Strain variation in fold-and-thrust belts: Implications for construction of retrodeformable models

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Deformation in fold-and-thrust belts such as the Himalayas can be represented by the displacement vector field. The strain component of the displacement vector field across the fold-and-thrust belt varies from near zero in external thrust sheets to a significant part of the field in internal thrust sheets. In addition, strain exhibits three-dimensional patterns in parts of internal sheets, near fault zones, and in the overturned limbs of fault-related folds due to superposition of penetrative-strain producing deformation events. This paper examines superposition of these strain producing deformation events in some detail and points out situations in fold-and-thrust belts wherein the finite strain becomes three-dimensional. This suggests that the plane-strain assumption used in the construction of retrodeformable models of fold-and-thrust belt evolution breaks down in these situations and the models lose their validity. Therefore, current techniques used for construction of retrodeformable models in fold-and-thrust belts need to be modified and three-dimensional models which include three-dimensional finite and incremental strain data need to be constructed for an accurate study of the evolution of geometry and kinematics in fold-and-thrust belts.

1. Introduction

Fold-and-thrust belts (FTBs) are an important component of most major convergent mountain belts such as the Himalayas (figure 1). FTBs consist of several thrust sheets that are transported along associated thrust faults. Consequently, a thrust sheet (and its associated thrust fault) can be regarded as the basic structural unit in an FTB. Detailed study of individual thrust sheets from an FTB can be compiled to obtain the overall deformation characteristics of the FTB. This approach is very efficient for the study of “continuous” FTBs like the Himalayas where a continuous hinterland to foreland succession of thrust sheets is seen with little post-emplacement disturbance.

Using the above approach, deformation in a thrust sheet can be represented by the total displacement vector field which can be further broken down into several components (figure 2). The translation component of the displacement vector field is obtained by estimating the net slip vector for motion along the thrust fault associated with the thrust sheet, the rotational component is estimated from the change in dips of the planes due to tilting or folding and the strain component is estimated from computations of strain axial ratios from finite or incremental strain markers found in rocks. Working out the geometry of structures in the field and the laboratory qualitatively and quantitatively involves estimation of the translational and rotational components of the total displacement field while strain and strain variation estimates contribute towards understanding the kinematics of structures.

The geometric and kinematic evolution of FTBs have been studied using retrodeformable models constructed from surface and subsurface (well and seismic) observational data. The main objective of a retrodeformable model is step-wise removal or the inversion of the total displacement vector field representing the deformation. Ideally, the retrodeformation should be carried out mathematically as an inverse problem but given the paucity of suitable geologic data a semi-quantitative approach is the best available

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Figure 1. A schematic (not to scale) cartoon of the Himalayan fold-and-thrust belt to illustrate some of the terminology used in the paper.

Figure 2. Main components of the total displacement vector field for the case of emplacement of a thrust sheet (Mitra 1994). The pure strain component is usually not removed in the construction of retrodeformable cross-sections.

conserved between the deformed and the undeformed sections (Woodward et al 1989; Mitra and Namson 1989). Therefore, assumed deformation within a thrust sheet is limited to flexural-slip for one-dimensional bodies (linear or curvilinear) and plane strain for two- or three-dimensional bodies. Removal of the strain component of the total displacement vector field is, however, a problem that is still being addressed.

1.1 Inclusion of strain in retrodeformable models of FTB evolution

A complete description of the total displacement vector field (translation, rotation and strain) from each thrust sheet in an FTB (along with path indicators which document incremental strains at all spatial and temporal changes in the deformation path) would be ideal. This information could then be used to construct the deformation path and to step-wise invert the deformation from each thrust sheet in the FTB to its initial undeformed configuration (i.e. the sedimentary prism or wedge) from which the FTB evolved. However, in the real world, information is always incomplete and although translation and rotation components are more easily quantifiable and obtainable, strain data are usually much harder to obtain. Various scenarios arise during the construction of retrodeformable models in the real world based on the nature of the total displacement vector and the available data. These scenarios are discussed in Table 1.
Mittra (1994) investigated construction of retrodeformable models without including strain data and pointed out that not including strain in the construction of retrodeformable models of FTB evolution leads to large errors in the model because the strain component is not negligible in most natural thrust sheets. For further accuracy the three-dimensional volume construction and restoration must also be included (Von Winterfield and Oncken 1995). As strain in general is heterogeneous, ideally, a complete mapping of three-dimensional strain in a thrust body is required. Due to the impracticability of this demand, techniques of extrapolation or prediction of strain are needed to approximate natural strain distributions (Von Winterfield and Oncken 1995). Mukul (1998) realised this and used spatial statistics to extrapolate and predict strain from measured strain data in an internal thrust sheet and then study strain variation qualitatively and quantitatively (Mukul and Mitra 1998).

Attempts to include strain in retrodeformable models of FTB evolution has raised several important questions. Can the plane strain assumption still be made in the construction of retrodeformable models for all parts of FTBs? If not, why? Is it possible to construct accurate retrodeformable models for FTBs by making the plane strain assumption? If no, then how and why does this assumption fail? The objective of this paper is to examine these questions in the light of recent data (McNaught and Mitra 1996; Yonkee 1996; Patel and Jain 1997; Mukul and Mitra 1998) and discuss the new implications based on interpretation of these data.

1.2 Spatial variation of strain in FTBs

There are two main orders of spatial variation in FTBs. First, because the basal decollement (detachment or the sole fault) climbs progressively upsection from the hinterland to the foreland of an FTB (figure 1) (Dahlstrom 1970; Boyer and Elliott 1982), rocks in the external thrust sheets of an FTB deform at lower P-T conditions than rocks in the internal sheets (especially rocks close to the thrust). Consequently, rocks in the external sheets are likely to deform in the brittle to brittle-ductile regimes and exhibit low strain/penetrative deformation than rocks in the internal thrust sheets which are likely to deform in brittle-ductile to ductile regimes and exhibit higher strain/penetrative deformation. This is corroborated by the fact that penetrative cleavage is more commonly observed in internal sheets. Therefore, a first-order decrease in strain would be observed in FTBs from the hinterland (internal) sheets to the foreland (external) sheets. Second, strain variation in thrust sheets is also controlled by the original sedimentary basin taper of the FTB (Boyer 1995; Mitra 1997). Low strain values are observed in thrust sheets from FTBs that evolved from a sedimentary prism with a high initial wedge taper because the prism (or the part of the sedimentary prism to which a thrust sheet belongs) does not have to undergo internal shortening to build critical taper (Boyer 1995). Conversely, high pure strain values indicate that internal shortening has to occur for the sedimentary prism to build critical taper for motion and that the initial wedge taper was low (Boyer 1995). Therefore, the overall finite strain variation patterns in FTBs is mainly due to a combination of the above two factors although compaction or local strain patterns related to folding in the thrust sheets may further complicate them (Coward and Kim 1981).

2. Modelling strain results from fold-and-thrust belts

In order to understand and model the finite strain patterns from FTBs we must work out the strain-producing deformation events in FTB settings. For mathematical simplicity we will consider the Cartesian co-ordinate system (XYZ) parallel to the principal axes and consider the strain matrices in their diagonal form (figure 3). The first such event is most likely to be compaction due to overburden resulting in the development of initial shape fabric in sedimentary rocks. Most of the vertical shortening during compaction is taken up in closing the pore spaces (accompanied by loss of fluids) present in the sediment column during deposition of the sediments (figure 4(a)). Due to lateral constraint of the surrounding material, there is no horizontal extension, just vertical shortening accompanied by volume loss. Consequently, compaction results in a uniaxial strain with only one non-zero principal extension i.e. a shortening (e3 < 0) normal to bedding (S0) and essentially amounts to decrease in volume (Sanderson 1976). Elongations are considered positive and shortening negative here. The second strain-producing deformation event that occurs in a deforming sedimentary prism (which evolves into an FTB) is layer parallel shortening (LPS). LPS strain ellipsoids are oriented such that the e1e3 plane is parallel to the transport plane (XZ), the e3 (maximum shortening) axis of the ellipsoid is parallel to the transport direction (X) (e.g. Mitra 1994). The e2 (intermediate elongation) is the pole to the transport or the e1e3 plane and parallel to Y. As e1, e3 >> e2 the LPS deformation is plane strain (figure 4(b)). The third strain-producing deformation event in FTBs is top-to-the-f oreland fault-parallel shear (γ) associated with thrust faults usually parallel to the fault propagation direction (X) and in the plane normal to the fault (XZ) (figure 4(c)). Finally, overburden thickness results in a load stress which causes flattening particularly along fault zones which usually have lower strengths than the surrounding, undeformed, rocks (figure 4(d)). However, the load stress generated due to overburden is likely to be lithostatic
<table>
<thead>
<tr>
<th>Displacement vector field for FTB deformation</th>
<th>Available data</th>
<th>Part of FTB likely to exhibit stated characteristics</th>
<th>Plane strain condition validity</th>
<th>Restoration procedure needed for the best possible retrodeformable model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Translation and rotation</td>
<td>(i) Finite state information only</td>
<td>External thrust sheets or sheets from part of FTB with high initial wedge taper</td>
<td>High</td>
<td>One step retrodeformation in 2D or 3D will work.</td>
</tr>
<tr>
<td></td>
<td>(ii) Finite and intermediate state data (e.g. rotated or folded synorogenic conglomerates, overprinting multiple slickenlines on fault plane)</td>
<td></td>
<td>High</td>
<td>Multi-step retrodeformation in 2D or 3D will be required.</td>
</tr>
<tr>
<td>2. Translation, rotation and 2D pure strain</td>
<td>(i) Translation and rotation (finite ± incremental state data)</td>
<td>External, transitional and internal (?) thrust sheets or sheets from part of FTB with low to moderate wedge taper</td>
<td>High</td>
<td>One- or multi-step retrodeformation in 2D depending on absence or presence of incremental state data. Accuracy of restored section low since strain data absent (Mitra 1994).</td>
</tr>
<tr>
<td></td>
<td>(ii) Translation and rotation (finite ± incremental state data) and finite strain data</td>
<td></td>
<td>High</td>
<td>One- or multi-step retrodeformation in 2D. Finite strain data must be included in the restoration process. For multi-step retrodeformation when incremental state translation and rotation data exist a model path is required for producing observed finite strain data.</td>
</tr>
<tr>
<td></td>
<td>(iii) Translation, rotation and 2D strain (finite ± incremental strain data)</td>
<td></td>
<td>High</td>
<td>Multi-step retrodeformation which includes finite and incremental stage data. Forward model of the evolution of thrust belt using computer simulation and constrained by real data can be attempted. Accuracy of restored section high.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Restoration possible with existing techniques (e.g. Woodward et al 1989; McNaught and Mitra 1996).</td>
</tr>
</tbody>
</table>
Table 1. (Continued.)

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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>3. Translation, rotation and 3D pure strain</td>
<td>(i) Translation and rotation (finite ± incremental state data)</td>
<td>Transitional and internal thrust sheets or sheets that have evolved from part of FTB with low wedge taper</td>
<td>Low</td>
<td>3D retrodeformation needs to be carried out. Techniques for this are still being developed (e.g. Winterfeld and Ondcken 1995). In the absence of strain data accuracy of restored section will be low (Mitra 1994).</td>
</tr>
<tr>
<td></td>
<td>(ii) Translation, rotation and 3D strain (finite ± incremental)</td>
<td></td>
<td>Low</td>
<td>3D retrodeformation techniques need to be developed which include strain data in the restoration process. Define model paths and use computer simulation techniques to arrive at a possible deformation path constrained by finite and incremental state data. Accuracy of this restoration technique will be high because it will have to take into account most amount of data and will be the most generalized technique.</td>
</tr>
</tbody>
</table>
Figure 3. The co-ordinate reference frame chosen to set up the idealised mathematical model of penetrative deformation from thrust sheets in parts of fold-and-thrust belts. The X axis is parallel to the transport direction and the Y axis is coplanar with X, but perpendicular to the overall transport direction in the fold-and-thrust belt. The Z co-ordinate axis is perpendicular to the transport direction. The $XZ$ plane is the transport plane. The pure strain component of the total displacement vector, when present, may be the result of compaction, layer parallel shortening (LPS), fault-parallel shear and overburden load stress or various combinations of these penetrative-strain producing deformation mechanisms.

outside the fault zone. This inference is based on strain data from several fault related deformation zones (e.g. Yonkee 1996; Patel and Jain 1997; Mukul and Mitra 1998) which all indicate flattening in these zones. This deformation is characterised by $e_3 < 0$ and $e_1, e_2 > 0$; $e_1 = e_2$ in the case of uniaxial flattening and $e_1 \neq e_2$ for general flattening.

The next question that needs to be addressed is that given the different components of strain (figure 3) and the different scenarios that arise in the construction of retrodeformable models using strain data (table 1), what are the different circumstances which lead to the violation of the plane strain assumption? The answer probably lies in the manner in which the different strain components are superposed sequentially during deformation in an FTB. We investigate this further in the following sections. The superposition of strains is achieved by sequential premultiplication of the matrices representing strain transformations (Sander-son 1976). For example, a compactional strain ($S_1$) followed by a tectonic strain ($S_2$) gives a total strain ($S$) where:

$$S = S_2 \times S_1.$$
Figure 4. Schematic, notional, figures constructed using Bezier curves (DePaor 1996) to depict approximate strain ellipse shapes generated in the transport (XZ) and XY planes during major penetrative-strain producing deformation events in fold-and-thrust belts. (a). Compaction: Plane strain deformation with negligible strain along X and Y directions; (b). Layer parallel shortening: Plane strain deformation with negligible strain along Y direction; (c). Fault-parallel shearing: Plane strain deformation with negligible strain along Y direction; (d). Overburden load: Uniaxial or general flattening deformation in a weak fault zone near the middle of the figure.

Fault plane) rotational and translational data can easily be removed from a two- or three-dimensional retrodeformable model using one- or multi-step retrodeformation (e.g. Royse et al 1975). The plane strain assumption is valid in these situations and the retrodeformation can be carried out using standard techniques (Geiser 1988; Mitra and Namson 1989; Woodward et al 1989).

Some external thrust sheets, however, do exhibit strain (e.g. 10–20%, Darby-Prospect and Absaroka
thrust sheets in the Sevier FTB, western USA (Mitra 1994)). This occurs if the external thrust sheets are part of the FTB with low initial wedge taper (such as the shelf sequences which form a thin, weak wedge with low initial taper of about $1^\circ-2^\circ$) and a layer parallel shortening (LPS) is required to build critical taper for thrusting (Boyer 1995; Mitra 1997). Therefore, assuming that compaction did occur and the LPS strain was superposed coaxially on compaction, the finite strain matrix is given by:

\[
\begin{pmatrix}
1 - e_3 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 + e_1 + \Delta
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 + \Delta
\end{pmatrix}
\]

where $\Delta$ is the dilatation (change in volume/initial volume).

The finite strain matrix is still plane strain and no deformation occurs perpendicular to the transport plane (figure 5(a)). Therefore, the plane strain assumption is still valid for construction of retrodeformable models using standard procedures (table 1) except that unless the finite strain component of the displacement vector field is removed accuracy of the restored model will suffer (Mitra 1994). If we consider non-coaxial superposition of strains then we have to introduce a rotation $\phi$ (which is not equal to 0 or $\pi/2$) about some axis which would make the principal strain axes of compaction coincident with the superposed LPS strain axes (Sanderson 1976). Assuming further that the misorientation between the principal strain axes of the two penetrative-strain producing events occurs only in the $XZ$ plane and the intermediate strain axes ($e_2 = 1$) of both the deformations is parallel

XZ (TRANSPORT) PLANE

Figure 5. Schematic figures constructed using Bezier curves (DePaor 1996) to depict approximate strain ellipse shapes generated in the transport ($XZ$) plane during superposition of strain produced as a result of layer parallel shortening (LPS) on compaction in fold-and-thrust belts. (a) Coaxial superposition of a planar LPS on compaction in the transport ($XZ$) plane (equation 1) results in plane strain deformation with negligible finite strain along $Y$ direction; (b) Non-coaxial superposition of a planar LPS on compaction in the transport ($XZ$) plane (equation 2) results in plane strain deformation with negligible finite strain along $Y$ direction. The misorientation between the principal strain axes of the two penetrative-strain producing events occurs only in the $XZ$ plane and the resultant finite strain is rotational. Compaction in the model was followed by counter-clockwise rotation, superposition of LPS strain and then clockwise rotation to obtain the finite configuration shown in the figure.
to the $Y$ direction, the finite strain matrix is given by:

$$
\begin{pmatrix}
\cos \phi & 0 & \sin \phi \\
0 & 1 & 0 \\
-\sin \phi & 0 & \cos \phi
\end{pmatrix}
\begin{pmatrix}
1 - e_3 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 + e_3 + e_3
\end{pmatrix}
$$

\times

$$
\begin{pmatrix}
\cos \phi & 0 & -\sin \phi \\
0 & 1 & 0 \\
\sin \phi & 0 & \cos \phi
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 + \Delta
\end{pmatrix}
$$

$$
\begin{pmatrix}
1 + e_1 \sin^2 \phi - e_3 \cos^2 \phi & 0 & (e_1 + e_3)(1 + \Delta) \\
0 & 1 + e_3 & 0 \\
(e_1 + e_3) \sin \phi \cos \phi & 0 & \sin^2 \phi)(1 + \Delta)
\end{pmatrix}
$$

(2)

where $\phi$ = the required rotation ($\neq 0$ or $\pi/2$).

The plane strain condition is not violated in the above non-coaxial superposition of compaction and LPS. The finite strain is still confined to the $XZ$ (transport plane) (figure 5(b)). However, it is no longer irrotational. In the general case of non-coaxial superposition, however, all the three LPS strain axes are inclined to the compaction strain axes which are parallel to the co-ordinate axes and the finite strain matrix is given by:

$$
\begin{pmatrix}
l_1 & n_1 & l_3 \\
m_1 & m_1 & m_3 \\
n_1 & n_2 & n_3
\end{pmatrix}
\begin{pmatrix}
[1 - e_3] = A \\
0 & 1 & 0 \\
0 & 0 & [1 + e_1] = B
\end{pmatrix}
$$

\times

$$
\begin{pmatrix}
l_2 & n_1 \\
m_2 & m_2 \\
n_3 & m_3
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & [1 + \Delta] = C
\end{pmatrix}
$$

$$
\begin{pmatrix}
A^2 l_1^2 + l_2^2 + Bl_3^2 & Am_1 l_1 + l_2 m_2 + Bl_3 m_3 \\
Al_1 m_1 + l_2 m_2 + Bl_3 m_3 & Am_1^2 + m_2^2 + Bm_3^2 \\
Al_1 n_1 + l_2 n_2 + Bl_3 n_3 & Am_1 n_1 + m_2 n_2 + Bm_3 n_3
\end{pmatrix}
$$

$$
C[Al_1 n_1 + l_2 n_2 + Bl_3 n_3] \\
C[Am_1 n_1 + m_2 n_2 + Bm_3 n_3] \\
C[Am_1^2 + m_2^2 + Bm_3^2]
$$

(3)

where $l$, $m$, and $n$ are direction cosines of the axis of rotation along which the rotation is introduced.

The finite strain matrix in the most generalised non-coaxial superposition of LPS strain on compaction is, therefore, three-dimensional and does not satisfy the plane strain condition and in this case the retrodeformable model constructed will be erroneous (table 1). However, most of the geological evidence from external sheets (e.g. Mitra 1994; Wojtal 1986) suggests that the deformation in the external sheets is plane strain. Therefore, retrodeformable models constructed from external thrust sheets using standard methodology would mostly be largely valid (table 1).

### 2.2 Transitional thrust sheets

Sheets which are transitional between the external and internal parts of FTBs (e.g., the Meade thrust sheet in the Idaho-Wyoming salient of the Sevier FTB in the western United States) exhibit brittle-ductile to ductile, penetrative deformation and strain. It is important to note that the definition of a transitional sheet is as dependent on lithology as on location. Thus, a thrust sheet carrying carbonate units is likely to exhibit more ductile deformation than a sheet carrying quartzites at the same location.

The Meade thrust cut through the ductile core and the forelimb of a synform-antiform fold pair which was formed as a fault propagation fold (McNaught and Mitra 1993). The antiform of the fold pair was preserved in the hanging wall of the Meade thrust and the synform was preserved in the footwall of the thrust. This footwall synform was then rotated due to the formation of a younger (Sheep Creek) antiform (McNaught and Mitra 1993). Finite strain ellipses calculated using deformed ooids and deformed Pentacrinites ossicles from the Meade sheet were explained (McNaught and Mitra 1996) as a result of inhomogeneous fault-parallel shear (at least two, non-coaxial orientations were estimated) modifying original LPS ellipses on bedding. Finite-strain ellipsoids from the Meade sheet vary in shape from triaxial prolate to plane strain. This and the orientations of calcite veins suggested a component of non-plane strain in the Meade sheet during early LPS (McNaught and Mitra 1996). Although the modification of LPS ellipsoids by fault-parallel shear is established, it is difficult to determine the mechanism by which the strain becomes non-planar from the limited three-dimensional data given. McNaught and Mitra (1996) suggest that there was some thickening and out-of-plane strain during early increment of LPS. If so, the subsequent superposition of strains on the non-planar LPS strain would result in a three-dimensional finite strain distribution and the finite strain matrix would be given as:

$$
\begin{pmatrix}
1 & 0 & \gamma \\
0 & 1 & 0 \\
0 & 0 & 1 + e_2
\end{pmatrix}
\begin{pmatrix}
1 - e_3 & 0 & 0 \\
0 & 1 - e_2 & 0 \\
0 & 0 & 1 + e_1
\end{pmatrix}
$$

$$
\begin{pmatrix}
1 - e_3 & 0 & \gamma(1 + e_1) \\
0 & 1 - e_2 & 0 \\
0 & 0 & (1 + e_1)
\end{pmatrix}
$$

(4)

The finite strain matrix is three-dimensional even if a simple homogeneous fault-parallel shear is superimposed on the LPS strain matrix and the plane strain condition is violated.

Alternatively, non-coaxial, inhomogeneous fault-parallel shear strain increments superposed on an earlier planar LPS strain would also result in non-planar strain ellipsoids. In this case, superimposing two non-coaxial fault parallel shear increments on a
planar LPS the finite strain matrix is given as:

\[
\begin{bmatrix}
  l_1 & m_1 & n_1 \\
  l_2 & m_2 & n_2 \\
  l_3 & m_3 & n_3 \\
\end{bmatrix}
\begin{bmatrix}
  1 & 0 & \gamma_2 \\
  0 & 1 & 0 \\
  0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
  1 - e_3 & 0 & 0 \\
  0 & 1 - e_3 & 0 \\
  0 & 0 & 1 + e_1 \\
\end{bmatrix}
\begin{bmatrix}
  l_1 & m_1 & n_1 \\
  l_2 & m_2 & n_2 \\
  l_3 & m_3 & n_3 \\
\end{bmatrix}
\]

(5)

where

\[
A_{11} = (1 - e_3)\left[l_1^2 + l_2^2 + l_3^2 + \gamma_2 l_3\right],
A_{12} = m_1l_1 + m_2l_2 + m_3l_3 + \gamma_2 m_3l_1,
A_{13} = (1 + e_1)\left[\gamma_1(l_1^2 + l_2^2 + l_3^2) + \gamma_2(l_1n_3 + \gamma_1n_1l_3) + \gamma_3n_1l_3\right],
A_{21} = (1 - e_3)[m_1l_1 + m_2l_2 + m_3l_3 + \gamma_2 m_3l_1],
A_{22} = n_1l_1 + n_2l_2 + n_3l_3 + \gamma_2 n_3l_1,
A_{23} = (1 + e_1)\left[\gamma_1(m_1l_1 + m_2l_2 + m_3l_3) + \gamma_2(m_1n_3 + \gamma_1n_1l_3) + \gamma_3m_3n_3\right],
A_{31} = (1 - e_3)[n_1l_1 + n_2l_2 + n_3l_3 + \gamma_2 n_3l_1],
A_{32} = m_1n_1 + m_2n_2 + m_3n_3 + \gamma_2 m_3n_1,
A_{33} = (1 + e_1)\left[\gamma_1(m_1n_1 + n_2l_2 + n_3l_3) + \gamma_2(n_1n_3 + \gamma_1n_1l_3) + (n_1^2 + n_2^2 + n_3^2)\right].
\]

Validity of the plane strain condition in this situation depends on the magnitude of the intermediate strain axis \(e_2\) of the non-planar early LPS strain axis. If \(e_2 \approx 0\), then the plane strain condition remains valid and the finite strain matrix approximates equation (7). Additional three-dimensional strain data from the Meade thrust sheet are probably needed to understand the strain distribution in the Meade sheet better and to determine which of the situations discussed above apply. The questions which remain are the nature of the distribution of the magnitudes of the intermediate strain axis \(e_2\) of the non-planar LPS strain in the Meade sheet and whether or not the two non-coaxial increments of fault-parallel strain superposed on the LPS strain are really coplanar as assumed.

The validity of the plane strain condition in transitional thrust sheets like the Meade depends on the nature of the penetrative deformation in the sheet. Typically, a single penetrative-deformation event (early LPS) is recorded in such thrust sheets and the plane strain condition is not violated (e.g. in the upright limb of the hanging wall antiform of the Meade thrust). However, if another non-coaxial penetrative-deformation event is superposed on the LPS event (such as fault parallel shear near fault zones or in overturned forelimbs of fault propagation folds), chances are that the plane strain condition might be violated. Nevertheless, such superposition of strains might be local and retrodeformable models for the overall thrust sheet would still work assuming plane strain (table 1).

2.3 Internal thrust sheets

Internal thrust sheets in a fold-and-thrust belt are present in the interior part of the belt (figure 1). They mostly exhibit ductile deformation and penetrative cleavage which may be related to more than one deformation event. They also transport significant overburden (sometimes over 15 km) which may affect the finite strain in the thrust sheet. Multiple penetrative-strain producing deformation events are more probable in internal thrust sheets because internal sheets get deformed each time a younger thrust is emplaced towards the foreland (figure 1). It is likely that some of these events are penetrative-strain producing and will contribute towards the configuration of the finite strain ellipsoid. Also, chances of non-coaxial superposition of strains is far more likely in internal thrust sheets because the principal strain axes of later deformation events are likely to be misoriented with respect to the principal strain axes of the then finite strain ellipsoid.

2.3.1 Strain data from the Zanskar sheet in NW-Himalayas

Patel and Jain (1997) looked at the finite-strain data from the Panjal Traps, which consist of Permian
volcanogenic assemblages within the lower parts of the Tethyan Sedimentary Zone in the Zanskar region of the NW-Himalayas of Jammu and Kashmir state in India. Panjal Traps lie very close to the Zanskar shear zone which is a thrust fault transporting rocks from the Tethyan Sedimentary Zone over the Higher Himalayan Crystallines (Patel and Jain 1997). Deformed amygdules in Panjal Traps were used as strain markers by Patel and Jain (1997) to study the finite strain variation. Finite strain ellipsoid shapes in the Panjal Traps range from triaxial oblate to near plane strain; the magnitude of finite strain and the flattening increases close to the shear zone. Patel and Jain (1997) attributed the finite strain ellipsoid pattern to a non-coaxial superposition of two penetrative-strain producing deformation events; a plane, (extension along Y and shortening along Z), constant-volume, flattening strain (from increase of overburden thickness and resulting load stress due to previous thrusting within the Tethyan Sedimentary zone) was superposed on fault parallel shear (in the XZ plane) associated with the thrusting in the Zanskar shear zone. The finite strain ellipsoids exhibited a three-dimensional pattern and the plane strain condition was violated for almost the entire unit. The finite strain matrix in this case would be given by:

$$
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 + e_1 & 0 \\
0 & 0 & 1 - e_3
\end{pmatrix}
\begin{pmatrix}
1 + e_2 & 0 & 0 \\
0 & 1 + e_1 & 0 \\
0 & 0 & 1 - e_3
\end{pmatrix}
= \begin{pmatrix}
1 & 0 & \gamma(1 + e_2) \\
0 & 1 + e_1 & 0 \\
0 & 0 & (1 - e_3)
\end{pmatrix}
$$

For maximum elongation ($e_1$) = intermediate ($e_2$), the finite strain matrix would be:

$$
\begin{pmatrix}
1 + e_1 & 0 & 0 \\
0 & 1 + e_1 & 0 \\
0 & 0 & 1 - e_3
\end{pmatrix}
\begin{pmatrix}
1 + e_2 & 0 & 0 \\
0 & 1 + e_1 & 0 \\
0 & 0 & 1 - e_3
\end{pmatrix}
= \begin{pmatrix}
1 + e_1 & \gamma(1 + e_2) & 0 \\
0 & 1 + e_1 & 0 \\
0 & 0 & (1 - e_3)
\end{pmatrix}
$$

The finite strain matrix is three-dimensional in all the three cases which suggests that overburden load stress causes the plane strain condition to break down whenever it becomes a significant component of the total finite strain matrix. This would in turn mean that the plane strain condition is likely to be violated in internal thrust sheets and near faults zones where the overburden thickness is high.

2.3.2 Finite strain data from the Willard sheet, Idaho-Wyoming salient of the Sevier FTB, USA

Finite-strain patterns in the hanging wall of the Willard thrust (Yonkee 1996), which is an internal thrust sheet in the Idaho-Wyoming-Northern Utah salient of the Sevier FTB in the western United States, indicate that the LPS fabric is modified by fault-parallel shear and general flattening close to the Willard thrust resulting in flattened finite strain ellipsoids near the fault zone and the violation of the plane-strain condition (figure 7(a)). Assuming plane strain LPS strain, the finite strain matrix near the fault zone in the Willard sheet would then be given as:

$$
\begin{pmatrix}
1 + e_1' & 0 & 0 \\
0 & 1 + e_2' & 0 \\
0 & 0 & 1 - e_3'
\end{pmatrix}
\begin{pmatrix}
1 + e_2 & 0 & 0 \\
0 & 1 + e_1 & 0 \\
0 & 0 & 1 - e_3
\end{pmatrix}
\begin{pmatrix}
1 + e_1' & 0 & 0 \\
0 & 1 + e_1 & 0 \\
0 & 0 & 1 - e_3
\end{pmatrix}
= \begin{pmatrix}
1 + e_1' & \gamma(1 + e_2') & 0 \\
0 & 1 + e_1 & 0 \\
0 & 0 & (1 - e_3)
\end{pmatrix}
$$

where $e_i$'s are related to LPS strain and $e_i'$'s are related to the general flattening due to overburden load stress.
The finite strain matrix is three-dimensional near the fault zone. The flattening component of strain, however, would decrease away from the fault zone and the finite strain matrix in that case would be:

\[
\begin{pmatrix}
1 & 0 & \gamma \\
0 & 1 & 0 \\
0 & 0 & 1 + e_1
\end{pmatrix}
\begin{pmatrix}
1 - e_3 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 + e_1
\end{pmatrix}
= \begin{pmatrix}
(1 - e_3) & \gamma(1 + e_1) & 0 \\
0 & 1 & 0 \\
0 & 0 & (1 + e_1)
\end{pmatrix}
\]

Thus, if LPS is plane strain, the plane strain assumption will not be violated away from the fault zone in the Willard thrust sheet (figure 7(b)). However, if the LPS strain is non-planar or if the fault parallel shear is superposed in a non-coaxial manner, the finite strain matrix will be non-planar as seen in equation (5).

Further strain work in the Willard thrust sheet is underway (Yonkee 1996) and should be indicative of the nature of the strain in the rest of the Willard thrust sheet.

**2.3.3 Finite strain data from the Sheeprock thrust sheet, Provo salient, Sevier FTB, western USA**

The Sheeprock thrust cut through the ductile core and the forelimb of a synform-antiform fold pair which was formed as a fault propagation fold (Mukul and Mitra 1998). The antiform of the fold pair was preserved in the hanging wall of the Sheeprock thrust and the synform was preserved in the footwall of the thrust. The entire structure was then rotated due to the formation of a fault bend antiform related to a younger (Midas?) thrust (Mukul and Mitra 1998). The finite strain ellipsoids from the Sheeprock thrust exhibit three-dimensional variation in axial ratios as well as orientation (Mukul and Mitra 1998) and, clearly, the plane-strain condition is violated for the
finite deformation in the Sheeprock thrust sheet. The finite strain ellipsoids near the Sheeprock thrust result from superposition of general flattening due to overburden load stress (resulting from about 15 km of overburden transported by the Sheeprock thrust) and fault-parallel shear on initial layer parallel shortening (LPS) strain (Mukul and Mitra 1998). Also, the nature of superposition of the overburden load stress on the LPS and the fault-parallel shear strain varies in the Sheeprock sheet. Assuming that the strain superposition occurs coaxially and that the LPS strain is plane strain, the superposition of fault-parallel shear on LPS strain results in the finite strain matrix given in equation (14). The magnitude of $\gamma$ is highest in the overturned limb of the fault-propagation fold and near the fault zone. Consequently, the finite strain ellipsoid near the central part of the thrust sheet away from these zones is mostly plane strain and related to LPS strain (Mukul and Mitra 1998); this also justifies the planar LPS strain assumption. Close to the fault zone and in the overturned limb of the fault propagation fold, general flattening related to overburden load stress is superposed on the finite strain matrix in equation (14) resulting in a finite strain matrix given in equation (13):
parallel retrodeformable models can be constructed

\[
\begin{pmatrix}
(1 + e_1')(1 - e_3) & 0 & \gamma (1 + e_1')(1 + e_1) \\
0 & (1 + e_2') & 0 \\
0 & 0 & (1 - e_3')(1 + e_1)
\end{pmatrix}.
\]

(15)

However, the overburden flattening is superposed in
such a way that the direction of the maximum
principal elongation \( (e_1) \) direction of the finite strain
ellipsoid in equation (15) is perpendicular to the trans-
port plane near the back-end of the Sheeprock thrust
sheet and lies in the transport plane near the front-end
of the sheet. This implies that the overburden load
stress results in general flattening in the Sheeprock
sheet.

3. Conclusions

Data observed from fold-and-thrust belts discussed
above seem to suggest that the plane-strain assump-
tion breaks down whenever there are more than one
penetrative-strain producing deformation events in
the thrust sheet. Typically two situations may result
in the violation of the plane strain condition:

(i) A non-coaxial superposition of a penetrative-
strain producing event on another will result in a
three-dimensional strain ellipsoid and violation of
the plane strain condition (e.g. equations (3) and
(5)) except in cases where the two events are co-
planar, plane strain events (e.g. if an initial planar
LPS strain is superposed by strain related to
fault-parallel shear in the transport plane as
illustrated in equations (2) and (7)).

(ii) Finite strain ellipses are also three-dimensional
and the plane strain condition is also violated
when one or more of the penetrative-strain pro-
ducing events are non-planar. For example, if a
non-planar LPS strain (e.g. equation (4)) or a
uniaxial or general flattening resulting from over-
burden stress is part of the finite strain ellipsoid
(e.g. equations (8), (11), (12) and (13)).

Both these criteria are most likely to be satisfied in
the penetrative deformation of internal sheets so it is
probably safe to assume that in most natural fold-and-
thrust belts, the plane strain condition is likely to be
violated in the internal thrust sheets. However, most
retrodeformable models from fold-and-thrust belts are
constructed along the transport (XZ) plane. There-
fore, it is useful to look at finite strain magnitudes in
the transport-perpendicular \( (Y) \) direction. If these
magnitudes are of the same order as those measured in
the transport plane, transport-parallel retrodeform-
able models will not work and 3D retrodeformable
techniques have to be developed (table 1). On the
other hand, if they are negligible when compared with
strain magnitudes in the transport plane, transport-
parallel retrodeformable models can be constructed
using standard techniques (table 1). The bottom-line
is that existing techniques for construction of retro-
deformable models will work without too many
problems in the external and transitional parts of
most FTBs and are likely to fail in the internal parts.

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