Computation of regional gravity anomaly – A novel approach

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A new scheme is put forward based on the shape function concept of finite element approximation to compute regional gravity anomaly. The uniqueness of this approach is that excepting eight (or twelve) discrete gravity measurements coinciding with the eight (or twelve) nodes of a quadratic (or cubic) isoparametric element superimposing the map space, often very large, no other observed gravity data are invoked to compute the regional. Tests on two Bouguer gravity fields, an oil prospect and a rift valley confirmed better resolution by this approach. This technique is straight forward, uses simple mathematics to be easily automated and yields repeatable results.

1. Introduction

The essential step in the processing of Bouguer gravity data is the regional-residual separation. Its importance can be judged from the fact that gravity anomaly resolution continues to be the topic of investigations ever since the graphical method and Griffin's (1949) centre-point-and-ring method were suggested about fifty years back from today. Methods based on trend surface and spectral analysis are the two main techniques. With adequate geological control direct computation of regional has gained importance in recent times (Pilkington *et al* 1995). In a new approach Subba Rao (1996) has suggested the computation of local anomaly above the structures that produce zero free-air anomaly and subsequently to obtain residual by deducting it from the free-air anomaly.

This paper presents a new and simple procedure to compute the regional based on the shape function concept used in finite element analysis. This technique requires just a few discrete gravity values at stations coinciding with the nodes of an element, eight in the case of quadratic and twelve in the case of cubic, superimposing the survey space. Mallick (1991) first applied this technique to recompute the residual anomaly and ore estimation of a chromite deposit in Cuba (Davis *et al* 1957). Here, we chose two gravity surveys, well known Mykawa Gravity Minimum in Texas (Elkins 1951) to recompute the residual to resolve the salt domes, and Recõncavo Basin, a rift valley in Brazil (Leão *et al* 1996) to recompute the regional to illustrate the efficacy of the method.

2. The method

The present approach utilizes the concept and properties of the basis or shape functions (Zienkiewicz 1979; Cheung and Yeo 1979) used in finite element analysis. We state in the very beginning that we are only taking advantage of the very useful interpolating properties of the isoparametric elements and not dealing with the solution of any differential equation as one is accustomed to by finite element analysis (for example, Mallick and Sarma 1992; Karkhanis *et al* 1997). We explain the methodology with the help of figure 1(a) and (b). The Mykawa gravity survey area in figure 1(b), the *x-y* space, is superimposed by an element. The nodes in filled circles are numbered 1 through 8. The gravity values at these nodes lying on the peripheries are assumed to represent the regional. With simple substitutions $\xi = (x - x_c)/a$ and $\eta = (y - y_c)/b$, where x_c, y_c define the centre of the element, and 2a and 2b are the sides of the rectangle, the element in figure 1(b) is represented by a reference

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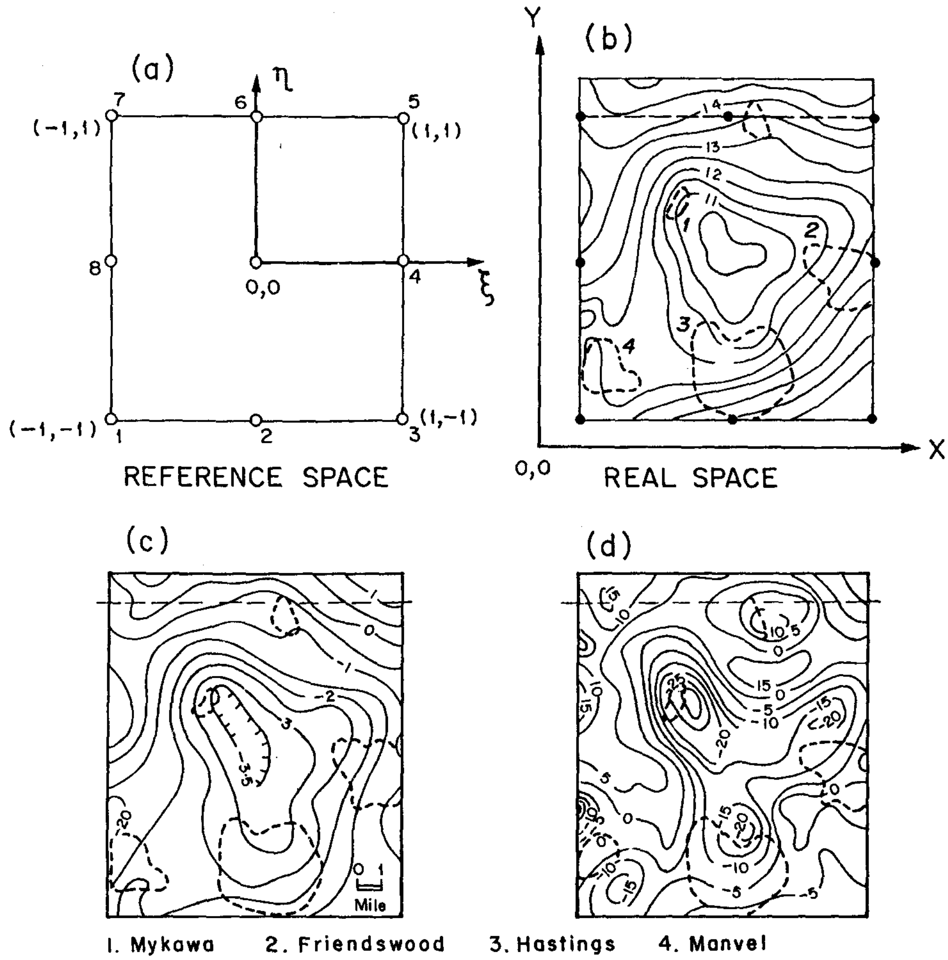

Figure 1. Mykawa Gravity Minimum. (a) Reference element, (b) the Bouguer gravity, (c) the residual and (d) the second-derivative maps (after Elkins 1951).

element in figure 1(a) defined by non-dimensional coordinates ξ and η varying between -1 and 1. The element can be a square, a rectangle or a quadrilateral. We have chosen a square. The nodes 1 through 8 in reference element correspond to the nodes of the element in x-y space.

The field variable, the regional gravity in this case, at any point (ξ, η) in the reference element can be expressed by a series, which in fact is a weighted sum of the nodal gravity values, g_i,

$$g(\xi,\eta) = \sum N_i(\xi,\eta) g_i \quad i = 1,2,\ldots,8, \tag{1}$$

where the weights, $N_i(\xi,\eta)$, are the shape functions of the element. Equation 1 refers to a quadratic element and the shape functions are given by:

$$\begin{aligned} N_i(\xi,\eta) &= (1+\xi\xi_i)(1+\eta\eta_i) \\ &\quad \times (\xi\xi_i + \eta\eta_i - 1) \quad i = 1,3,5,7, \\ N_i(\xi,\eta) &= (1-\xi^2)(1+\eta\eta_i) \quad i = 2,6, \\ N_i(\xi,\eta) &= (1+\xi\xi_i)(1-\eta^2) \quad i = 4,9, \end{aligned} \tag{2}$$

where ξ_i and η_i are the nodal coordinates. It may be noted that $N_i(\xi,\eta) = 1$ at ith node and zero elsewhere. Furthermore, $\Sigma N_i(\xi,\eta) = 1$. Cheung and Yeo (1979) have described other properties of the shape functions and their expressions for cubic and other elements.

The regional anomalies computed by equation 1 in the reference element need to be translated to the real map space. This is carried out by a geometric transformation similar to equation 1.

$$\begin{aligned} x(\xi,\eta) &= \Sigma M_i(\xi,\eta) x_i, \\ y(\xi,\eta) &= \Sigma M_i(\xi,\eta) y_i. \end{aligned} \tag{3}$$

In case of isoparametric elements used here, $M_i(\xi,\eta) = N_i(\xi,\eta)$. x_i and y_i are the nodal coordinates shown in figure 1(b). With this brief account on finite element approximation we proceed to illustrate its application to two field cases.

3. Mykawa Gravity Minimum, Gulf Coast, Texas

Elkins (1951) in his classical paper described the Mykawa Gravity Minimum, shown in figure 1(b), as the sum total of the effects of four salt domes, namely Mykawa, Friendswood, Hastings and Manvel. Elkins

(1951) has demonstrated that while the residual gravity anomaly in figure 1(c) did not resolve the salt domes, the second derivative map in figure 1(d), on the other hand, did separate the responses of different salt domes.

4. Computational procedure

We adopted the following steps to recompute the residual anomaly by the new scheme.

- An eight-noded square element in real x-y space, approximately 15 mi × 15 mi, was superimposed on Mykawa Gravity Minimum as shown in figure 1(b). The observed gravity values, i.e., 2.5, 4.0, 6.0, 2.9, 4.2, 3.5, 4.0, 3.3 mgal read at nodes 1 through 8 in figure 1(b) were assigned to the corresponding nodes 1 through 8 in the reference element. These eight discrete gravity values at the peripheries of the survey area represent the regional.
- The regional gravity anomalies at different points in reference element were computed by equation 1. As an example, the regional at the origin of the reference element is:

$$g_d(0,0) = N_1(0,0)g_1 + N_2(0,0)g_2 + \cdots + N_3(0,0)g_8 = 2.675\,\text{mgal},$$

where $[N_i(0,0)] = \{-0.25, 0.5, -0.25, 0.5, -0.25, 0.5, -0.25, 0.5\}$, g_1 through g_8 are given in step 1, and d refers to the effects of the deeper structures.
- In the next step, the computed regional value, 2.675 mgal, is transferred to x-y space by equation 3 and with the help of the same weights, defined in step 2. On carrying out the simple calculation, one finds $x = 9.0$ mi and $y = 8.23$ mi, which correspond to the centre of the element superimposing the Mykawa Gravity map.

To put the computational procedure into proper perspective, let us consider the regional at another point, ($\xi = 0.5$ and $\eta = 0.5$). It turns out to be $g_d(0.5, 0.5) = 2.78$ mgal. This point in reference element corresponds to $x = 12.6$ mi and $y = 12.3$ mi, i.e. the centre of the NE quadrant of the element superimposing the survey space.
- In the last step, the residual was computed by the relation (Pawlowski 1994).

$$g_s(x,y) = g(x,y) - g_d(x,y) \tag{4}$$

where s stands for effect of shallow structures.

The computations in step 3 bring out one important fact, among others, that no other gravity values from the survey space enter into the regional computation. It makes this approach unique and different from all other earlier techniques.

5. Regional and residual maps

The basic gravity data have been obtained by digitizing the Bouguer contours in figure 1(b). The regional computed by the present technique is shown in figure 2(a) and the residual in figure 2(b). The residuals in figure 2(b) and those of Elkins in figure 1(c) show close resemblance both in pattern and residual values. These maps, as Elkins correctly observed, give a combined response and do not resolve the salt domes. This is because there is still some residual left in the regional, most likely manifested by the contour closure in the regional map. There is no regional map of Elkins for comparison. However, a close look at the gravity map in figure 1(b) shows that some of the salt domes lie in the periphery and the choice of regional values, $g_i (i = 1, 2, \ldots, 8)$ assumed at element nodes need to be modified. This is carried out by plotting the Bouguer gravity profiles on the four sides of the element and removing the near surface effects. The gravity values at nodes 4 and 8 were corrected and the regional was recomputed.

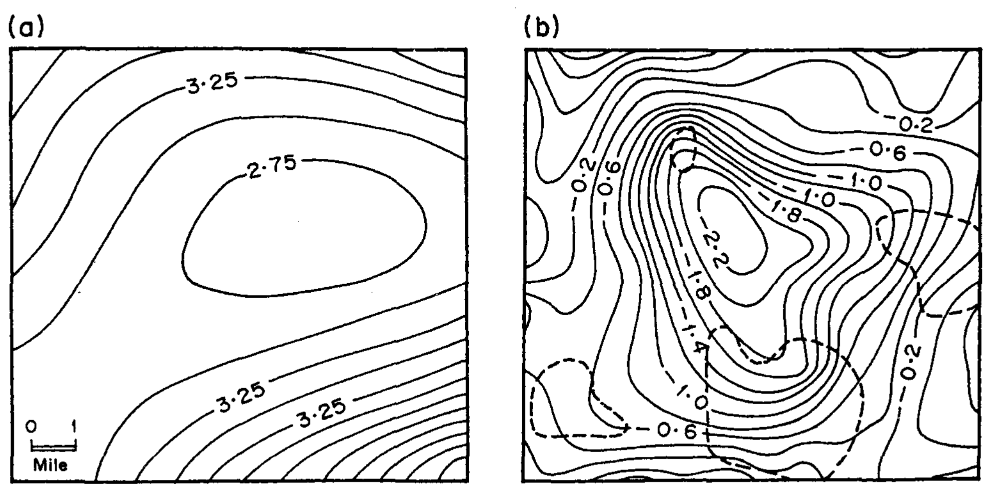

Figure 2. Regional-residual separation by the new technique (a) The regional and (b) the residual maps.

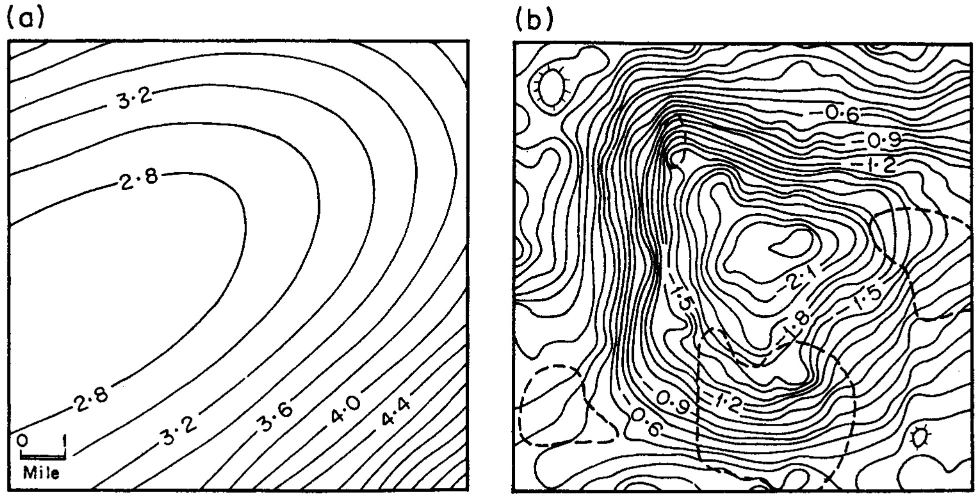

Figure 3. Improved regional-residual separation by 8-noded element. (**a**) The regional and (**b**) the residual maps. The residual map shows further resolution.

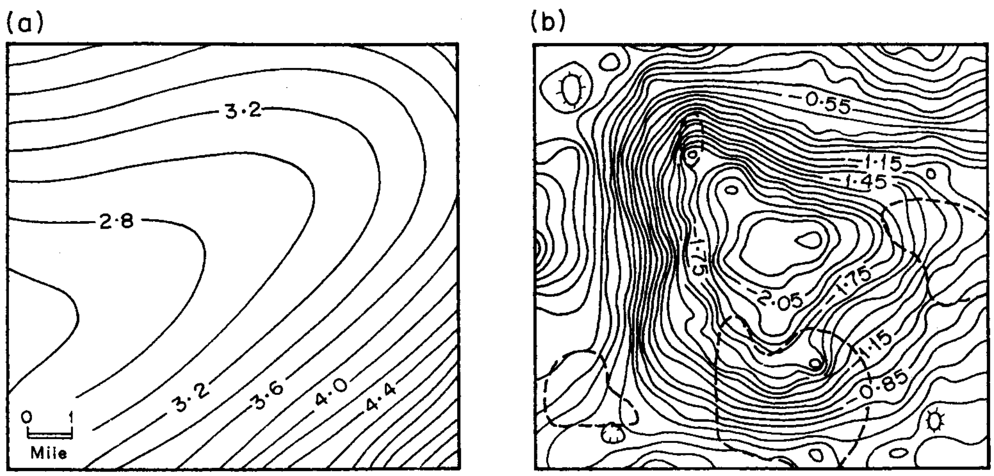

Figure 4. Regional-residual separation by 12-noded element. (**a**) The regional and (**b**) the residual maps. The residual map shows further resolution.

Figure 3(a) shows the smooth regional map, and the corresponding residual in figure 3(b) has resolved the Mykawa, northern part of the Hastings and to some extent Manvel salt domes. It is of interest to note that this new residual map has a good resemblance to the second derivative map of Elkins shown in figure 1(d).

6. Use of 12-noded cubic element

Next we have superimposed the Bouguer gravity map by a third-order 12-noded isoparametric cubic element. As in the case of the eight-noded element, we have followed the same procedure to obtain the regional shown in figure 4(a) and the residual in figure 4(b). The Manvel, Mykawa and Hastings salt domes are resolved. Furthermore, as in the second derivative map in figure 1(d), there is a residual closure just above the Friendswood salt dome. It can be observed that, besides the resolution of the effects of shallower and deeper structures, the new technique provides lateral resolution as well.

7. Recõncavo Basin, Brazil

Leão *et al* (1996) have obtained by polynomial fitting method the regional gravity anomaly of Recõncavo Basin, Brazil with the purpose of obtaining basement configuration. We have recomputed by the new technique the regional anomaly for parts of the same basin for ready comparison.

The Recõncavo Basin forms the southern part of the 100 km-wide north-south trending rift valley. The sedimentary rocks from Jurassic to Tertiary, at places 6000 m thick, are deposited in the valley. The basement is segmented by several north-east and north-west trending normal faults causing lateral density

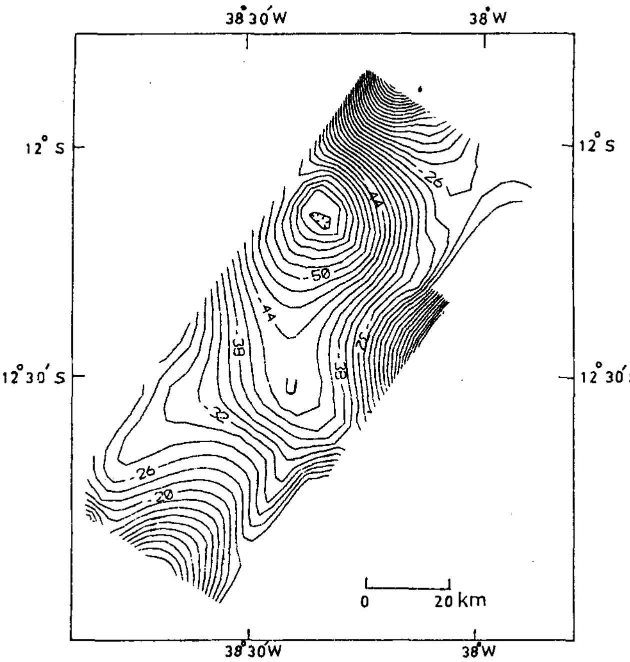

Figure 5. Regional map of Recôncavo Basin computed by superimposing 12-noded elements on the Bouguer gravity map.

variation. There is an eastward crustal thinning, too. We have obtained the basic data by digitizing the Bouguer anomaly map (Leão *et al* 1996, figure 6). Since the basin is elongated in NE-SE direction we have divided the region into three square blocks to separately compute the regional. A composite map was thereafter prepared for the entire region.

The regional map computed employing thirteenth-order polynomial by Leão *et al* (1996) has utilized information on density variation, eastward crustal thinning and available bore hole data. With no such constraints we have computed the regional by our procedure using twelve-noded third-order element. The regional map in figure 5 compares well with that of Leão *et al* (1996, figure 9), in particular the resolution of Camacari Low, marked by U in figure 5.

The digitization of the contours, particularly in regions of high gradient, limits our accuracy.

8. Conclusions

The regional and residual anomalies recomputed by our approach for Mykawa oil prospect and Recôncavo rift valley confirm the following:

- The technique is suitable to compute regional and residual of small and large areas, has vertical and lateral resolutions and does not require *a priori* information on geology of the area. The weighting factors are defined without ambiguity. The computation of the regional does not require gravity values lying inside the survey area, thereby eliminating the effects of shallow and local structures.
- Approximation of regional at eight or twelve discrete points lying on four sides of the survey area is significant in the sense that it is possible to compute the regional of an inaccessible area. It is therefore important from the defence point of view as well.
- This approach is more objective than other methods.

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