

Flow laws in polymineralic aggregates deformed by a combination of diffusion creep and dislocation creep

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Microstructures in naturally deformed rocks in the upper crust demonstrate that creep strain in nature may be accommodated by a combination of dislocation creep, diffusion/dissolution processes and microcracking. A theoretical approach towards deriving an aggregate flow law is presented, where the strain in the constituent phases is assumed to occur by simultaneous operation of diffusive mass transfer and crystal plastic mechanisms (dislocation creep). Both uniform stress and uniform strain rate situations are considered.

1. Introduction

Steady-state flow in rocks, as reflected in the flow law has been extrapolated from laboratory data on the rheology of natural aggregates or their synthetic equivalents (Carter 1976; Carter and Tsenn 1987; Paterson 1987). The majority of published data pertains to flow in monomineralic aggregates deforming under a dislocation creep regime (Carter *et al* 1964; Ave'Lallemant and Carter 1970; Carter and Ave'Lallement 1970; Chopra and Paterson 1981, 1984; Shea and Kronenburg 1992). Some experimental data on dislocation creep behaviour of polymineralic rocks like granulite, peridotite and aplite are also available (Tullis and Yund 1977; Dell'Angello and Tullis 1986; Wilks and Carter 1990). The problem of extrapolation of laboratory data to earth has been discussed by a number of workers (e.g., Carter and Tsenn 1987; Paterson 1987; Ross and Lewis 1989). Some approaches towards a synthetic flow law in polyphase aggregates have been made through the use of model calculations (Ji and Zhao 1993) and finite element modelling (Tullis *et al* 1991). However, the applicability of these flow laws as a general flow law for upper crustal deformation is restricted by the constraint of a dislocation creep regime.

Microstructures in naturally deformed rocks (e.g. Ross and Lewis 1989; Saha 1995) demonstrate that creep strain in nature, particularly in the upper crust

is accommodated by a combination of dislocation creep (crystal plastic mechanisms, CPM), diffusive mass transfer (DMT) or dissolution processes, and 'brittle' microcracking. If the ambient conditions are favourable cracking on its own is a fast process. However, coupled microcracking and dissolution-precipitation as in crack-seal mechanism (Ramsay 1980; Cox and Etheridge 1983) require that crack void filling by dissolution-precipitation controls growth of individual crack, its closure and subsequent evolution of a new crack (Cox and Etheridge 1989). For such a combination dissolution is the real rate controlling process. In this paper an approach towards deriving an aggregate flow law in polyphase aggregates is presented, where the strain in the constituent phases is assumed to occur by simultaneous operation of DMT and CPM.

The main assumptions in the present analytical derivation of an aggregate flow law from end member flow laws are as follows. (a) A steady state flow law is applicable to the deformation behaviour of the polyphase aggregate. (b) The form of the flow laws corresponding to DMT or CPM remain unchanged during the course of steady state flow of the aggregate. (c) Plane strain deformation operates throughout the flow. The constitutive equation for DMT includes a microstructural factor (say grain size in Rutter 1983). In the case of CPM too, microstructure and texture may influence aggregate flow behaviour. The present

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analysis does not take into account such effects of microstructural or textural variability assuming that a stable microstructure is attained during steady state flow.

2. Flow laws in monomineralic aggregates

2.1 Dislocation creep regime

The general form of the flow law corresponding to monomineralic aggregates in dislocation creep regime is given by

$$\dot{\epsilon} = A\sigma^n \exp(-Q/RT), \quad (1)$$

where $\dot{\epsilon}$ = strain rate, σ is flow stress, n is an exponent, A is pre-exponential factor, T is temperature, R = Boltzman's constant, Q = molar activation enthalpy. Equation (1) can be further condensed to the simple power law form

$$\dot{\epsilon} = B_\beta \sigma^n, \quad (1a)$$

by combining pre-exponential factor with $\exp(-Q/RT)$ term in a new 'constant' B_β (cf Tullis *et al* 1991).

2.2 Diffusion creep regime

Natural deformation under low ambient temperature as in the upper crust is often accommodated by a combination of several deformation mechanisms as demonstrated by a number of workers (e.g., Mitra 1976; Etheridge *et al* 1984; O'Hara 1990; Saha 1995). Diffusive mass transfer (DMT) is an important deformation mechanism (Eliot 1973; Rutter 1983; Groshong 1988; Knipe 1989). The flow law corresponding to DMT has been derived theoretically by a number of workers (e.g., Herring 1950; Coble 1963; quoted in Eliot 1973), and is of the form

$$\dot{\epsilon} = B_\alpha \sigma, \quad (2)$$

where the factor B_α is dependent, among other things, on diffusion coefficient (which has different values for grain boundary and lattice diffusion) and grain size. Although the detailed form and value of B_α is important in actual aggregate flow law calculations, in the following discussion B_α is treated as a 'constant', different from B_β in equation (1a) for a single constituent phase.

2.3 Dissolution creep regime

Etheridge *et al* (1984) consider a mode of strain accommodation by solution transfer in the cleavage seam (parallel to XY plane) and reprecipitation in extension veins perpendicular to X . For this model deformation strain rate is related to the differential in fluid pressure by

$$\dot{\epsilon}_{zz} = (\kappa/\nu)(\delta P - \rho g)(\delta C/W\rho_s), \quad (3)$$

where $\dot{\epsilon}_{zz}$ = strain rate in Z direction; κ = permeability; ν = viscosity; δP = differential in fluid pressure between a newly opened microfracture site and its surrounding; ρ = density of the migrating fluid; ρ_s = density of the solute; δC = chemical potential gradient, i.e., the mean difference in the concentration of solute component(s) between cleavage zones and extension sites; W = vein half spacing (Etheridge *et al* 1984).

The above relationship between fluid pressure and strain rate has some similarity with the flow law corresponding to DMT, differential in fluid pressure being comparable to differential stress or flow stress, albeit the obvious difference in numeric value. But Etheridge *et al* (1984) consider δC to be dependent on δP . Therefore, in the subsequent analysis the simpler form of flow law corresponding to DMT as in equation (2) will be utilized.

3. Strain or strain rate in polyphase aggregate

To arrive at a flow law applicable to polyphase rocks deformed by a combination of diffusive mass transfer (DMT) and crystal plastic mechanisms (CPM), we need to consider first the nature of strain rate in a polyphase aggregate.

3.1 One dimensional case

Let the aggregate length AB be a sum of length $AC(X_i)$ corresponding to phase- i and length $CB(X_j)$ corresponding to phase- j (A, B, C collinear, figure 1). If the final lengths after deformation are X_{fi} and X_{fj} respectively, then strain over segments AC and CB are given by

$$\epsilon_i = (X_{fi} - X_i)/X_i \quad (4a)$$

and

$$\epsilon_j = (X_{fj} - X_j)/X_j, \quad (4b)$$

respectively.

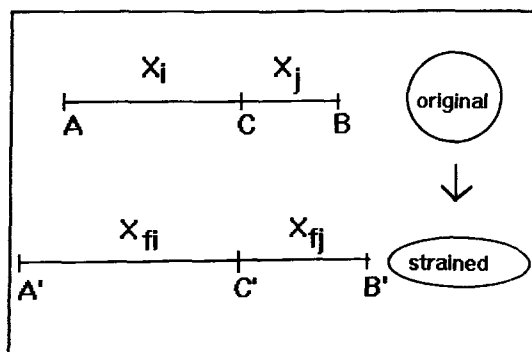


Figure 1. Defining change in length of two collinear segments of phases i and j .

The strain over domain AB is given by

$$\varepsilon_t = [(X_{f_i} + X_{f_j}) - (X_i + X_j)] / (X_i + X_j). \quad (4c)$$

Let us define coefficients f_i and f_j as follows:

$$f_i = X_i / (X_i + X_j) \text{ and } f_j = X_j / (X_i + X_j).$$

Then the equations (4a)–(4c) can be rearranged as

$$\varepsilon_t = f_i \varepsilon_i + f_j \varepsilon_j. \quad (5a)$$

If there were k adjoining segments corresponding to k different phases, then by induction

$$\varepsilon_t = \sum_{i=1}^k f_i \varepsilon_i. \quad (5b)$$

Differentiating both sides of equation (5b) with respect to time one can get an expression for aggregate strain rate:

$$\dot{\varepsilon}_t = \sum_{i=1}^k f_i \dot{\varepsilon}_i, \quad (5c)$$

where $\dot{\varepsilon}_t$ and $\dot{\varepsilon}_i$ represent aggregate strain rate and strain rate corresponding to i th phase respectively.

Being in agreement with works by Gifkins (1970) and Tullis *et al* (1991), it is proposed that the relationship given by equation (5c) can be extended to 3-dimensional case (axisymmetric and/or plane strain cases). Thus the aggregate strain rate may be obtained by taking a volume average of strain rates in constituent phases (see also Ji and Zhao 1993).

4. Combining diffusion creep with dislocation creep

In the deformation of a polyphase aggregate the stress supported by different constituent phases usually lies between two bounds, corresponding to *isostress* (all phases supporting equal stress) and *isostrain rate* (all phases having the same strain rate) situations (Tullis *et al* 1991; Ji and Zhao 1993; Handy 1996). While deriving an aggregate flow law in a polyphase rock affected by simultaneous diffusion creep and dislocation creep *isostress* and *isostrain rate* criteria are considered separately.

4.1 Isostress criterion

For a monomineralic aggregate deformation by an association of DMT and CPM, the aggregate strain rate is given by

$$\ln \dot{\varepsilon} = f_\alpha \ln \dot{\varepsilon}_\alpha + f_\beta \ln \dot{\varepsilon}_\beta, \quad (6a)$$

where $\dot{\varepsilon}$ is aggregate strain rate; $\dot{\varepsilon}_\alpha$ and $\dot{\varepsilon}_\beta$ are strain rates corresponding to DMT and CPM respectively; f_α and f_β are volume fractions affected by two different mechanisms respectively. By converting the strain rate to stress in the R.H.S of equation (6a) through

the use of equations (1a) and (2), one obtains

$$\ln \dot{\varepsilon} = \ln(B_\alpha^{f_\alpha} B_\beta^{f_\beta}) + (f_\alpha + n f_\beta) \ln \sigma. \quad (6b)$$

Let us now consider two different constituent phases i, j in the aggregate, affected by simultaneous DMT and CPM. By induction,

$$\ln \dot{\varepsilon} = f_{\alpha i} \ln \dot{\varepsilon}_{\alpha i} + f_{\alpha j} \ln \dot{\varepsilon}_{\alpha j} + f_{\beta i} \ln \dot{\varepsilon}_{\beta i} + f_{\beta j} \ln \dot{\varepsilon}_{\beta j}, \quad (7a)$$

where the first subscript refers to deformation mechanism and the second to the constituent phase. Therefore rewriting the R.H.S of equation (7a) in stress terms,

$$\ln \dot{\varepsilon} = (f_{\alpha i} \ln B_{\alpha i} + f_{\alpha j} \ln B_{\alpha j} + f_{\beta i} \ln B_{\beta i} + f_{\beta j} \ln B_{\beta j}) + (f_{\alpha i} + f_{\alpha j} + f_{\beta i} n_{\beta i} + f_{\beta j} n_{\beta j}) \ln \sigma. \quad (7b)$$

Where there are m different phases,

$$\ln \dot{\varepsilon} = \ln \prod_{i=1}^m (B_{\alpha i}^{f_{\alpha i}} B_{\beta i}^{f_{\beta i}}) + \ln \sigma \sum_{i=1}^m (f_{\alpha i} + f_{\beta i} n_{\beta i}). \quad (8)$$

4.2 Isostrain rate criterion

Since different phases deform by the same strain rate, $\dot{\varepsilon}$, the stresses supported by them will be different. Let σ_i and σ_j be the stresses corresponding to phases i and j respectively. The aggregate stress is a volume average of stresses supported by different phases. Therefore,

$$\ln \sigma = f_i \ln \sigma_i + f_j \ln \sigma_j, \quad (9)$$

where subscripts i, j refer to two different phases respectively. Taking into account two deformation mechanisms as above, and substituting the stress terms in the R.H.S of equation (9) by strain terms from equations (1a) and (2) one obtains

$$\ln \sigma = \left(f_{\alpha i} + \frac{f_{\beta i}}{n_{\beta i}} + f_{\alpha j} + \frac{f_{\beta j}}{n_{\beta j}} \right) \ln \dot{\varepsilon} - \left(f_{\alpha i} \ln B_{\alpha i} + f_{\alpha j} \ln B_{\alpha j} + \frac{f_{\beta i}}{n_{\beta i}} \ln B_{\beta i} + \frac{f_{\beta j}}{n_{\beta j}} \ln B_{\beta j} \right). \quad (9a)$$

Changing sides, one obtains

$$\ln \dot{\varepsilon} = X + Y \ln \sigma, \quad (9b)$$

where

$$X = Y \left(f_{\alpha i} \ln B_{\alpha i} + f_{\alpha j} \ln B_{\alpha j} + \frac{f_{\beta i}}{n_{\beta i}} \ln B_{\beta i} + \frac{f_{\beta j}}{n_{\beta j}} \ln B_{\beta j} \right) \quad (9c)$$

and

$$Y = \left(f_{\alpha i} + \frac{f_{\beta i}}{n_{\beta i}} + f_{\alpha j} + \frac{f_{\beta j}}{n_{\beta j}} \right)^{-1}. \quad (9d)$$

For m different phases, by induction

$$\ln \dot{\varepsilon} = \frac{\ln(\prod_{i=1}^m B_{\alpha i}^{f_{\alpha i}} B_{\beta i}^{f_{\beta i}/n_{\beta i}}) + \ln \sigma}{\sum_{i=1}^m (f_{\alpha i} + f_{\beta i}/n_{\beta i})}. \quad (10)$$

6. Concluding remarks

A log-log linear relationship between aggregate strain rate and flow stress is the essence of a theoretical flow law applicable to the deformation of a polyphase aggregate undergoing simultaneous diffusive mass transfer and crystal plastic mechanisms. The volume fraction of any constituent phase affected by either DMT or CPM is an important parameter in the make-up of coefficients of the log-log linear relationship. Comparison of coefficients in equations (8) and (10) shows that the upper bound of flow stress corresponds to an assumption of uniform strain rate in the aggregate, and a lower stress bound is provided by the assumption of uniform stress.

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