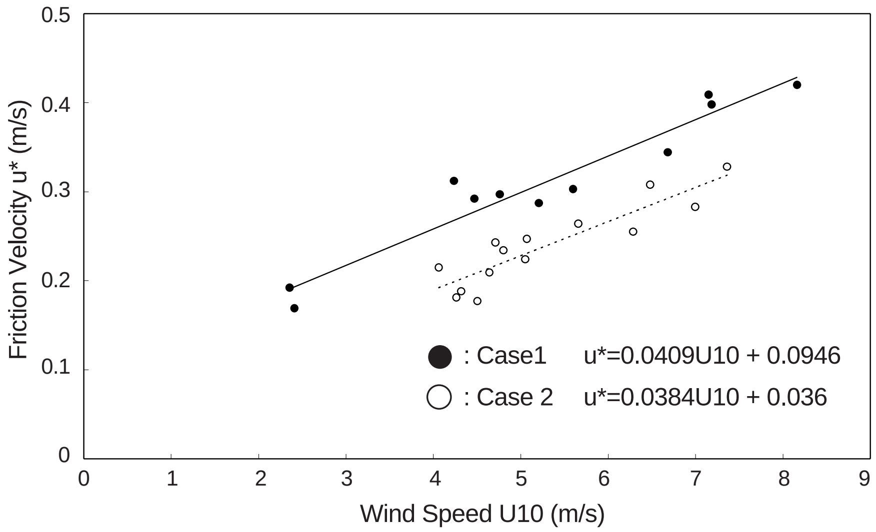


Figure 7. Relation between friction velocity u_* and wind speed above 10 m at sea surface U_{10}.
●: Case 1 (only two peaks in spectrum (swell and wind wave)).
○: Case 2 (more than two peaks in spectrum).

wind stress and directional wave spectra. The main purpose of this study is to find the cause of the very large scatter of non-dimensional roughness length in the open oceans.

The eddy correlation method was utilized to derive the wind stress from wind velocity components measured by a sonic anemometer installed on the tower. The roughness length was calculated based on the logarithmic law of the wind profile with a stratification correction. The angular frequency of wind-wave spectral peak was derived by the FFT algorithm from the time series of the sea surface displacement measured by a wave gauge. The EMEP method (Hashimoto 1997) was used to estimate the wave directional spectra from the time series of the sea surface displacement measured by four wave gauges. The data which satisfied criteria for near-neutral stratification was processed in this study. The local equilibrium between wind and waves and the aerodynamically rough flow must be satisfied according to Toba *et al* (1990).

The directional spectra observed at the Hiratsuka tower can be classified into two groups. Case 1 is with swell and wind waves clearly, and case 2 contains a lot of component waves.

These data are plotted on the diagram showing the relationship between the non-dimensional roughness length gz_0/u_*^2 and inverse of wave age $\sigma_P u_*/g$. Based on our data analysis, the large variation of non-dimensional roughness may be due to multi-directional component waves (case 2). It is found that for case 1 the frequency of the wind wave can be used to describe the wave age but for case 2 this process may not be useful.

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