An analytical source function for a coupled hybrid wave model

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An analytical form for the source function is formulated by comparing the fetch-limited approximation of the Ocean Wave Transport equation and the empirical equation for the fetch-dependent wave forecast nomograms. The source function thus generated has been utilised in the numerical model based on Toba's formulation of wave transport equation and tested for the seas around the Indian subcontinent (5°S to 25°N latitude; 45°E to 100°E longitude). The grid averaged hindcast wave heights are found to be moderately matching with the GEOSAT altimeter measured significant wave heights of the 1987–1989 period, particularly for waves higher than 1 meter.

1. Introduction

With the advancement in ocean wave measurement techniques, wave theories and computers, considerable progress in the area of Numerical Wave Modelling has taken place in the recent past (SWAMP 1985). The definition of an appropriate source function is very critical for the performance of any wave model, as it is the source function which describes all the wave processes namely, wave generation, wave dissipation and non-linear wave-wave interactions. The different terms in the source functions used in the current wave models are largely developed by making use of recent wave measurements and numerical integrations of wave transport equations. The dissipation source function in the third generation WAM (WAve Modeling) model, for example, was thus developed and tuned to reproduce the observed fetch-limited wave growth and the fully developed Pierson-Moskowitz spectrum (The WAMDI Group 1988). The present study adopts a similar approach on a simpler wave model which describes spectrally averaged wind wave energy transport and attempts to develop an analytical function for the total source term by making use of empirically developed fetch-limited wave nomograms. The model forecast significant wave heights (SWH) with this source function are compared with those measured by satellite altimeter in the Indian Ocean.

2. The wave model

The wave forecast/hindcast model in our work is based on the single parameter wind-wave transport equation, originally formulated by Toba (1978) for open ocean. The source function used in this formulation parametrically combines the effects of wind forcing, dissipative losses and non-linear wave-wave interactions which physically represents the fraction of the momentum flux from wind fields to the waves out of the total momentum transferred from the winds to the sea surface. Following Toba (1978), Kawai *et al* (1979) and Toba *et al* (1985), one can write the transport equation as

$$\frac{\partial E^{2/3}}{\partial t} + A(U_*)E^{1/3}\hat{V}_g \cdot \vec{\nabla} E^{2/3} = S \qquad (1)$$

where

E = spectrally integrated energy of wind waves,

$A(U_*) = 1.35 g^{2/3} U_*^{(-1/3)}$,

$U* = U(0.0008 + 0.000065U)^{1/2}$ = friction velocity,

U = wind speed (m/s) at 10 m height from sea surface,

g = acceleration due to gravity,

$\hat{V}_g$ = unit vector in the group velocity direction,

S = the Source Function.

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For S, a stochastic form in terms of the Complementary Error Function was suggested by Toba (1978):

$$S = P(U_*)\{1 - \mathrm{erf}\,[Q(U*)E^{1/3}]\} \quad (2)$$

where P and Q are functions of friction velocity.

In our earlier work (Mohan *et al* 1994; Sarkar *et al* 1997), a source function (equation 3) for the duration limited case, was developed and the model generated wave heights were compared with GEOSAT altimeter measured mean significant wave heights over the seas around India.

$$S = G\alpha\{1 - \tan h^{\delta}[(\beta b E^{1/3})^{\Gamma}]\} \quad (3)$$

where

$$G = 2.4 \times 10^{-4} U_*^{5/3} g^{-1/3},$$
$$b = 0.12 U_*^{-4/3} g^{2/3}.$$

α, β, Γ and δ are wind dependent coefficients.

Since in a realistic sea, both duration-limited and fetch-limited cases are encountered, the present work aims to formulate a source function based on the wave growth description for fetch-limited case.

3. Derivation of the source function

Empirical relationships between wave heights, periods, fetch and duration and their graphical representations are widely used in operational wave forecasting. With known wind speed and duration, one can read out the forecast wave height in duration limited case. Similarly, with known fetch, one can find out the wave height in the fetch-limited case from these diagrams, known also as wave nomograms (CERC 1984; WMO 1988).

The fetch-limited wave energy growth in shallow waters according to the CERC nomograms (1984) can be expressed as:

$$E = [C_1 U_A^2 T/(4g)]^2 \tan h^2 [C_2 (gx/U_A^2)^{1/2}/T] \quad (4)$$

where,

$T = \tan h[C_3(gd/U_A^2)^{3/4}]$,

$U_A = 0.71 U^{1.23}$ m/s,

d = depth,

x = distance in the direction of fetch (m),

C_{1-3} are numerical constants, viz.,

$C_1 = 0.283$,

$C_2 = 0.565 \times 10^{-2}$,

$C_3 = 0.53$.

Differentiating this with respect to fetch-length x and expressing the result in terms of E,

$$\mathrm{d}E/\mathrm{d}x = pE^{1/2}(1 - \varepsilon^2)/(\tan h^{-1}\varepsilon)$$

or

$$E^{1/3}\mathrm{d}(E^{2/3})/\mathrm{d}x = (2/3)pE^{1/2}(1 - \varepsilon^2)/(\tan h^{-1}\varepsilon) \quad (5)$$

where,

$p = C_1 C_2^2/(4T)$,

$\varepsilon = 10\sqrt{2}.E^{1/2}g/(U_A^2 T)$.

Comparison of expression (5) with equation (1) in the fetch-limited case (i.e., by putting the $\partial E^{2/3}/\partial t$ term equal to zero in equation 1), leads to the following analytical form for the source function:

$$S_1 = (2/3)A(U_*)pE^{1/2}(1 - \varepsilon^2)/(\tan h^{-1}\varepsilon). \quad (6)$$

This expression, though derived from the fetch-limited case, is being tried in the present work to be used as a general source function to describe wave growth under nonhomogeneous and time varying wind fields. According to this source function, when $\varepsilon = 1$ there is no further wave growth corresponding to the saturation wave energy of the fully developed sea.

In the case of deep ocean ($d \to \infty$), $T = 1$ and the source function reduces to:

$$S_{1d} = (2/3)A(U_*)p_d E^{1/2}(1 - \varepsilon_d^2)/(\tan h^{-1}\varepsilon_d) \quad (7)$$

where,

$p_d = C_1 C_2^2/4$,

$\varepsilon_d = 10\sqrt{2}E^{1/2}g/(U_A^2)$.

4. Model runs

The new source function S1 (equation 6) was introduced into the wave transport equation (1) in place of S and the wave model was run with analysed surface wind vectors in the area encompassing the Arabian Sea, the Bay of Bengal and a part of the Indian Ocean. This area (viz., 5° to 25°N latitude; 45° to 100°E longitude) is represented by a rectangular grid of 250 × 250 km spacings with appropriately defined coastal and deep ocean boundaries. The time step size of numerical integration is chosen to be 30 minutes. Analysed surface wind fields at each integration time step were created by linear interpolation of the twelve hourly analysis generated by the European Centre for Medium-Range Weather Forecasts (ECMWF). Beyond the southern boundary of the model area (i.e., south of 5°S latitude) wind fields are assumed to be equal to those in the neighbouring grid points north of them. At the sea-land boundaries, if the wind is sea-ward, the wave height is made equal to zero at every model time step, while if the wind is land-ward, no such restriction is applied. Thus a series of conditions on wind direction for wave growth decided by the shape of the land-sea boundary contour was introduced for each grid point adjacent to land.

Table 1. *Best fit lines and correlation coefficients for all SWH and for SWH > 1 m. x is GEOSAT wave height and y is model predicted wave height. The total number of boxes (refer text) available for the correlation computation in each year is indicated in brackets.*

Year	For all SWH		For SWH > 1 m	
	Best fit line	Correlation	Best fit line	Correlation
1987	$y = 0.814 \times -0.433$	0.60 (6854)	$y = 1.063 \times -0.169$	0.66 (1832)
1988	$y = 0.760 \times -0.361$	0.58 (6054)	$y = 1.089 \times -0.335$	0.66 (1701)
1989	$y = 0.922 \times -0.472$	0.63 (2876)	$y = 1.115 \times -0.309$	0.68 (1103)

The mechanism of swell generation by wind waves and its propagation was incorporated in the model, by making use of the angular difference between waves and winds (Joseph *et al* 1981).

5. Results and discussions

As explained above, the model was run in hindcast mode with the ECMWF analysed 12-hourly ocean surface winds over the study area for every month of the period, 1987–89. An exercise to compare the model generated wave heights with those measured by GEOSAT altimeter during its Exact Repeat Mission (Cheney *et al* 1987), for the above mentioned period, over the study area was conducted.

Since the GEOSAT data were available only in the deep ocean areas, the deep ocean limiting case of the new source function (equation 7) was used in the model. The model predicted SWH were then correlated with the coincident GEOSAT altimeter SWH measurements. For this, a box of $2.5° \times 2.5°$ size was defined around each grid point and the mean of all the altimeter measurements within the box was taken as the wave height at that grid point. Care was taken to ensure statistical significance by considering only those boxes having a number of measurements greater than 30. The average number of measurements per box was $\cong 41$. The time of pass of the sub-satellite point through a box is approximated to the nearest time step of the model run. The year wise correlations between the model predictions and altimeter measurements and the Best Fit Regression Equations for the three years 1987, 1988 and 1989 are given in table 1.

It can be seen in table 1 that the number of boxes with measurement points greater than 30 were quite large in each year. The results of this exercise was to be viewed keeping in mind that the source function derived here is expected to be optimum for fetch-limited cases. Scatter plots of the model predicted wave heights and GEOSAT altimeter measured wave heights for the same years are shown in figure 1 (a–c). The scatter of points is broader at the base of the diagrams where the wave heights are small and is narrower for wave heights greater than 1 m. This same effect can be seen in the general increase of the correlation coefficients when only wave heights greater than 1 m are considered. Poor correlations for low wave heights can be partially explained by the fact that the measurement inaccuracy for the GEOSAT altimeter itself is 0.5 m. A comparison of the correlation coefficients obtained for the model runs with the same input data sets for the old source function given by equation (3) (Mohan *et al* 1994) and for the new source function (equation 6) is shown in table 2.

The new source function gives slightly better correlation for the case of all SWH. For the case of SWH >1 m, correlation coefficients of 0.66 and above, obtained for both the source functions can be considered moderately encouraging. In terms of

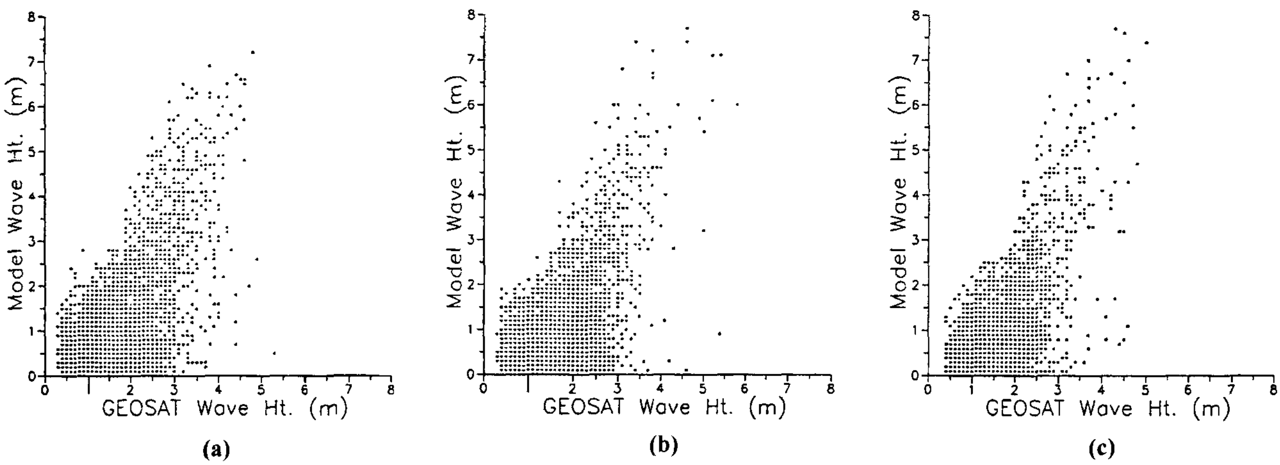

Figure 1. Scatter plot of model forecast vs. GEOSAT derived wave heights for (**a**) 1987 (**b**) 1988 and (**c**) 1989.

Table 2. *Correlation coefficients for the old and the new source functions for all SWH and for SWH > 1 m.*

Year	All SWH		SWH > 1 m	
	Old	New	Old	New
1987	0.58	0.60	0.68	0.66
1988	0.56	0.58	0.69	0.66
1989	0.61	0.63	0.68	0.68

performance, both the source functions are found to be at par.

Since the new source function is derived from empirical wave nomogram valid for shallow waters, it can be used in shallow water wave prediction experiments. But its performance needs to be tested with wave measurements at coastal areas with well charted bathymetry.

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References

CERC 1984 Shore Protection Manual, Waterways Experiment Station, US Army Corps of Engineers

Cheney R E, Douglas B C, Agreen R W, Miller L, Porter D L and Doyle N 1987 GEOSAT Altimeter Geophysical Data record: User Handbook; NOAA, Rockville

Joseph P S, Kawai S and Toba Y 1981 Prediction ocean waves based on the single parameter growth equation of wind waves (II) – Introduction of grid method; *J. Oceanogr. Soc. Japan* **37** 9–20

Kawai S, Joseph P S and Toba Y 1979 Prediction of ocean waves based on the single parameter growth equation of wind waves; *J. Oceanogr. Soc. Japan* **35** 151–167

Mohan M, Sarkar A and Kumar R 1994 A source function for wave forecast model: Duration limited case; *India J. Mar. Sci.* **23** 105–107

Sarkar Abhijit, Mohan M and Kumar R 1997 Inter-comparison of model predicted wave heights with satellite altimeter measurements in the North Indian Ocean; *Ocean Engineering* **24** 879–885

SWAMP Group 1985 *Ocean Wave Modeling* (New York: Plenum Press)

Toba Y 1978 Stochastic form of the growth of wind waves in a single parameter representation with physical implications; *J. Phys. Oceanogr.* **8** 469–507

Toba Y, Kawai S and Joseph P S 1985 *The TOHOKU Wave Model* in Ocean Wave Modelling (New York: Plenum Press) pp. 201–210

WAMDI Group 1988 The WAM Model: A third generation wave prediction model; *J. Phys. Oceanogr.* **18** 1775–1810

World Meteorological Organisation 1988 Guide to Wave Analysis and Forecasting; WMO-No. 702 (Switzerland: WMO)

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