Response of an anisotropic liquid-saturated porous medium due to two dimensional sources

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Eigenvalue approach, following Laplace and Fourier transforms, has been employed to find the general solution to the field equations in an anisotropic liquid-saturated porous medium, in the transformed domain. The results of isotropic liquid-saturated porous medium can be derived as a special case. A numerical inversion technique has been applied to get the solutions in the physical domain. To illustrate the utility of the approach, an application of infinite space with impulsive force at the origin has been considered. The results in the form of displacement and stress components have been obtained and discussed graphically for a particular model.

1. Introduction

The problems related to liquid-saturated porous medium are attracting more and more attention because of its significance in many practical investigations. An important aspect of these problems is the response of the media to arbitrary inputs. Therefore, many researchers have studied this aspect of the problem, e.g., Paul (1976a, b); Burridge and Vargas (1979); Pal (1983); Philippacopolous (1988); Sharma (1992) and Kumar *et al* (2000).

There are reasonable grounds for the assumption that anisotropy may exist in the continents. Anisotropy has significant effects on the characteristics of wave propagation. Therefore, many investigators have studied the problems related to anisotropic liquid-saturated porous media. Sharma and Gogna (1991) have studied the wave propagation in anisotropic liquid-saturated porous solids. The wave propagation theory in anisotropic periodically layered fluid-saturated porous media has been discussed by Sun *et al* (1993). Propagation of plane waves in transversely isotropic fluid-saturated porous medium was studied by Wang and Zhang (1997).

Here, in this investigation, we employ the eigenvalue approach following Laplace and Fourier transforms to study the response of an anisotropic liquid-saturated porous medium due to two dimensional sources.

2. Basic equations

In the absence of body forces, following Biot (1956a), the equations of motion for the liquid-saturated porous medium in the absence of dissipation are given as

$$\sigma_{ij,j} = \frac{\partial^2}{\partial t^2}\left(\rho_{11}u_i + \rho_{12}U_i\right), \tag{1}$$

$$\sigma_{,i} = \frac{\partial^2}{\partial t^2}\left(\rho_{12}u_i + \rho_{22}U_i\right), \quad (i = x, y, z), \tag{2}$$

where σ_{ij} are the stress components in the solid, $\sigma = -\beta f_p$ is the stress in the fluid (f_p is the pressure in the fluid and β is the porosity); u_i, $U_i (i = x, y, z)$ are the components of the displacement vectors in the solid and liquid parts, respectively of the porous medium; ρ_{11}, ρ_{12} and ρ_{22} are the dynamical

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coefficients and are related to the mass densities of the solid ρ_s and fluid ρ_f as

$$\rho_{11}+\rho_{12}=(1-\beta)\rho_s, \qquad \rho_{12}+\rho_{22}=\beta\rho_f, \tag{3}$$

so that the mass density of the bulk material is

$$\rho=\rho_{11}+2\rho_{12}+\rho_{22}=\rho_s+\beta(\rho_f-\rho_s). \tag{4}$$

The stress strain relations for the transversely isotropic liquid-saturated porous solid with symmetry about the z-axis are given by Biot (1956b) as

$$\begin{aligned}
\sigma_{xx} &= 2Ne_{xx}+A\left(e_{xx}+e_{yy}\right)+Fe_{zz}+M\varepsilon,\\
\sigma_{yy} &= 2Ne_{yy}+A\left(e_{xx}+e_{yy}\right)+Fe_{zz}+M\varepsilon,\\
\sigma_{zz} &= Ce_{zz}+F\left(e_{xx}+e_{yy}\right)+Q\varepsilon,\\
\sigma_{yz} &= Le_{yz},\\
\sigma_{xz} &= Le_{xz},\\
\sigma_{xy} &= Ne_{xy},\\
\sigma &= M\left(e_{xx}+e_{yy}\right)+Qe_{zz}+R\varepsilon,
\end{aligned} \tag{5}$$

where

$$e_{ij}=\begin{cases}\dfrac{\partial u_i}{\partial x_j}, & i=j,\\[2mm] \dfrac{\partial u_i}{\partial x_j}+\dfrac{\partial u_j}{\partial x_i}, & i\neq j,\end{cases}$$

$$\varepsilon=\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}, \tag{6}$$

A, N, F, M, C, Q, L and R are the elastic constants for transversely isotropic liquid-saturated porous solid. These elastic constants can be reduced to that of isotropic liquid-saturated porous solid through the relations

$$F=A, \qquad M=Q, \qquad L=N, \qquad C=A+2N. \tag{7}$$

3. Formulation of the problem

A transversely isotropic liquid-saturated porous medium with symmetry about z-axis, taken vertically downwards, has been considered. General solutions have been obtained for a two dimensional plain strain problem (the field components along y-direction are zero and others are independent of y-coordinate). In application, a concentrated force is applied at some interior point of the infinite medium, taken as origin, along vertical direction, i.e., force is applied at the origin of cartesian coordinate system along the z-axis.

4. Solution of the problem

For the two dimensional plain strain problem, we have

$$\vec{u}=(u,0,w), \qquad \vec{U}=(U,0,W), \tag{8}$$

where u, w and U, W represent the displacement components in the solid and liquid parts of the porous aggregate, respectively.

The field equations for the transversely isotropic liquid-saturated porous solid, in the component form, are obtained by substituting (5) and (6) in equations (1) and (2). For the two-dimensional plain strain problem, these field equations reduce to

$$\begin{aligned}
&(A+2N)\frac{\partial^2 u}{\partial x^2}+L\frac{\partial^2 u}{\partial z^2}+(F+L)\frac{\partial^2 w}{\partial x\partial z}\\
&\qquad +M\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 W}{\partial x\partial z}\right)=\rho_{11}\frac{\partial^2 u}{\partial t^2}+\rho_{12}\frac{\partial^2 U}{\partial t^2},\\
&L\frac{\partial^2 w}{\partial x^2}+C\frac{\partial^2 w}{\partial z^2}+(F+L)\frac{\partial^2 u}{\partial x\partial z}\\
&\qquad +Q\left(\frac{\partial^2 U}{\partial x\partial z}+\frac{\partial^2 W}{\partial z^2}\right)=\rho_{11}\frac{\partial^2 w}{\partial t^2}+\rho_{12}\frac{\partial^2 W}{\partial t^2},\\
&M\frac{\partial^2 u}{\partial x^2}+Q\frac{\partial^2 w}{\partial x\partial z}+R\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 W}{\partial x\partial z}\right)\\
&\qquad =\rho_{12}\frac{\partial^2 u}{\partial t^2}+\rho_{22}\frac{\partial^2 U}{\partial t^2},\\
&M\frac{\partial^2 u}{\partial x\partial z}+Q\frac{\partial^2 w}{\partial z^2}+R\left(\frac{\partial^2 U}{\partial x\partial z}+\frac{\partial^2 W}{\partial z^2}\right)\\
&\qquad =\rho_{12}\frac{\partial^2 w}{\partial t^2}+\rho_{22}\frac{\partial^2 W}{\partial t^2}.
\end{aligned} \tag{9}$$

The following non-dimensional quantities

$$\begin{aligned}
&u'=\frac{u}{h}, \qquad w'=\frac{w}{h}, \qquad U'=\frac{U}{h}, \qquad W'=\frac{W}{h},\\
&x'=\frac{x}{h}, \qquad z'=\frac{z}{h}, \qquad t'=\omega t,\\
&\sigma'_{zz}=\frac{\sigma_{zz}}{C}, \qquad \sigma'_{zx}=\frac{\sigma_{zx}}{C}, \qquad \sigma'=\frac{\sigma}{C},
\end{aligned} \tag{10}$$

where $\omega = \frac{1}{h}\sqrt{\frac{C}{\rho}}$ and 'h' has the dimensions of length, have been defined to reduce the field equations (9) in the non-dimensional variable form. Further, the coefficients in the non-dimensional form are defined as

$$a^2 = \frac{Q}{C}, \quad b^2 = \frac{R}{C}, \quad c^2 = \frac{A+2N}{C},$$
$$d^2 = \frac{L}{C}, \quad e^2 = \frac{F+L}{C}, \quad f^2 = \frac{M}{C},$$
$$R_{11} = \frac{\rho_{11}}{\rho}, \quad R_{12} = \frac{\rho_{12}}{\rho}, \quad R_{22} = \frac{\rho_{22}}{\rho}, \tag{11}$$

where

$$\rho = \rho_{11} + 2\rho_{12} + \rho_{22}. \tag{12}$$

The Laplace transformation with respect to 't' is defined as

$$\left\{\overline{u}(x,z,p), \overline{w}(x,z,p), \overline{U}(x,z,p), \overline{W}(x,z,p)\right\} = \int_0^\infty \{u(x,z,t), w(x,z,t), U(x,z,t), W(x,z,t)\} e^{-pt} dt, \tag{13}$$

and the Fourier transformation with respect to 'x' as

$$\left\{\hat{u}(q,z,p), \hat{w}(q,z,p), \hat{U}(q,z,p), \hat{W}(q,z,p)\right\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left\{\overline{u}(x,z,p), \overline{w}(x,z,p), \overline{U}(x,z,p), \overline{W}(x,z,p)\right\} e^{iqx} dx. \tag{14}$$

Using the non-dimensional quantities given by (10) and (11) in the field equations (9), and applying the Laplace and then the Fourier transformation on the resulting field equations after suppressing the dashes, finally, we obtain a system of equations, which is written in the matrix differential equation form as

$$A_1 \ddot{V} + B_1 \dot{V} + C_1 = 0, \tag{15}$$

where dot represents the differentiation with respect to z,

$$V = \left[\hat{u}, \hat{w}, \hat{U}, \hat{W}\right]^T,$$

$$A_1 = \begin{bmatrix} d^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & a^2 \\ 0 & 0 & 0 & 0 \\ 0 & a^2 & 0 & b^2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & -iqe^2 & 0 & -iqf^2 \\ -iqe^2 & 0 & -iqa^2 & 0 \\ 0 & -iqa^2 & 0 & -iqb^2 \\ -iqf^2 & 0 & -iqb^2 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -\left(q^2c^2 + H_{11}\right) & 0 & -\left(q^2f^2 + H_{12}\right) & 0 \\ 0 & -\left(q^2d^2 + H_{11}\right) & 0 & -H_{12} \\ -\left(q^2f^2 + H_{12}\right) & 0 & -\left(q^2b^2 + H_{22}\right) & 0 \\ 0 & -H_{12} & 0 & -H_{22} \end{bmatrix}, \tag{16}$$

and

$$H_{11} = R_{11}p^2, \; H_{12} = R_{12}p^2, \; H_{22} = R_{22}p^2. \tag{17}$$

Following the eigenvalue approach (Kumar *et al* 2000), to solve the system of equations (15), we assume

$$V(q,z,p) = X(q,p)e^{mz}, \tag{18}$$

which lead to the characteristic equation

$$\det(m^2 A_1 + mB_1 + C_1) = 0, \tag{19}$$

i.e.,

$$T_0 m^6 + T_1 m^4 + T_2 m^2 + T_3 = 0, \tag{20}$$

where

$$T_0 = -d^2 H_{22} X,$$
$$T_1 = T_{11} + T_{12} q^2,$$
$$T_2 = T_{21} + T_{22} q^2 + T_{23} q^4,$$
$$T_3 = T_{31} + T_{32} q^2 + T_{33} q^4 + T_{34} q^6,$$
$$T_{11} = ZX + d^2 H_{22} Y,$$
$$T_{12} = \left(XX_1' + b^2 X_1 + f^2 X_2\right) H_{22},$$
$$T_{21} = -(Y + d^2 H_{22})Z,$$

$$T_{22} = -\left[(X+X')H_{11}H_{22} + 2(a^2e^2 - f^2 - a^2c^2 + e^2f^2)H_{12}H_{22} + 2b^2d^2Z + \left(X_1 + c^2\right)H_{22}^2 - 2\left(b^2e^2 - a^2f^2\right)H_{12}^2\right],$$

$$T_{23} = -\left[\left(b^2X_1 + c^2X\right) + d^2X' + f^2X_2\right]H_{22},$$

$$T_{31} = Z^2,$$

$$T_{32} = \left(Y' + d^2H_{22}\right)Z,$$

$$T_{33} = X'Z + d^2H_{22}Y',$$

$$T_{34} = d^2H_{22}X',$$

and

$$X = b^2 - a^4, \qquad X' = b^2c^2 - f^4,$$
$$X_1 = d^4 - e^4, \qquad X_1' = c^2 + d^2,$$
$$X_2 = 2a^2e^2 - f^2, \qquad Z = H_{11}H_{22} - H_{12}^2,$$
$$Y = b^2H_{11} + H_{22} - 2H_{12}a^2,$$
$$Y' = b^2H_{11} + c^2H_{22} - 2H_{12}f^2. \qquad (21)$$

Equation (20) is cubic in m^2 and can be solved by Cardon's method. In order to use Cardon's method, the equation (20) is reduced to

$$(z^*)^3 + 3H^*z^* + G^* = 0, \qquad (22)$$

where

$$z^* = m^2 + \frac{T_1}{3T_0},$$

$$H^* = \frac{1}{3}\left(T_2' - \frac{1}{3}T_1'^2\right),$$

$$G^* = T_3' - \frac{1}{3}T_1'T_2' + \frac{2}{27}T_1'^3,$$

$$T_1' = \frac{T_1}{T_0}, \quad T_2' = \frac{T_2}{T_0}, \quad T_3' = \frac{T_3}{T_0}. \qquad (23)$$

The roots of the equation (22) are

$$z^* = r + s, \qquad (24)$$

where r and s are given by

$$r^3 = \frac{-G^* + \sqrt{(G^*)^2 + 4(H^*)^3}}{2},$$

$$s^3 = \frac{-G^* - \sqrt{(G^*)^2 + 4(H^*)^3}}{2}$$

satisfying

$$rs = -H^*.$$

Hence, roots of the equation (20) are given by

$$m_n^2 = z_n^* - \frac{T_1}{3T_0}, \quad n = 1, 2, 3. \qquad (25)$$

The eigenvectors $X(q,p)$ associated to different eigenvalues $\pm m_1, \pm m_2$ and $\pm m_3$ are obtained as

$$X_i^T = \left[-k_i', 1_i', -m_i', n_i'\right], \text{ for } m = m_i,$$

$$X_{i+3}^T = \left[-k_i', 1_i', -m_i', n_i'\right], \text{ for } m = -m_i,\ i = 1, 2, 3, \qquad (26)$$

where

$$n_i' = n_{i1}n_{i2} - n_{i3}n_{i4},$$

$$m_i' = n_{i1}m_{i1} - n_{i3}m_{i2},$$

$$l_i' = \frac{1}{n_{i1}}\left[n_{i4}m_i' - m_{i2}n_i'\right],$$

$$k_i' = \frac{1}{k_{i4}}\left[k_{i1}l_i' - k_{i2}m_i' + k_{i3}n_i'\right],$$

$$k_{i1} = a^2m_i^2 - H_{12},$$

$$k_{i2} = -iqm_ib^2,$$

$$k_{i3} = b^2m_i^2 - H_{22},$$

$$k_{i4} = -iqm_if^2,$$

$$m_{i1} = im_i\{f^2q(f^2q^2 + H_{12}) + b^2q(d^2m_i^2 - c^2q^2 - H_{11})\},$$

$$m_{i2} = f^2(a^2m_i^2 - H_{12}) - e^2(b^2m_i^2 - H_{22}),$$

$$n_{i1} = f^2(m_i^2 - d^2q^2 - H_{11}) - e^2(a^2m_i^2 - H_{12}),$$

$$n_{i2} = (f^2q^2 + H_{12})^2 + (b^2q^2 + H_{22})(d^2m_i^2 - c^2q^2 - H_{11}),$$

$$n_{i3} = im_i\{e^2q(f^2q^2 + H_{12}) + a^2q(d^2m_i^2 - c^2q^2 - H_{11})\},$$

$$n_{i4} = -iq(a^2f^2m_i - b^2e^2m_i), \quad i = 1, 2, 3. \qquad (27)$$

Thus, the general solution of plain strain problem in transversely isotropic liquid-saturated porous medium in the transformed domain can be written as

$$V(q,z,p) = \sum_{i=1}^{3}\left\{B_iX_i(q,p)e^{m_iz} + B_{i+3}X_{i+3}(q,p)e^{-m_iz}\right\}, \qquad (28)$$

where $B_i (i = 1, 2, 3, 4, 5, 6)$ are arbitrary constants to be determined from the boundary conditions.

Substituting (7) in equation (19), we get the characteristic equation of the corresponding problem in isotropic liquid-saturated porous medium. Hence, use of expressions (7) in other relevant expressions provide us the general solution of the plain strain problem in isotropic poroelasticity.

5. Application

An impulsive force of magnitude $F = -F_0\delta(x)\delta(t)$ acting at the origin of the cartesian coordinate system along z-axis in an infinite transversely isotropic liquid-saturated porous medium is considered. The appropriate boundary conditions in the present case at $z = 0$ are given as

$$\begin{aligned} u(x, 0^+, t) - u(x, 0^-, t) &= 0, \\ w(x, 0^+, t) - w(x, 0^-, t) &= 0, \\ W(x, 0^+, t) - W(x, 0^-, t) &= 0, \\ \sigma_{zz}(x, 0^+, t) - \sigma_{zz}(x, 0^-, t) &= -F_0\delta(x)\delta(t), \\ \sigma_{xz}(x, 0^+, t) - \sigma_{xz}(x, 0^-, t) &= 0, \\ \sigma(x, 0^+, t) - \sigma(x, 0^-, t) &= F_0\delta(x)\delta(t). \end{aligned} \tag{29}$$

Using (10), (13) and (14), these boundary conditions (29) can be written in the non-dimensional transformed form (after suppressing the dashes) as

$$\begin{aligned} \hat{u}(q, 0^+, p) - \hat{u}(q, 0^-, p) &= 0, \\ \hat{w}(q, 0^+, p) - \hat{w}(q, 0^-, p) &= 0, \\ \hat{W}(q, 0^+, p) - \hat{W}(q, 0^-, p) &= 0, \\ \hat{\sigma}_{zz}(q, 0^+, p) - \hat{\sigma}_{zz}(q, 0^-, p) &= -\frac{F_0}{2\pi}, \\ \hat{\sigma}_{xz}(q, 0^+, p) - \hat{\sigma}_{xz}(q, 0^-, p) &= 0, \\ \hat{\sigma}(q, 0^+, p) - \hat{\sigma}(q, 0^-, p) &= \frac{F_0}{2\pi}. \end{aligned} \tag{30}$$

Now, from (5) and (6), after reducing them to non-dimensional transformed form by using (10), (11), (13) and (14), alongwith (28), the displacement and stress components in the transformed domain (after suppressing the dashes) are given as

for z ≥ 0

$$\begin{aligned} \hat{u} &= -(B_4k_1'e^{-m_1z} + B_5k_2'e^{-m_2z} + B_6k_3'e^{-m_3z}), \\ \hat{w} &= (B_4l_1'e^{-m_1z} + B_5l_2'e^{-m_2z} + B_6l_3'e^{-m_3z}), \\ \hat{U} &= -(B_4m_1'e^{-m_1z} + B_5m_2'e^{-m_2z} + B_6m_3'e^{-m_3z}), \\ \hat{W} &= (B_4n_1'e^{-m_1z} + B_5n_2'e^{-m_2z} + B_6n_3'e^{-m_3z}), \\ \hat{\sigma}_{zz} &= B_4Q_1e^{-m_1z} + B_5Q_2e^{-m_2z} + B_6Q_3e^{-m_3z}, \\ \hat{\sigma}_{xz} &= B_4G_1e^{-m_1z} + B_5G_2e^{-m_2z} + B_6G_3e^{-m_3z}, \\ \hat{\sigma} &= B_4J_1e^{-m_1z} + B_5J_2e^{-m_2z} + B_6J_3e^{-m_3z}, \end{aligned} \tag{31}$$

for z ≤ 0

$$\begin{aligned} \hat{u} &= -(B_1k_1'e^{m_1z} + B_2k_2'e^{m_2z} + B_3k_3'e^{m_3z}), \\ \hat{w} &= (B_1l_1'e^{m_1z} + B_2l_2'e^{m_2z} + B_3l_3'e^{m_3z}), \\ \hat{U} &= -(B_1m_1'e^{m_1z} + B_2m_2'e^{m_2z} + B_3m_3'e^{m_3z}), \\ \hat{W} &= (B_1n_1'e^{m_1z} + B_2n_2'e^{m_2z} + B_3n_3'e^{m_3z}), \\ \hat{\sigma}_{zz} &= B_1P_1e^{m_1z} + B_2P_2e^{m_2z} + B_3P_3e^{m_3z}, \\ \hat{\sigma}_{xz} &= B_1R_1e^{m_1z} + B_2R_2e^{m_2z} + B_3R_3e^{m_3z}, \\ \hat{\sigma} &= B_1H_1e^{m_1z} + B_2H_2e^{m_2z} + B_3H_3e^{m_3z}, \end{aligned} \tag{32}$$

where

$$\begin{aligned} P_i &= l_i'm_i + iq(e^2 - d^2)k_i' + a^2n_i'm_i + iqa^2m_i', \\ Q_i &= -l_i'm_i + iq(e^2 - d^2)k_i' - a^2n_i'm_i + iqa^2m_i', \\ R_i &= -(k_i'm_i + iql_i')d^2, \qquad G_i = (k_i'm_i - iql_i')d^2, \\ H_i &= a^2l_i'm_i + iqf^2k_i' + b^2n_i'm_i + iqb^2m_i', \\ J_i &= -a^2l_i'm_i + iqf^2k_i' - b^2n_i'm_i + iqb^2m_i', \\ &(i = 1, 2, 3). \end{aligned} \tag{33}$$

Making use of the transformed displacements and stresses given by (31) and (32) in the transformed boundary conditions (30), we obtain a system of six equations in six unknowns B_1, B_2, B_3, B_4, B_5 and B_6, which on solving gives

$$\begin{aligned} B_4 = B_1 &= -s_1\frac{(t_3 + r_3)s_2 - (t_2 + r_2)s_3}{\Delta}\frac{F_0}{2\pi}, \\ B_5 = B_2 &= s_1\frac{(t_3 + r_3)s_1 - (t_1 + r_1)s_3}{\Delta}\frac{F_0}{2\pi}, \\ B_6 = B_3 &= -s_1\frac{(t_2 + r_2)s_1 - (t_1 + r_1)s_2}{\Delta}\frac{F_0}{2\pi}, \end{aligned} \tag{34}$$

where

$$\begin{aligned} \Delta &= (t_2s_1 - t_1s_2)(r_3s_1 - r_1s_3) \\ &\quad - (r_2s_1 - r_1s_2)(t_3s_1 - t_1s_3), \\ r_i &= Q_i - P_i, \quad s_i = G_i - R_i, \quad t_i = J_i - H_i. \end{aligned} \tag{35}$$

Thus, the expressions (31) and (32) give the displacement and stress components with the help of

(34)–(35) in the transformed domain in an infinite transversely isotropic fluid-saturated porous medium due to an impulsive force acting at the origin along z-axis. To get the displacement and stress components in the physical domain, we require to invert the Laplace and Fourier transforms in the transformed domain expressions. Therefore, we make use of a numerical inversion technique to get the displacements and stresses in the physical domain.

6. Numerical results and discussion

Using the numerical inversion technique described in Kumar *et al* (2000), the displacement and stress components in the solid and liquid parts of the porous aggregate have been computed separately, for a particular model. In the model considered, we choose the following values of elastic constants:

$$A = 4.43 \times 10^{10}\,\text{dyne/cm}^2, \quad F = fA,$$

$$Q = 0.743 \times 10^{10}\,\text{dyne/cm}^2, \quad M = mQ,$$

$$N = 2.765 \times 10^{10}\,\text{dyne/cm}^2, \quad L = lN,$$

$$R = 0.326 \times 10^{10}\,\text{dyne/cm}^2, \quad C = F + 2L.$$

So that for $f = m = l = 1.0$, these constants become the elastic constants for isotropic kerosene saturated sandstone (Fatt 1959).

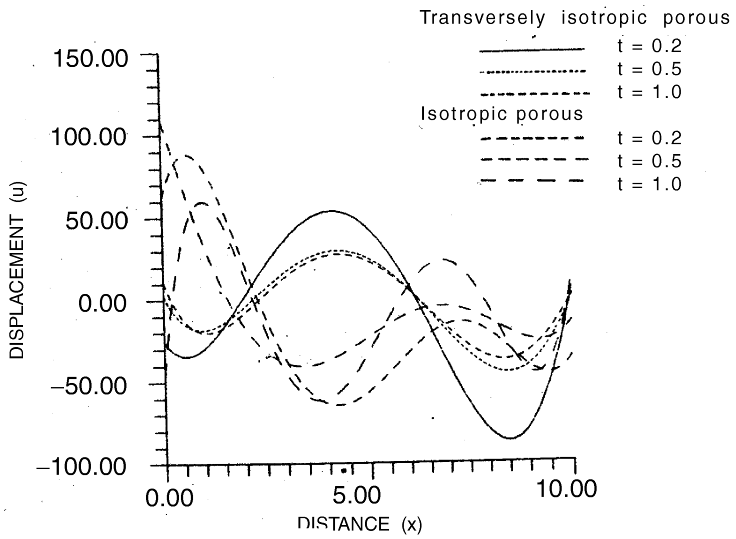

Figure 1. Displacement distribution due to impulsive source.

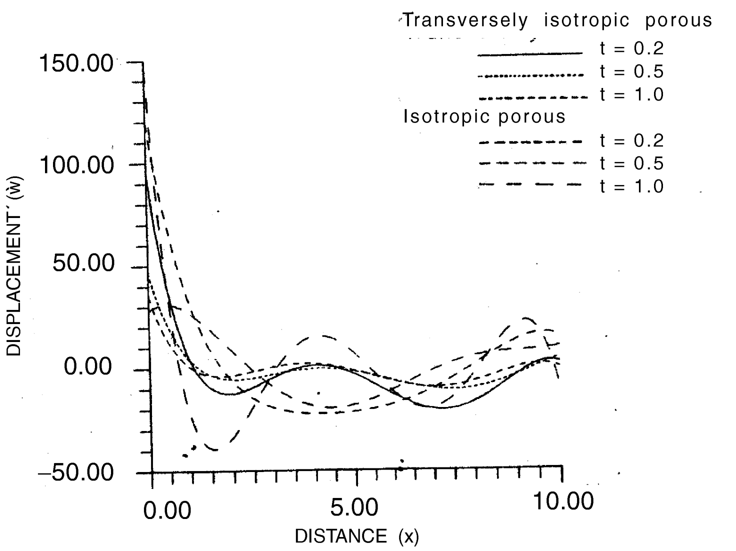

Figure 2. Displacement distribution due to impulsive source.

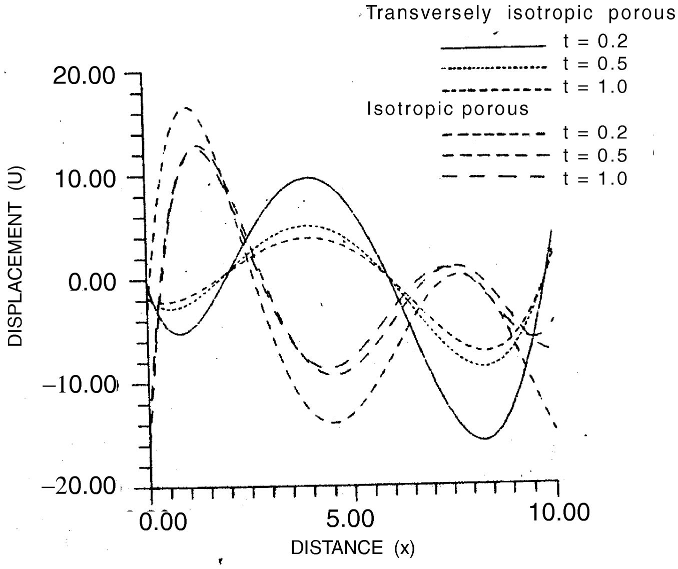

Figure 3. Displacement distribution due to impulsive source.

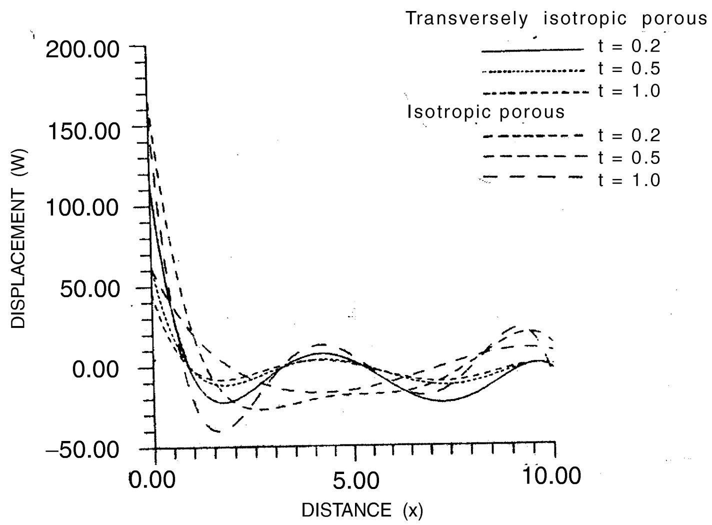

Figure 4. Displacement distribution due to impulsive source.

The dynamical coefficients are taken as

$$\rho_{11} = 1.926\,\text{gm/cm}^3, \qquad \rho_{12} = -0.00214\,\text{gm/cm}^3,$$

$$\rho_{22} = 0.21534\,\text{gm/cm}^3,$$

and porosity

$$\beta = 0.26.$$

Also, we have considered

$$\frac{F_0}{C} = 1.0.$$

Here, for numerical calculations, the anisotropy is considered due to

$$f = 1.5, \quad m = 1.2, \quad l = 2.0,$$

but, we can take some other set of values, also.

The responses represented by displacement and stress components are calculated on the plane $z = 1$, against distance 'x' for the following values of time $t = 0.2, 0.5$ and 1.0, taking the medium to be transversely isotropic liquid-saturated porous solid and isotropic liquid-saturated porous solid. The distribution curves are shown in figures 1–7. Figures 1–4 represent the distribution of displacement components due to impulsive source and figures 5–7 represent the distribution of stress components due to impulsive source.

It is observed that the maximum (absolute) displacements are occurring at the minimum time, i.e., the impact of an impulsive force is maximum, as soon as the force is applied. From figures 1–7, it is clear that if we fix the point of observation,

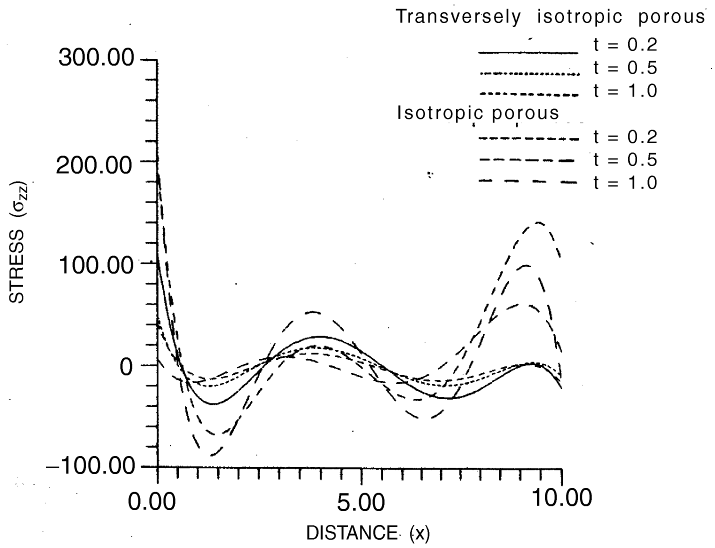

Figure 5. Stress distribution due to impulsive source.

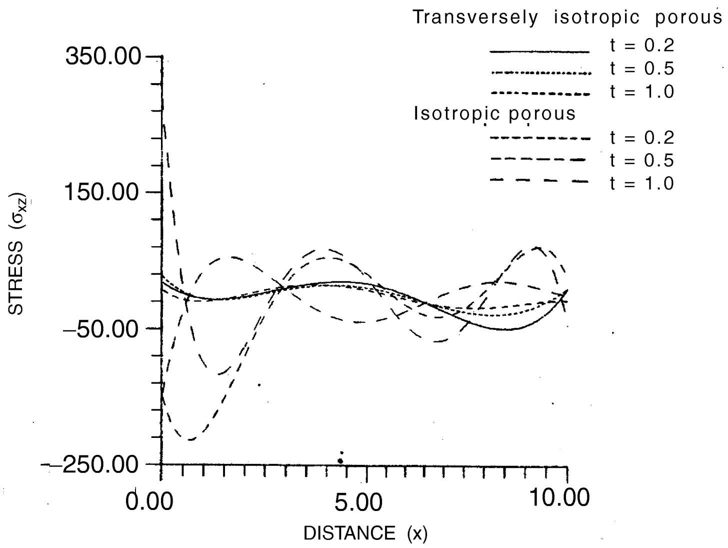

Figure 6. Stress distribution due to impulsive source.

i.e., the value of distance 'x', the displacement and stress components increase or decrease with the passage of time. With further increase in time, it reveals that the displacements and stresses, following an oscillatory pattern about zero value, become zero, ultimately. But, this oscillatory pattern is not uniform for all the distances 'x'. This is due to the fact that the medium considered is liquid-saturated porous, which is a two phase medium involving an elastic solid matrix with pores saturated with fluid. The disturbances travelling through these different constituents of the medium suffer sudden changes, resulting in an inconsistent/non-uniform pattern. Otherwise, when it passes through either solid or liquid, it shows a uniform pattern.

It is observed that the magnitude of displacement components increase or decrease along an oscillatory path with the increase in distance 'x'. With further increase in distance, it reveals that all the displacements become zero, ultimately. This means that all the displacements vanish as the point of observation is far away from the point of application of the force, which justifies the radiation conditions.

It is seen that the trend of curves in the two constituents of the liquid-saturated porous medium, solid and liquid, are almost the same with difference in the magnitudes of their values. This means that the disturbances in the two constituents are in the same phase.

Figures 1–7 also show the effect of anisotropy on the disturbances produced due to impulsive force. It is observed that anisotropy affects mainly the magnitudes of displacements and stresses. It is

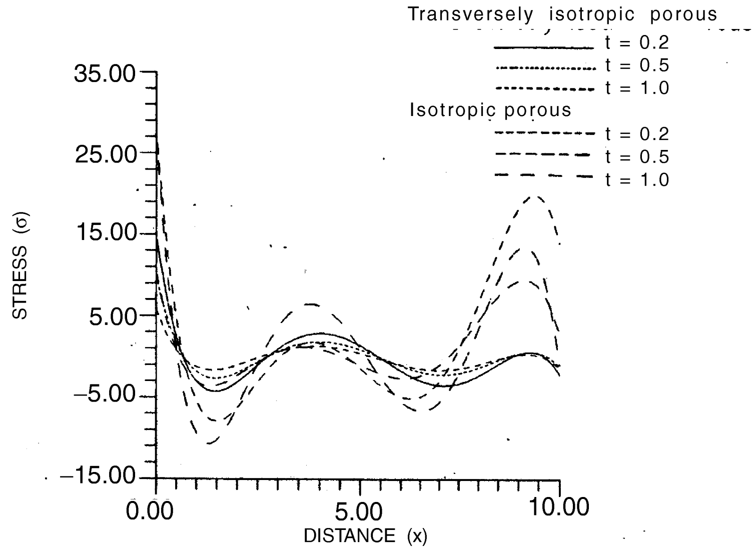

Figure 7. Stress distribution due to impulsive source.

clear from the figures that the values of displacement components are distributed in larger range for transversely isotropic liquid-saturated porous medium in comparison to isotropic liquid-saturated porous medium, i.e., anisotropic effect is to increase the magnitudes of displacements. Whereas, the values of stress components increase or decrease in different situations due to anisotropy of the medium. Thus, it reveals that the anisotropic effect is mainly quantitative in nature.

It is concluded that the trend of curves exhibits the properties of liquid-saturated porous medium and satisfies the requisite conditions of the problem. The disturbances produced in the liquid-saturated porous medium are affected by anisotropy of the medium, which in turn will effect the various phenomena, e.g., wave propagation.

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